Principles of Interferometry & Synthesis Imaging

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ALMA imaging workshop @ ASIAA 2026/09/02

Radio Astronomy Fundamentals

Interferometry basics

Single dish vs. Interferometer: Why interferometry?

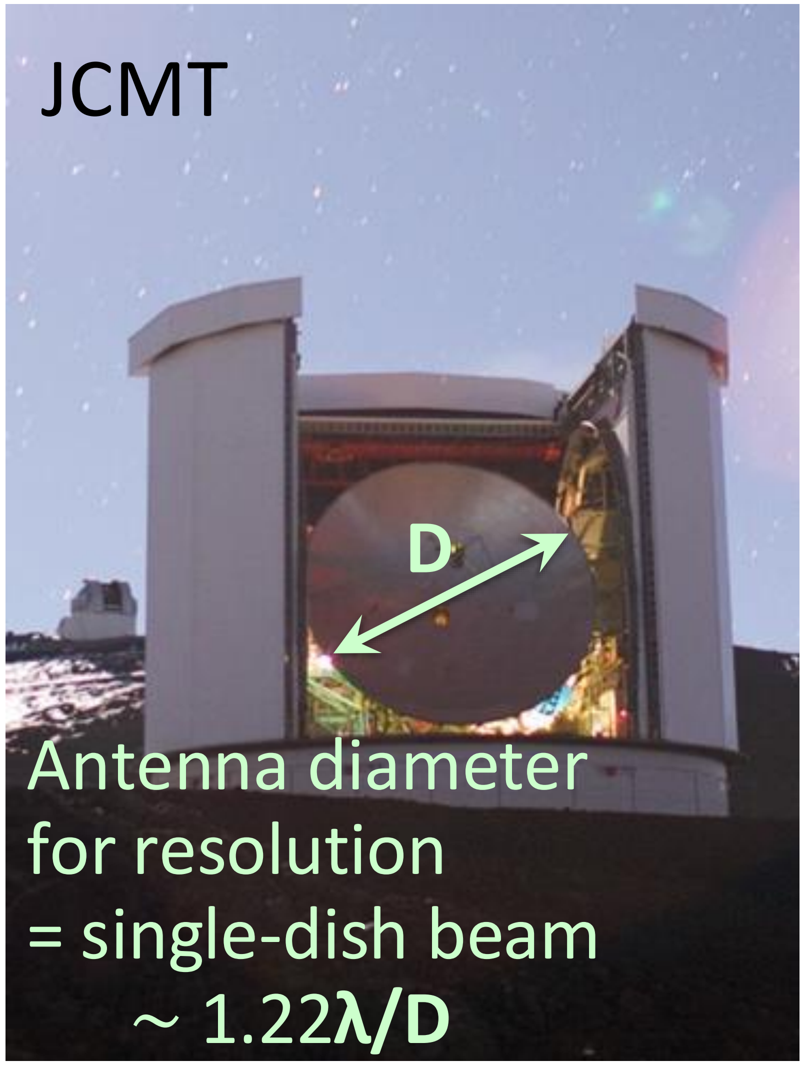


Image Credit: William Montgomerie (EAO)

A filled aperture

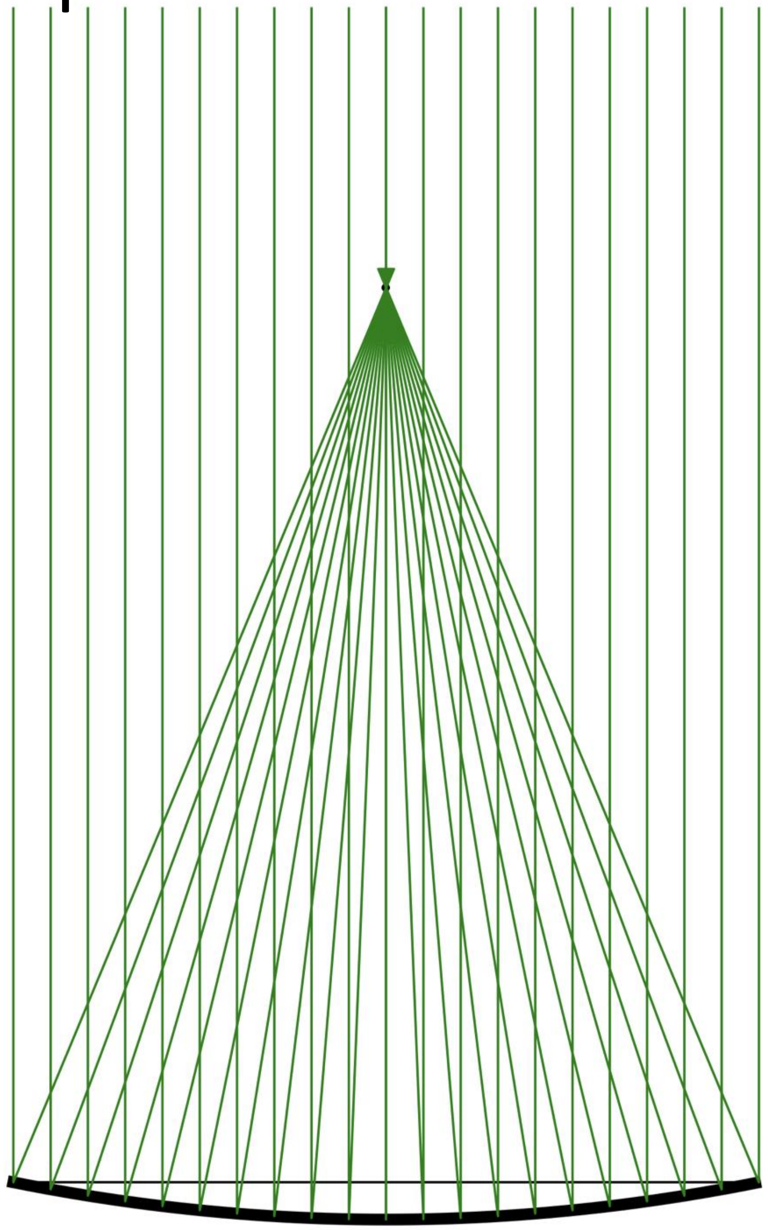
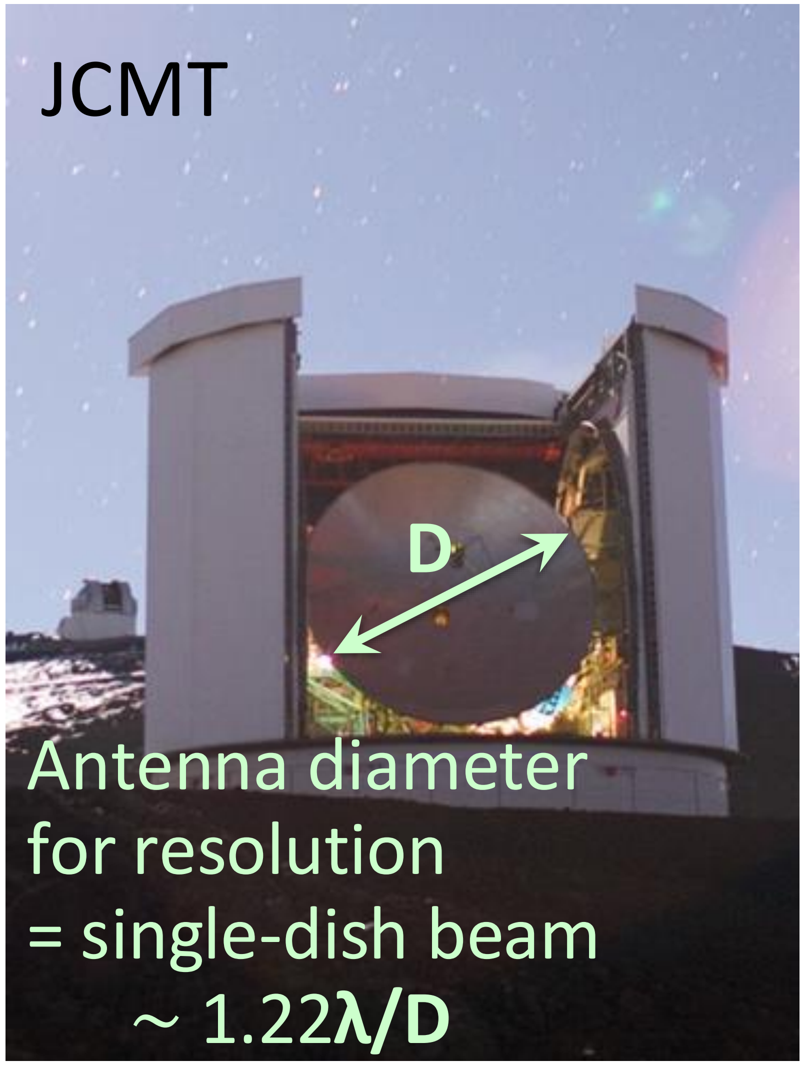


Image Credit:
George Moellenbrock
(NRAO)

Interferometry basics

Single dish vs. Interferometer: Why interferometry?



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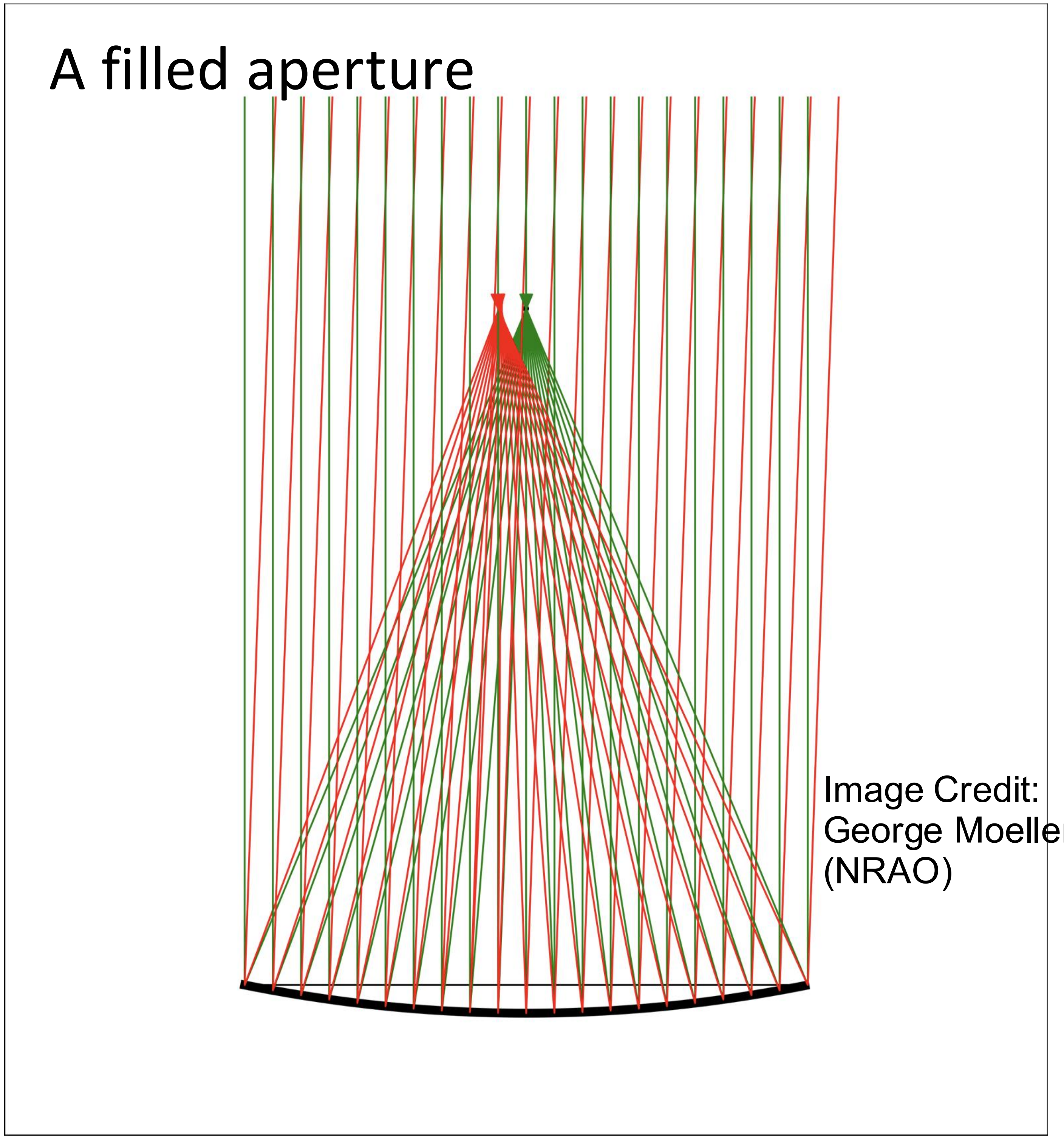

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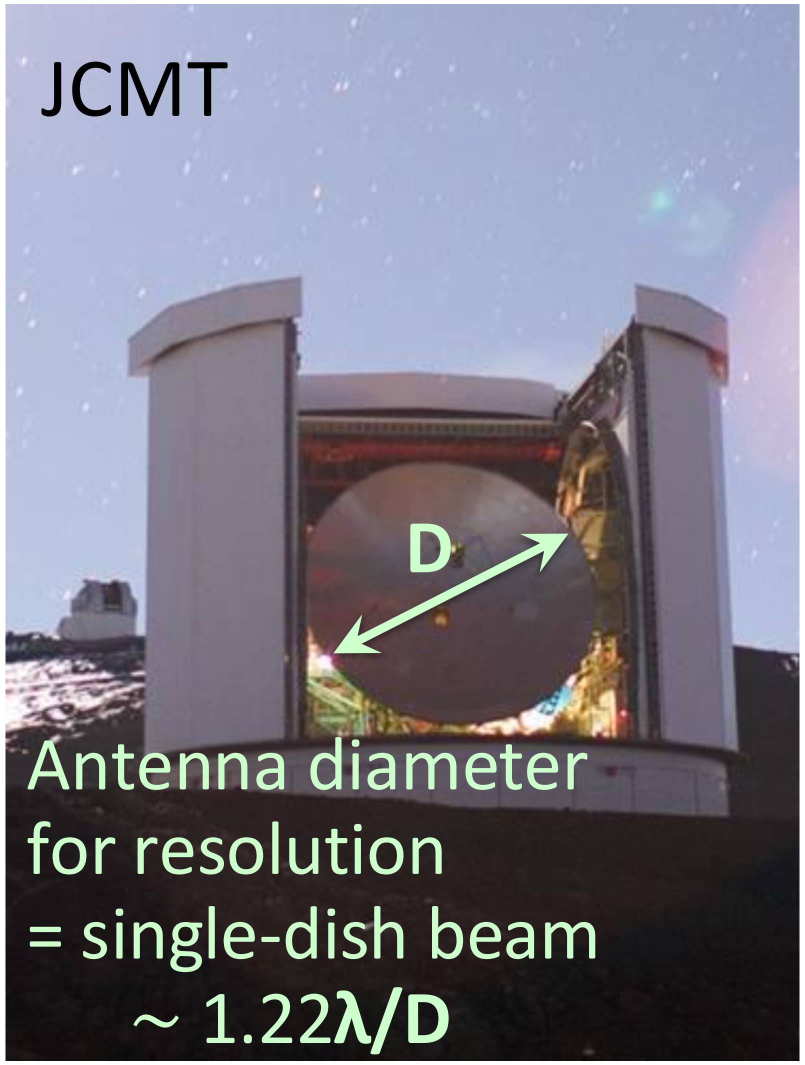
Interferometry basics

Single dish vs. Interferometer: Why interferometry?



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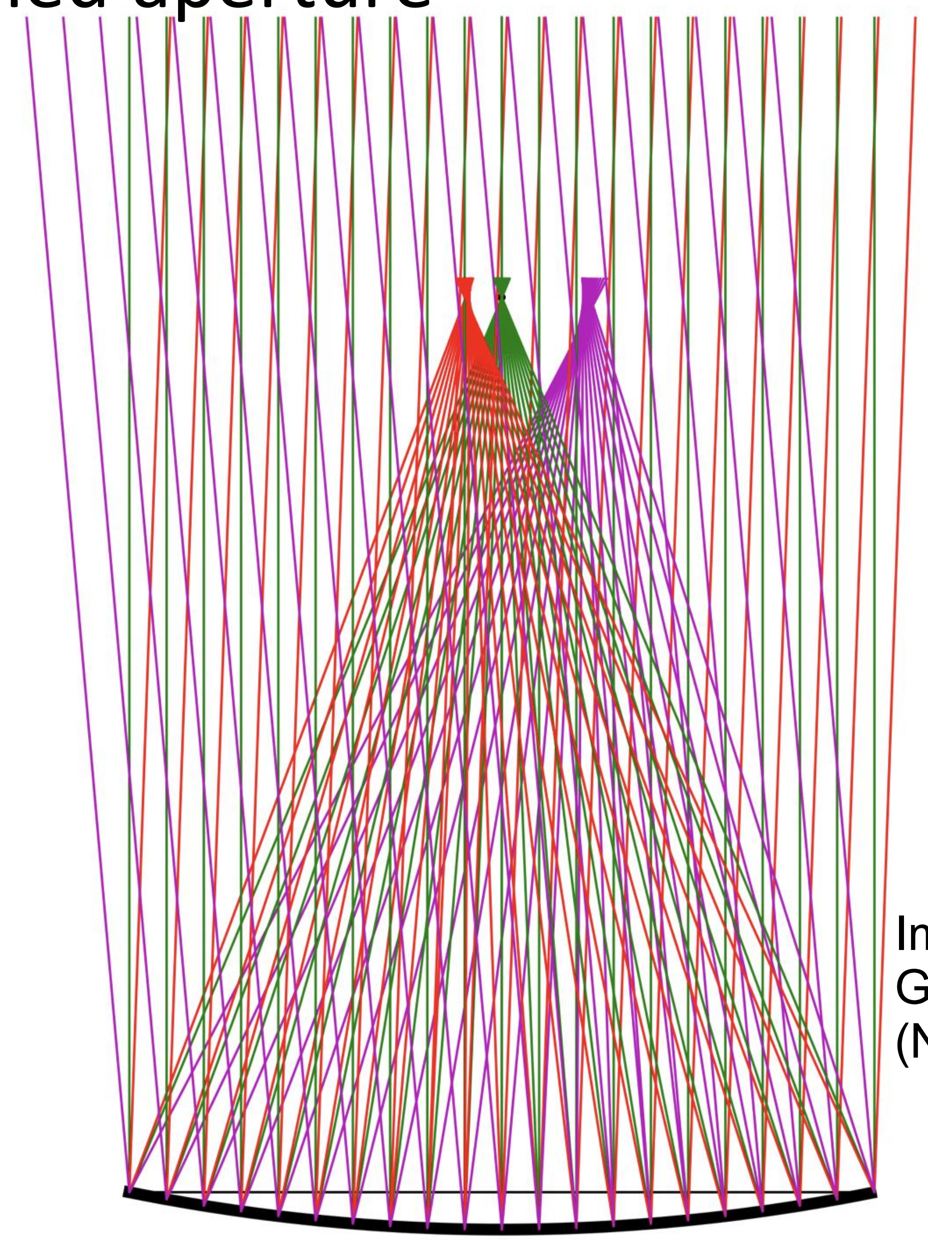

Image Credit:
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Interferometry basics

Single dish vs. Interferometer: Why interferometry?

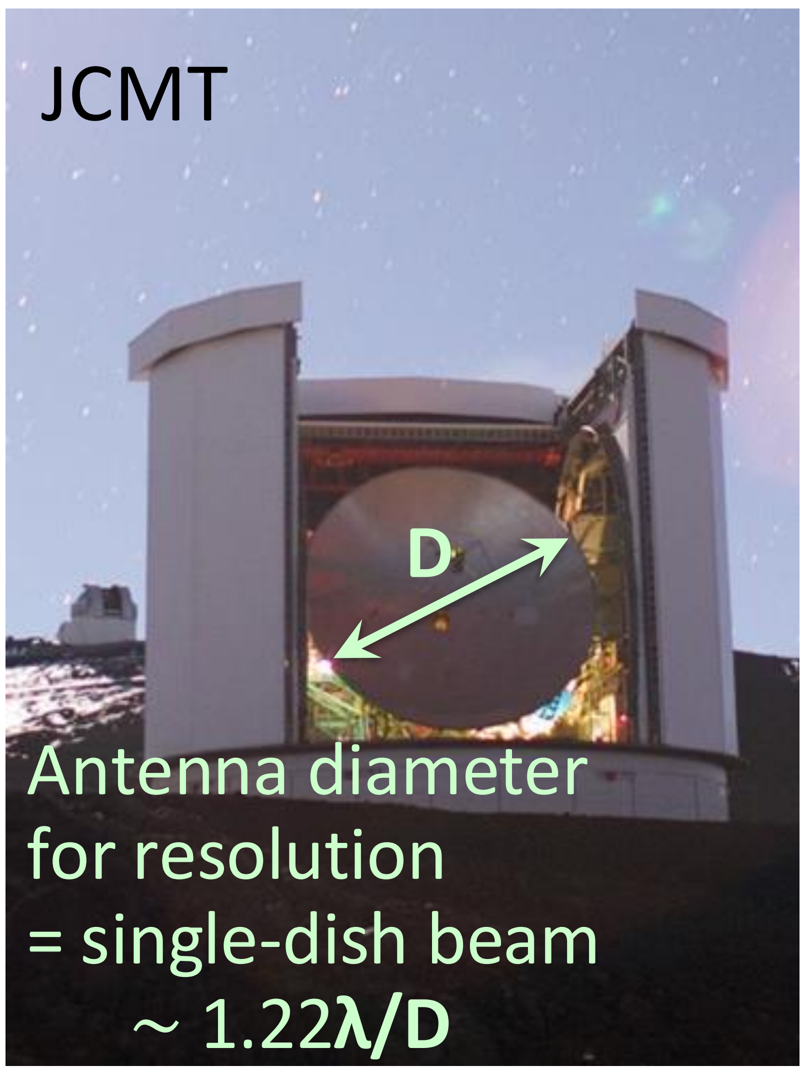

Image Credit: William Montgomerie (EAO)

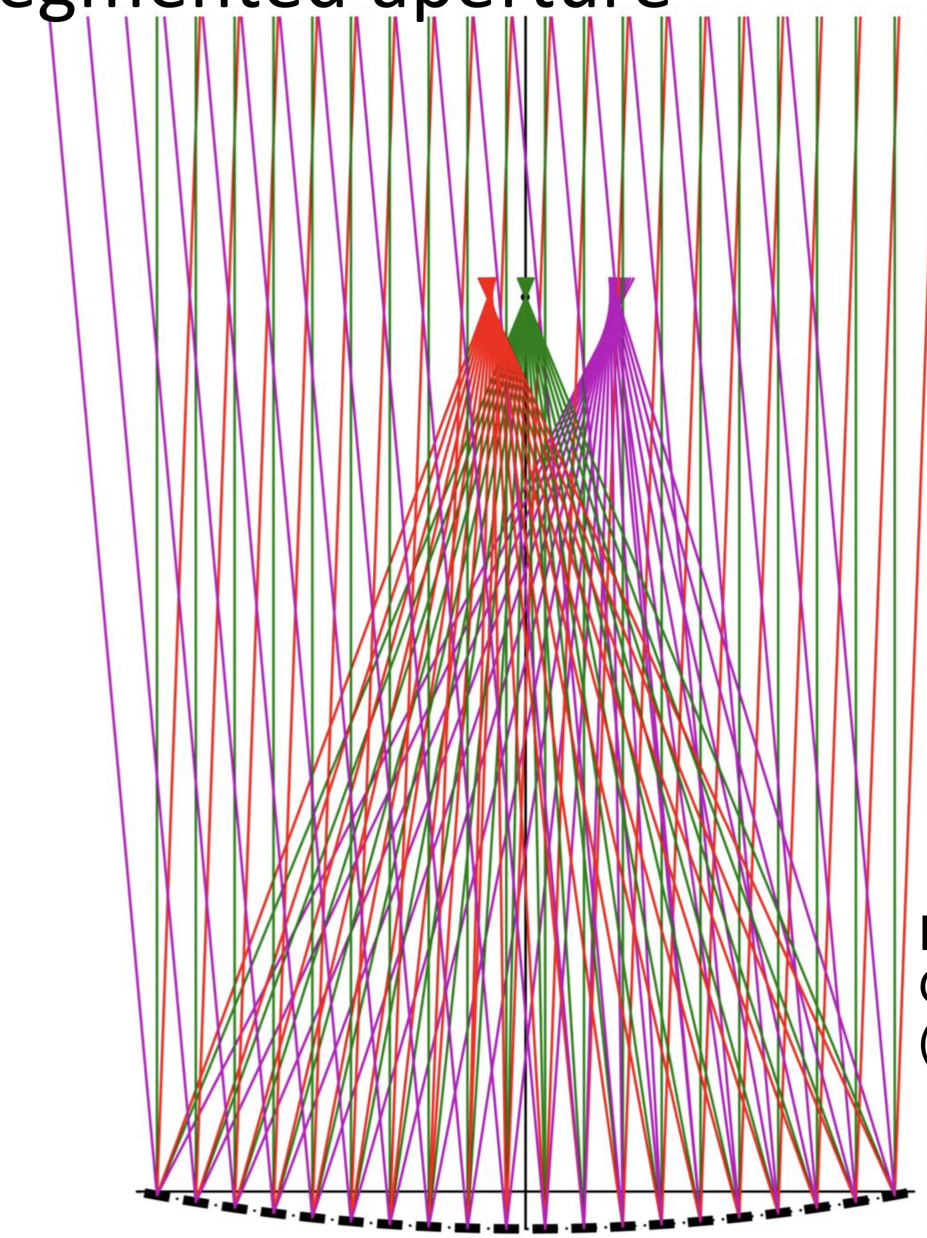

Image Credit:
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Interferometry basics

Single dish vs. Interferometer: Why interferometry?

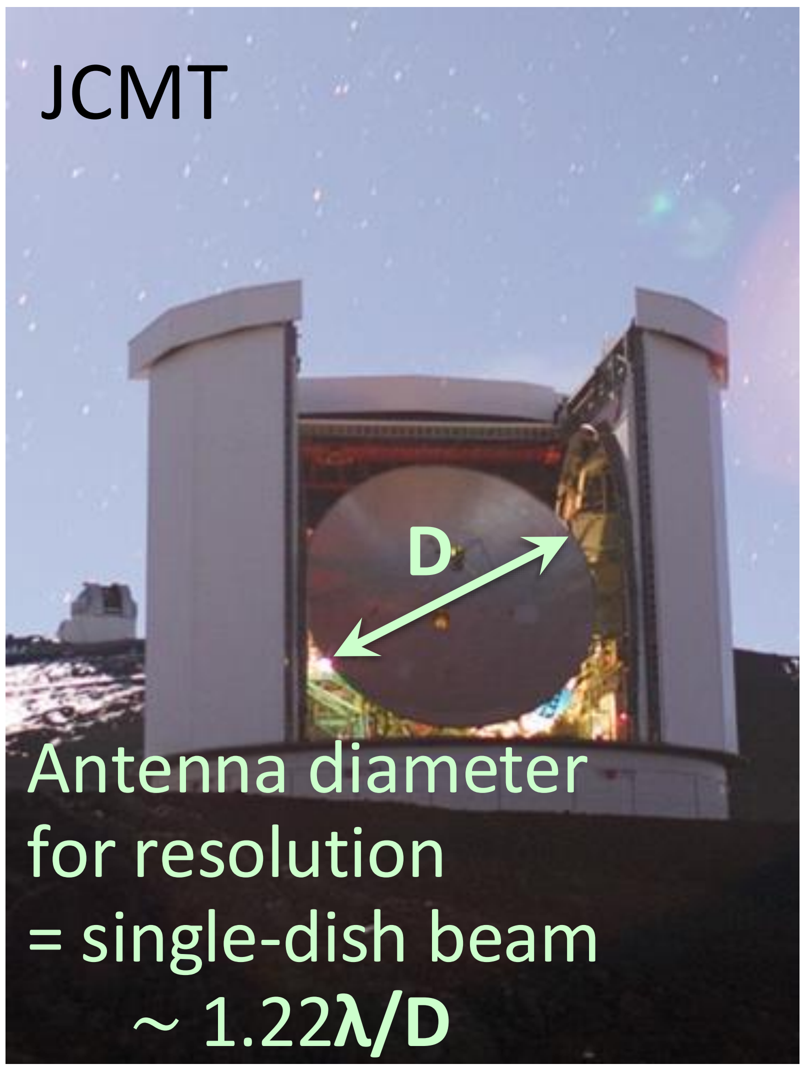
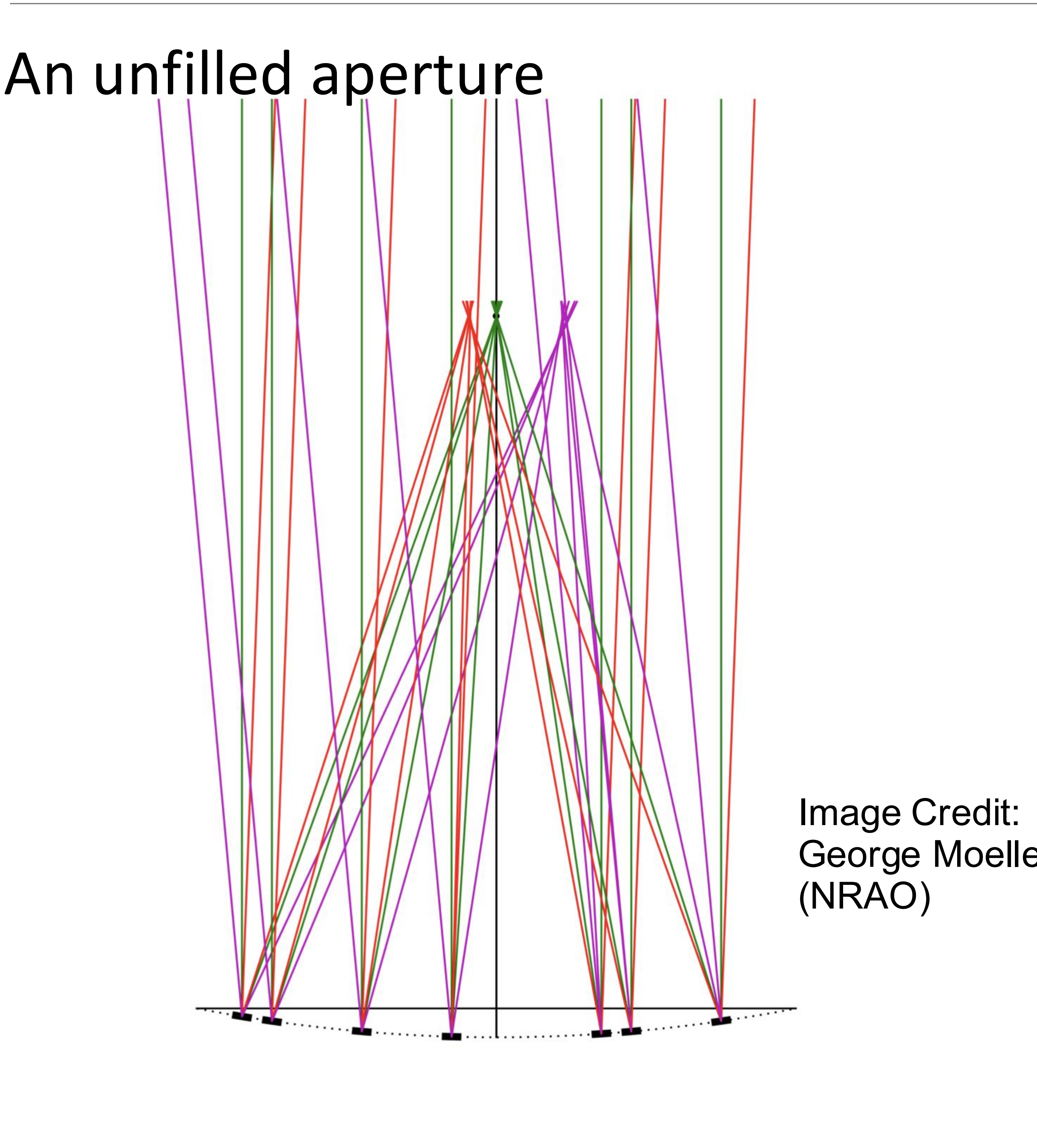


Image Credit: William Montgomerie (EAO)



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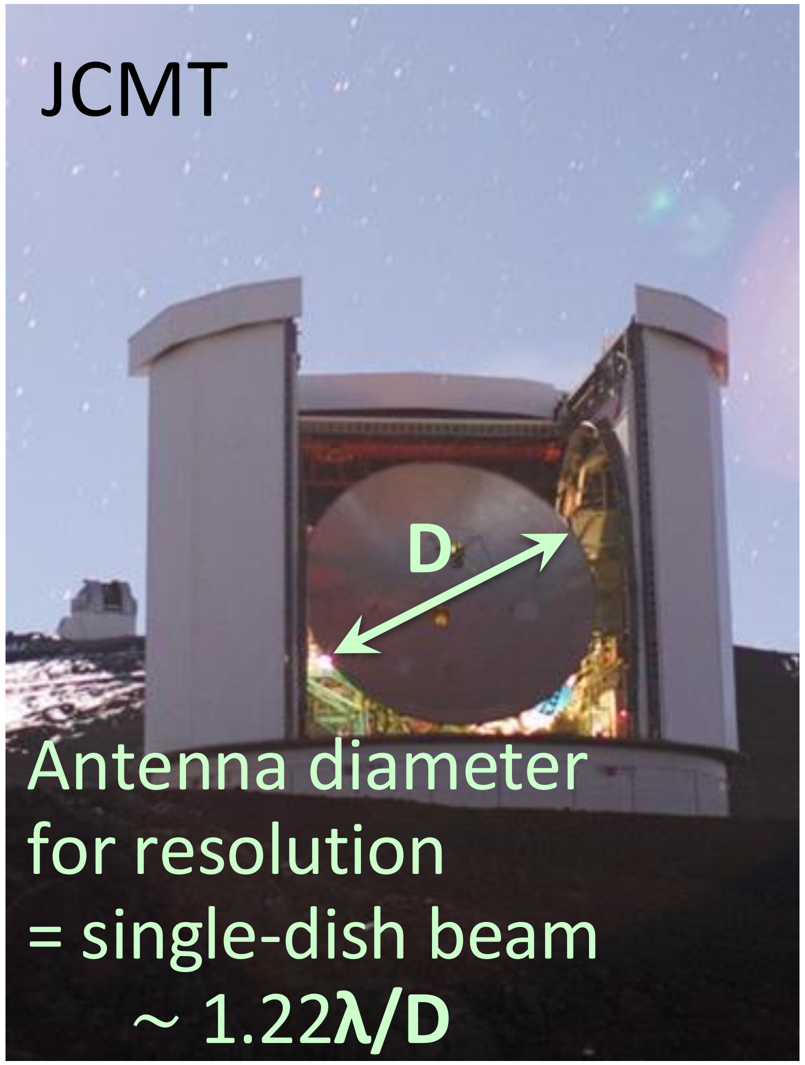
Interferometry basics

Single dish vs. Interferometer: Why interferometry?



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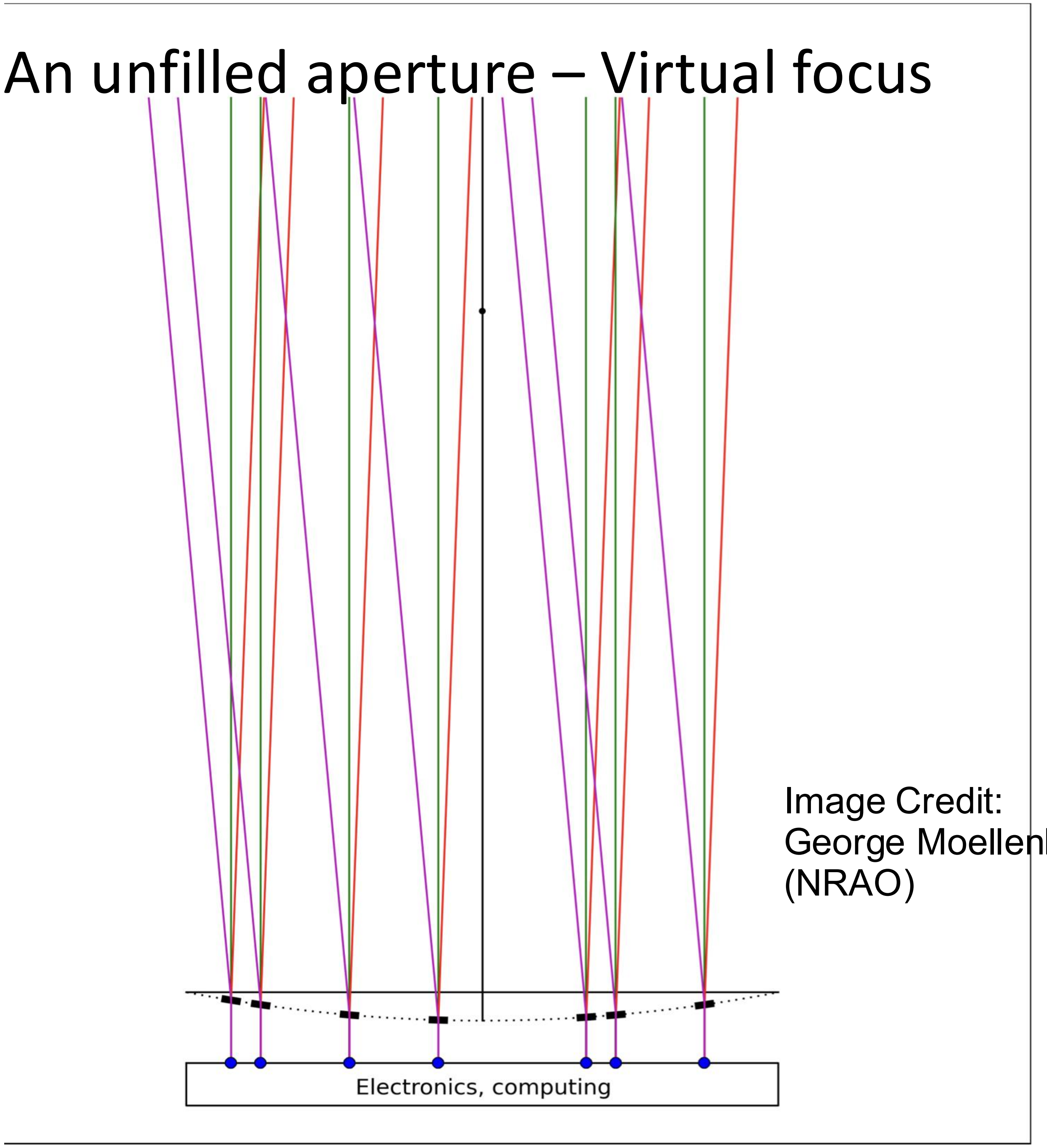


Image Credit:
George Moellenbrock
(NRAO)

Interferometry basics

Single dish vs. Interferometer: What's the Beam about?

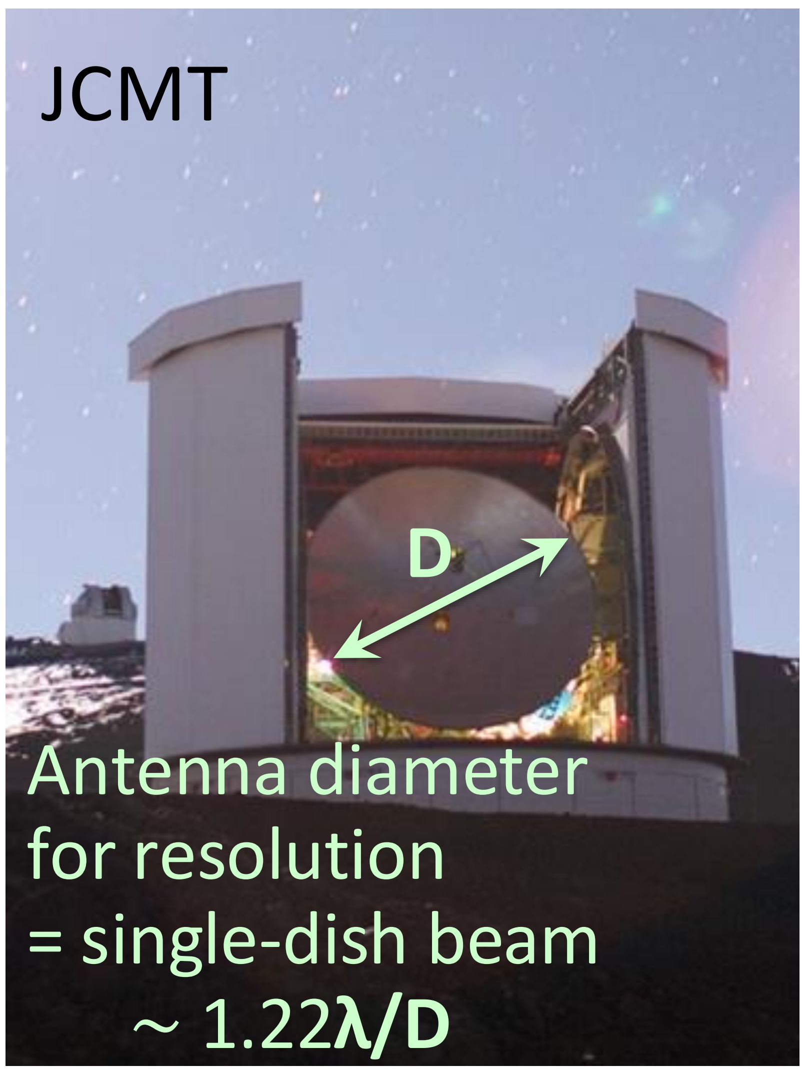
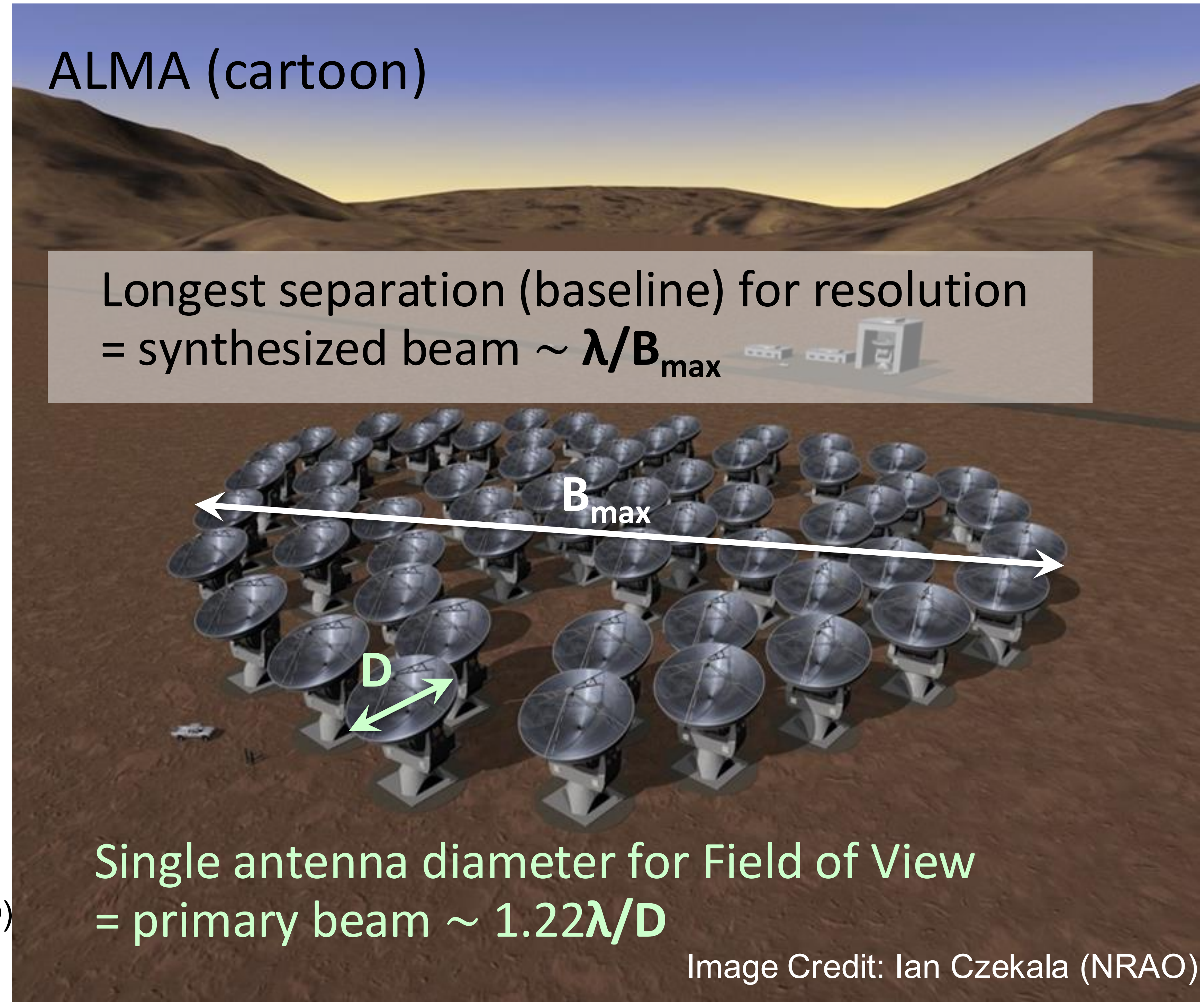


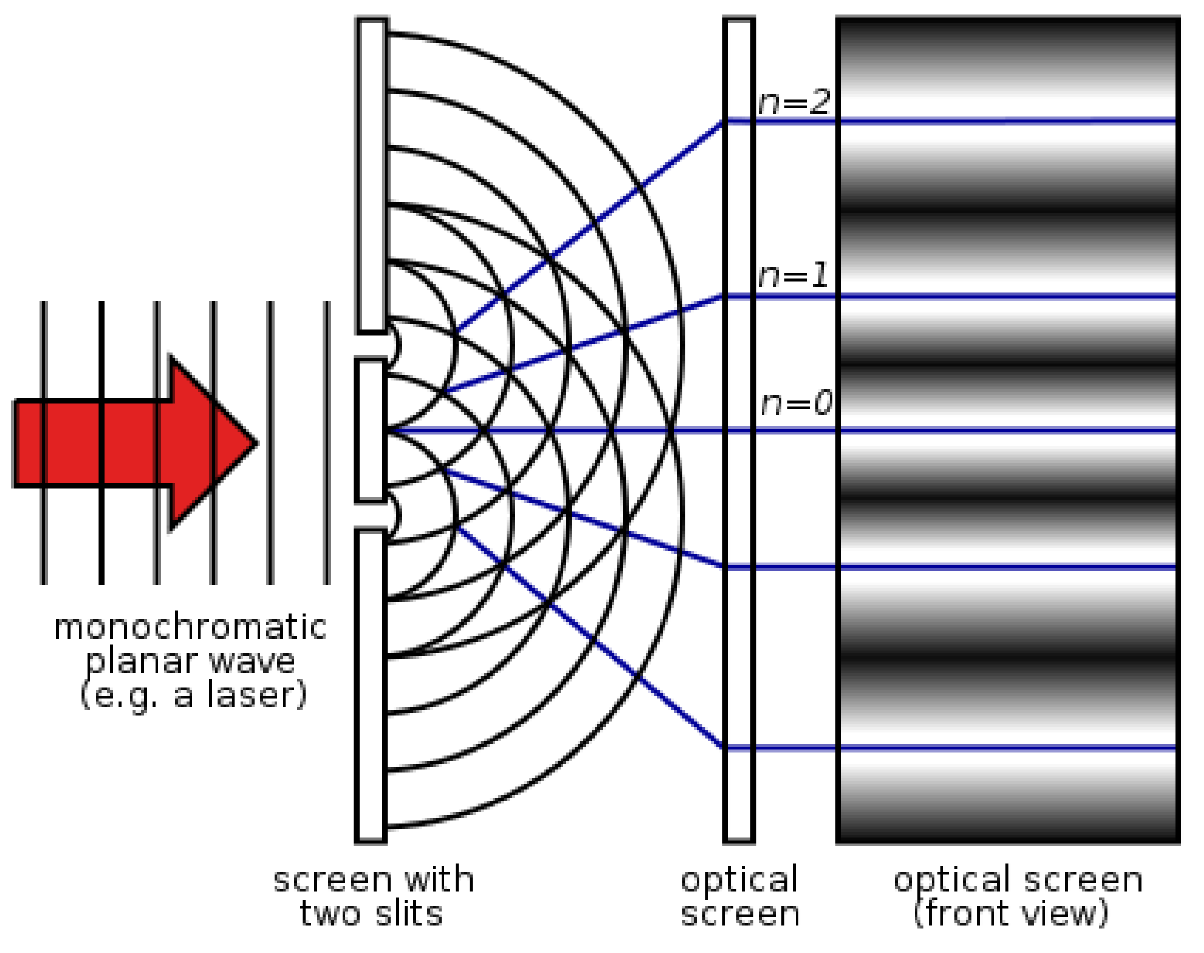
Image Credit: William Montgomerie (EAO)



Interferometry basics

What is an interferometer

An interferometer measures the interference pattern produced by multiple apertures, much like a 2-slit experiment.

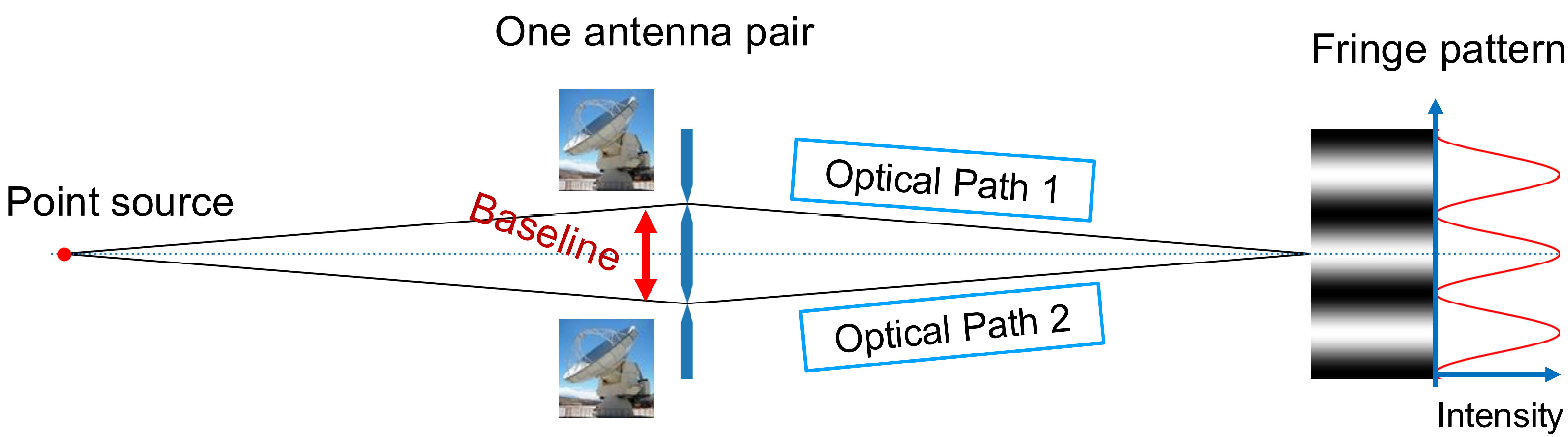


Credit: Ian Czekala (NRAO)

Interferometry basics

Young's double-slit experiment

An interferometer measures the **interference pattern (fringe)** observed by pairs of antennas.



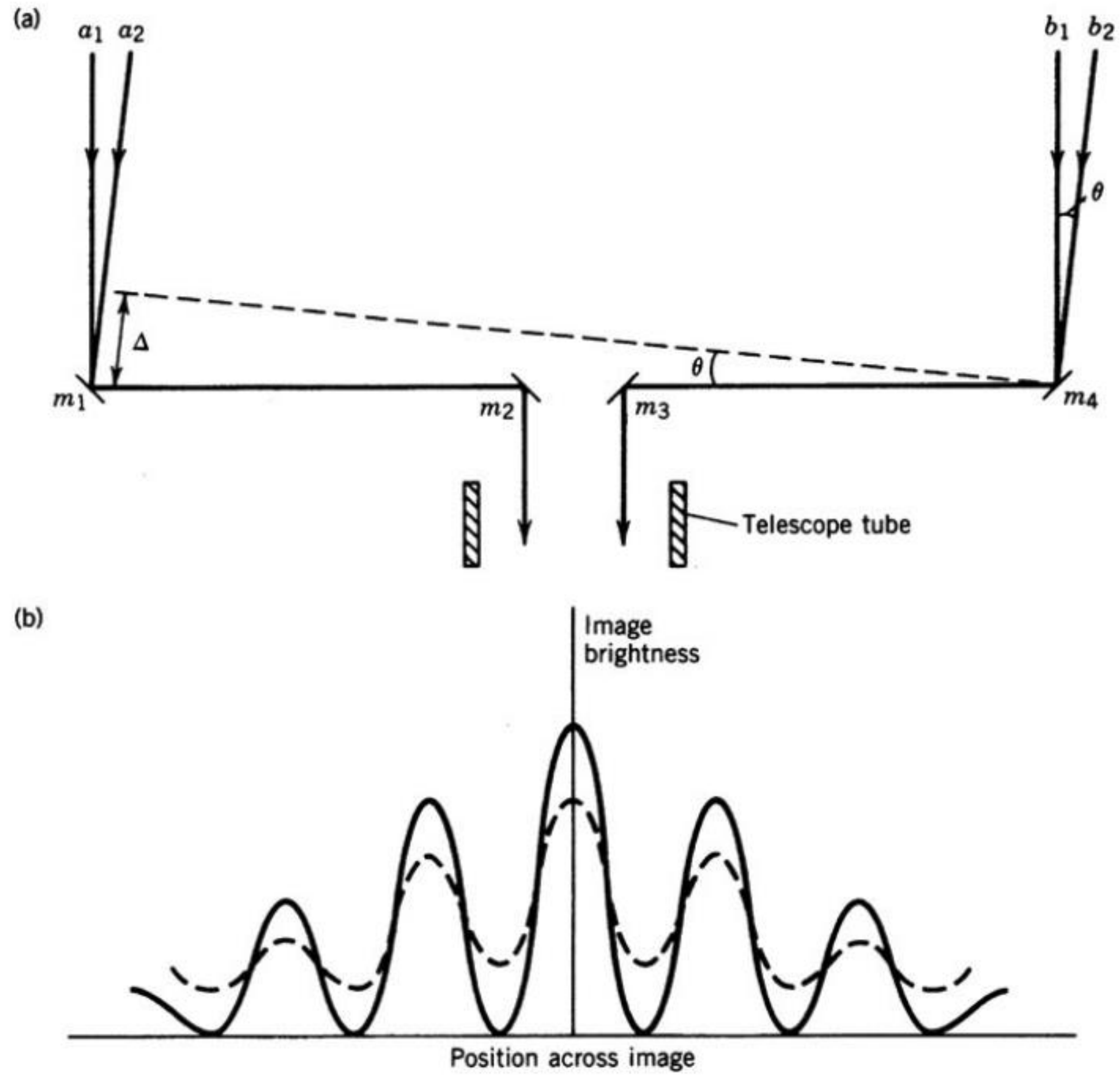
$$I = \mathrm{A}\left[1 + \cos\left(2\pi \frac{path1 - path2}{\lambda}\right)\right]$$

Code Credit: Oleg Smirnov

Interferometry basics

Michelson-Pease stellar interferometer

They were able to obtain sufficiently fine angular resolution to measure the diameters of some of the larger stars such as Arcturus and Betelgeuse.



Credit: Interferometry and Synthesis in Radio Astronomy

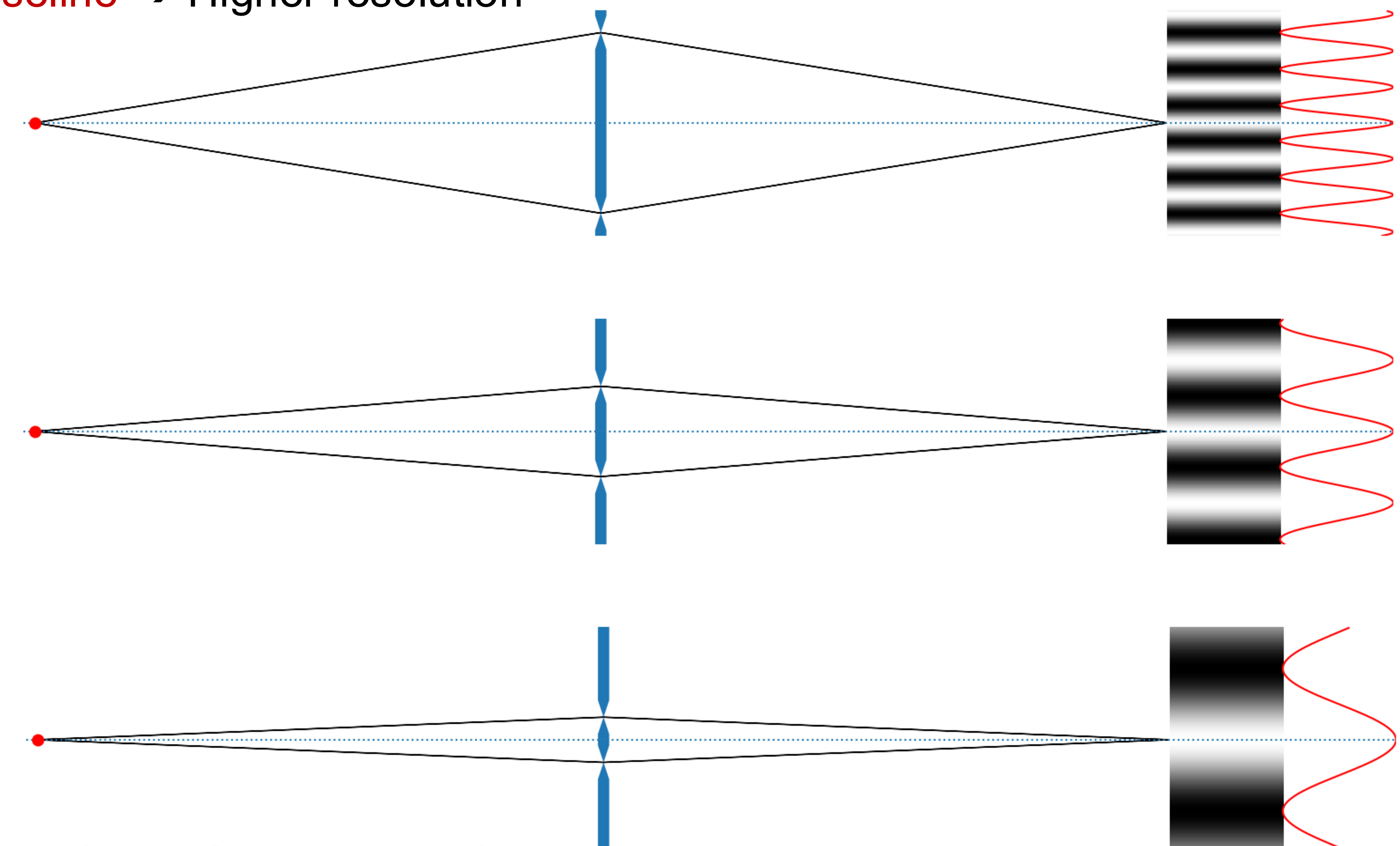
- "Visibility" was introduced to describe the contrast of the fringe pattern as $V_M=(I_{max}-I_{min})/(I_{max}+I_{min})$.
- I_{max} and I_{min} are the maximum and minimum of the envelope.
- $V_M = 1$ when $I_{min} = 0$.

Interferometry basics

Young's double-slit experiment

Each baseline sees one Fourier component of the sky brightness with one **spatial frequency**

Longer baseline ➔ Higher resolution

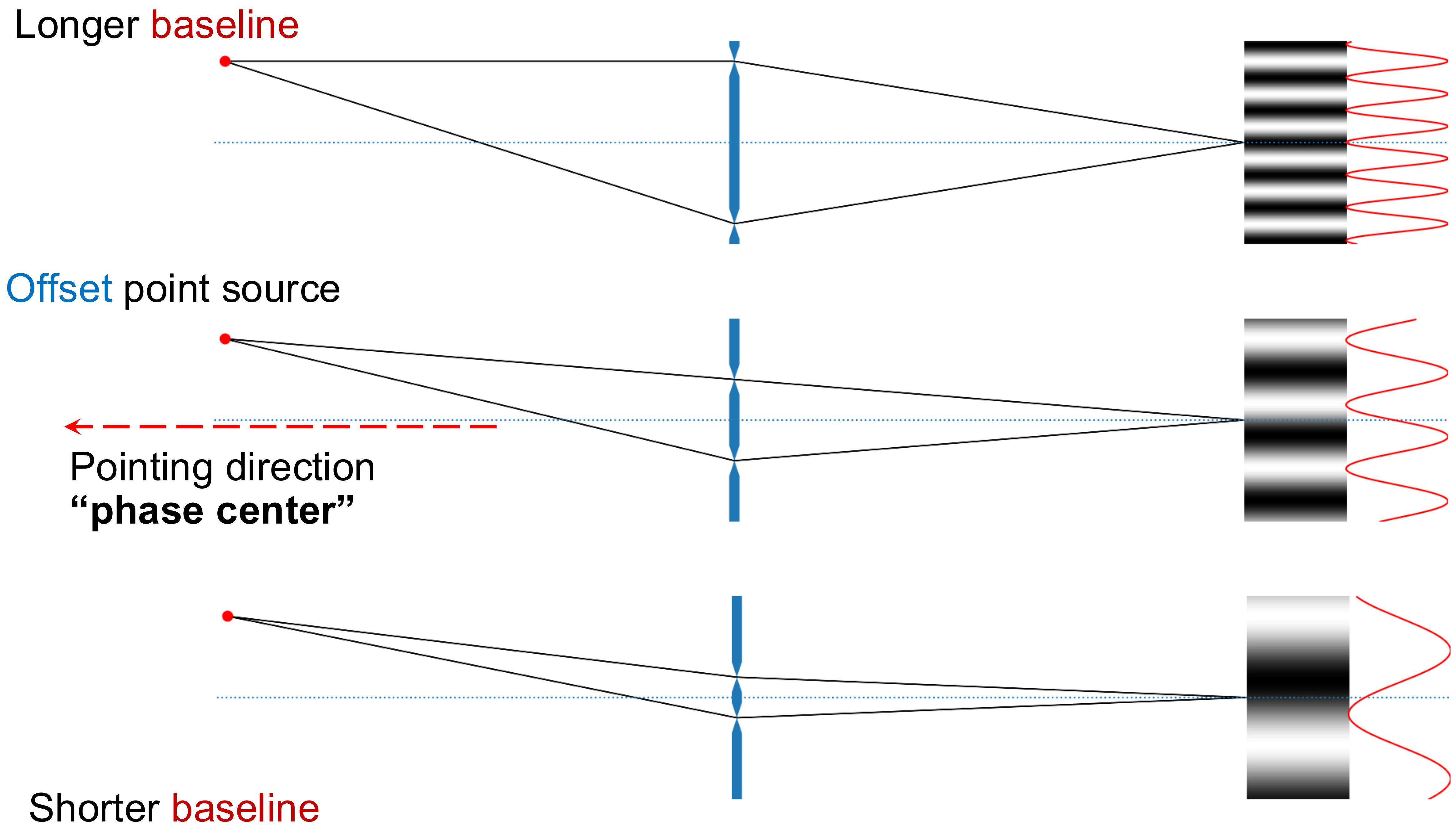


Shorter baseline ➔ Lower resolution

Interferometry basics

Young’s double-slit experiment

Same **source position** relative to the pointing direction results different **phases** seen by different antenna pairs (baselines)



Visibility function

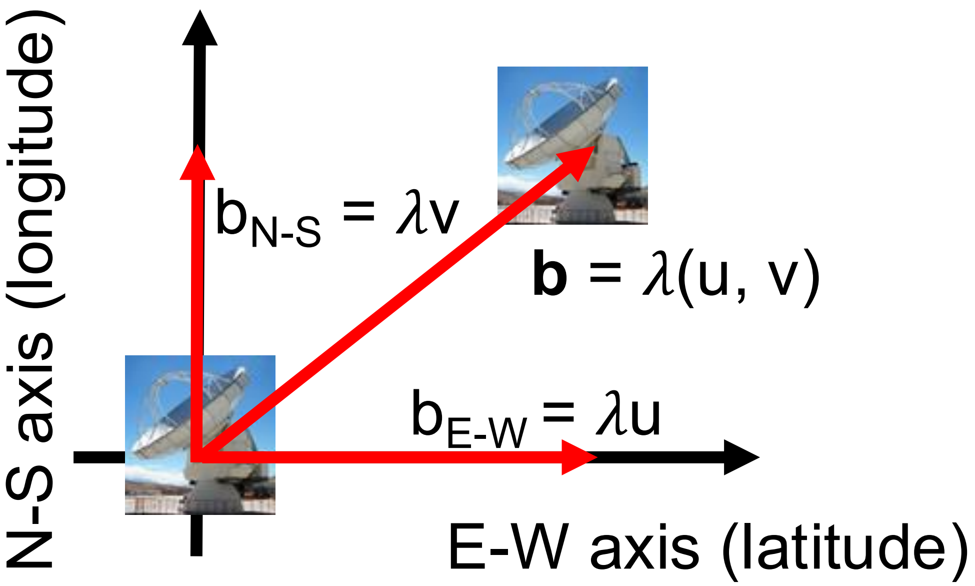
- "Visibility" was originally introduced to describe the contrast of the fringe pattern as $V_M=(I_{max}-I_{min})/(I_{max}+I_{min})$.

- Radio interferometry works with "complex visibilities"

Let $u = b_{E-W}/\lambda$, and $v = b_{N-S}/\lambda$ be spatial frequencies.
$V(u,v) = V_A e^{i\phi_V} = V_A(\cos\phi_V + i\sin\phi_V)$.

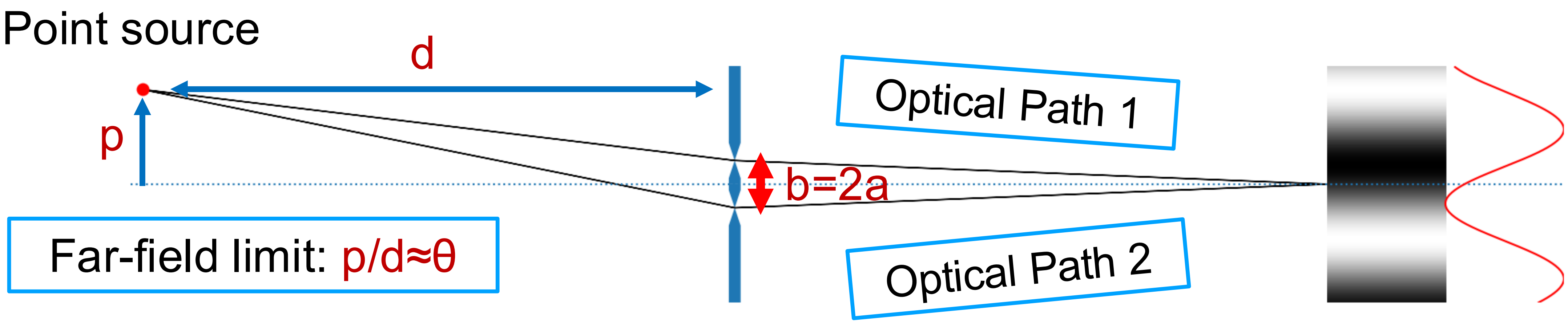
– Fringe amplitude → Visibility amplitude
Fluxes from a spatial frequency (u, v)

– Fringe phase → Visibility phase
"*Location" of a spatial frequency (u, v)*



Task 1: 1D Visibility

What is $\phi_V(u)$ in the double-slit experiment under the far-field limit?
Plot $\phi_V(u)$ in [0, 10]x[-180°, +180°].



Hint:

1. Let $u = b/\lambda$ be spatial frequencies.

$V(u) \equiv V_A e^{i\phi_V} = V_A(\cos\phi_V + i\sin\phi_V)$.

2. Given the far-field limit $(d \gg p \pm a)$ and $\sqrt{1+x} \approx 1 + \frac{x}{2}$ $as\ 1 \gg x$.

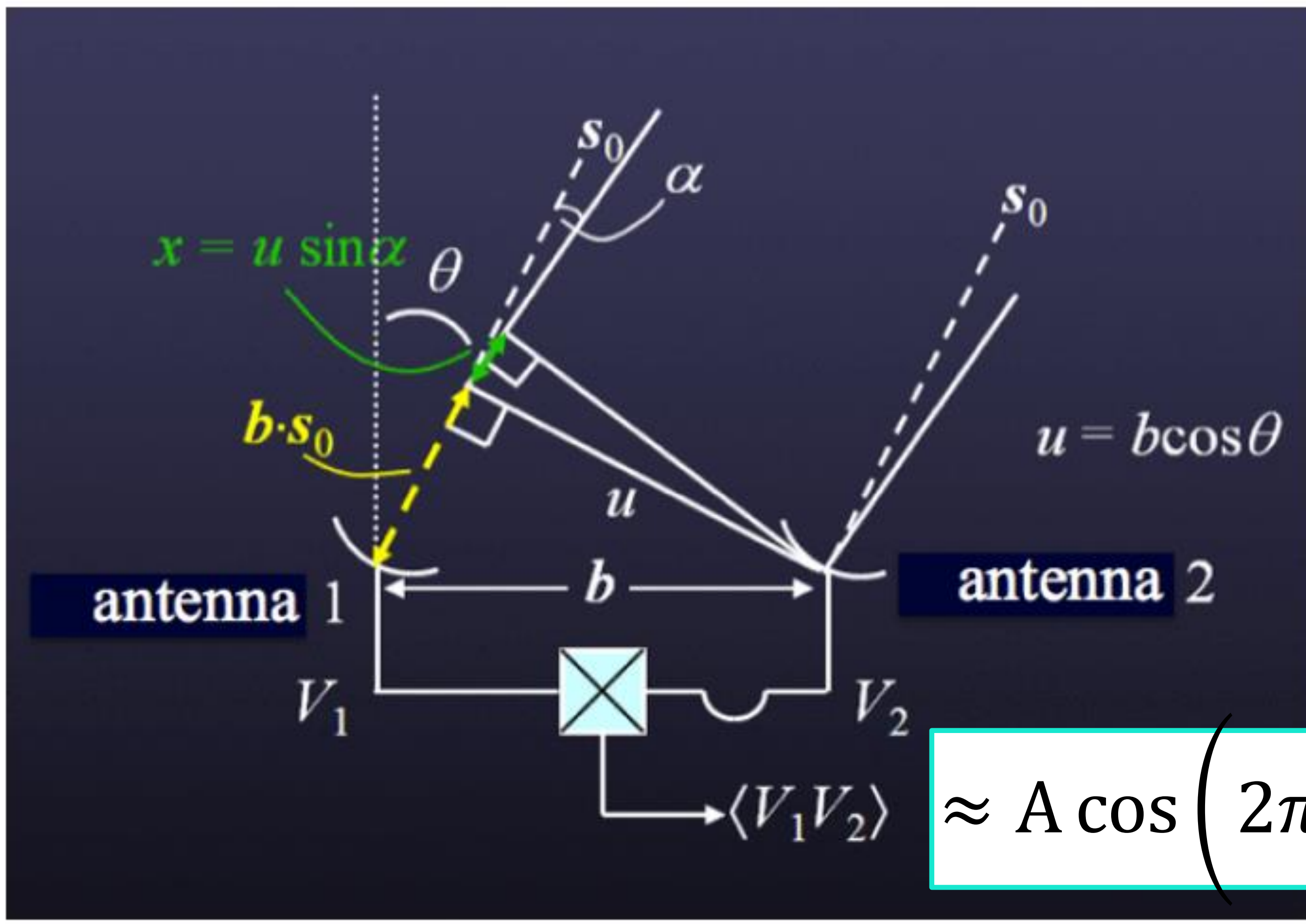
$\Delta L = \cdots$

3. $I \propto \cos\left(2\pi \frac{path1 - path2}{\lambda}\right) \approx \cos\left(\frac{2\pi}{\lambda}\Delta L\right) = \cdots$

$\Rightarrow \phi_V(u) = \cdots$

Interferometer correlator

- Double-slit Experiment (summation; Σ-interferometer):
  Interferes physical light waves directly on a screen.
- “Modern” Radio Interferometry (multiplication; Π-interferometer):
  Interferes alternating voltage signals in a **correlator**.



$$\approx \mathrm{A}\cos\left(2\pi \frac{path1\ -\ path2}{\lambda}\right)$$

Figure 3.4: An ideal 1-D interferometer consisting of two antennas, 1 and 2, separated by physical distance (i.e., a baseline) b. The antennas are both pointed towards a sky location given by s_o, which is at an angle θ from the meridian. The projected distance between the two antennas in that direction is thus $u = b\cos\theta$. The two antennas are connected to a correlator where the voltages detected from each are combined.

Credit:
ALMA Technical Handbook

Fourier transform

1D case

The Fourier transform of a function $f(x)$ can be written as

$$F(s) = \int_{-\infty}^{\infty} f(x)\, e^{-j2\pi sx} dx\ . \qquad \text{(A2.1)}$$

The inverse transform is

$$f(x) = \int_{-\infty}^{\infty} F(s)\, e^{j2\pi sx} ds\ . \qquad \text{(A2.2)}$$

The transform pair is written symbolically as

$$f(x) \longleftrightarrow F(s)\ . \qquad \text{(A2.3)}$$

If x has units of meters, then s has units of cycles/meter; if x has units of time, then s has units of cycles/second, i.e., hertz. The Fourier transform pair can also be written in the form normally used in the time-frequency domains as

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt\ , \qquad \text{(A2.4)}$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t} d\omega\ . \qquad \text{(A2.5)}$$

$$V(u) = \int_{-\infty}^{\infty} I(x) e^{-i2\pi ux} dx$$

$$I(x) = \int_{-\infty}^{\infty} V(u) e^{i2\pi ux} du$$

Credit: Interferometry and Synthesis in Radio Astronomy

Visibility function

van Cittert-Zernike theorem

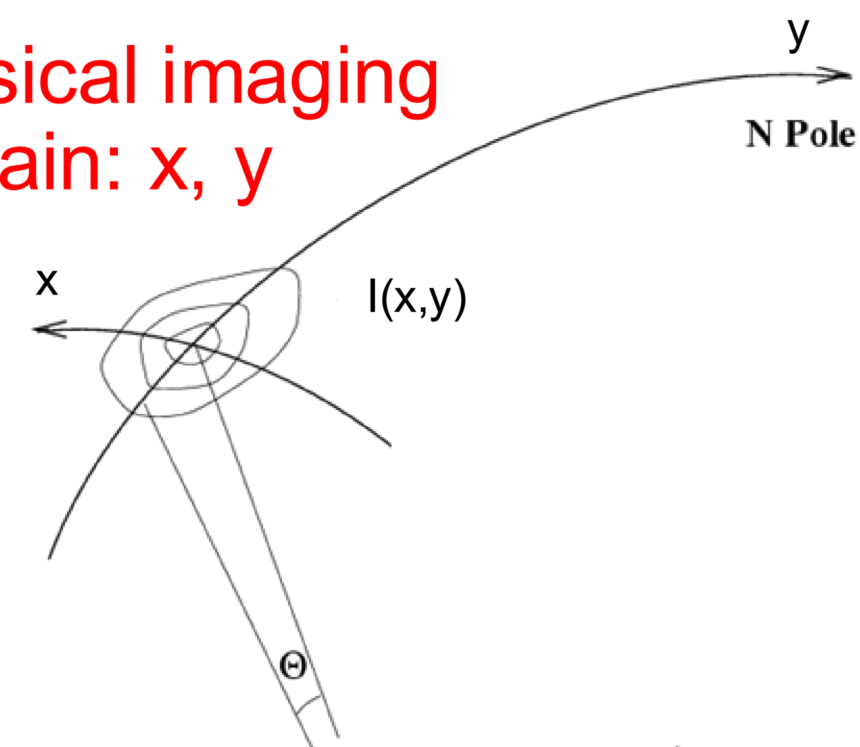
The interference pattern is directly related to the source brightness. The **complex visibility, *V(u,v)*,** is the 2D Fourier transform of the **sky brightness, *I(x,y)*.**

$$V(u,v) = \iint I(x,y)e^{-i2\pi(ux+vy)}dxdy$$

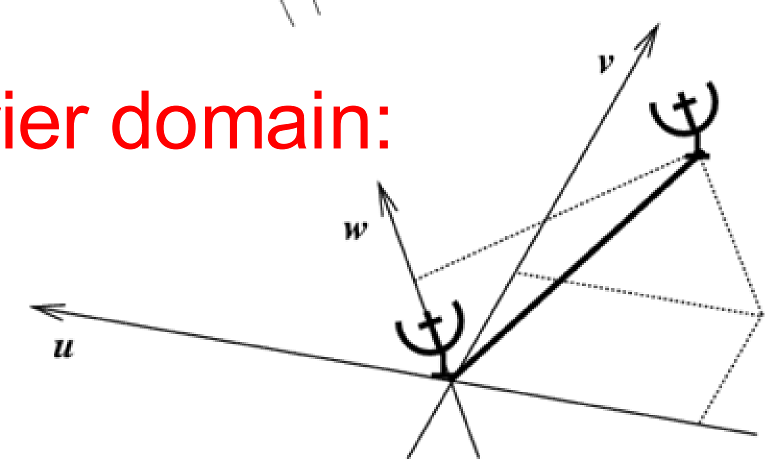
$$I(x,y) = \iint V(u,v)e^{+i2\pi(ux+vy)}dudv$$

u, v are E-W, N-S spatial frequencies [wavelengths]
x, y are E-W, N-S angles in the tangent plane [radians]

Physical imaging domain: x, y

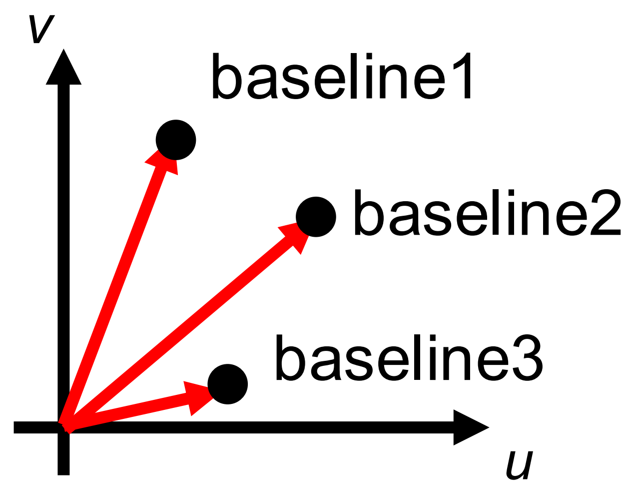


Fourier domain: u, v



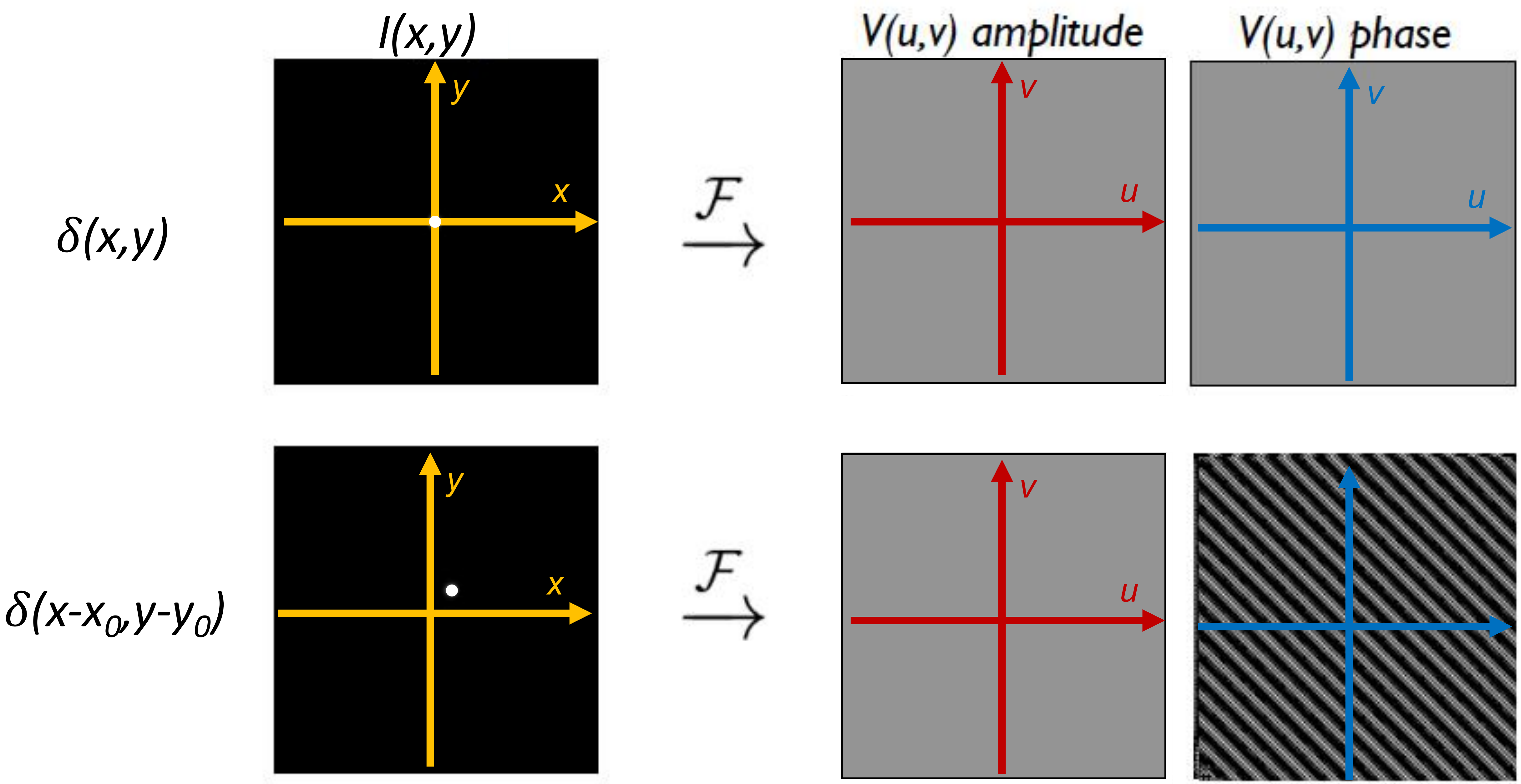
Fourier transform

2D case



$\mathcal{F}$ properties:

- $\mathcal{F}(\delta)$ *= constant for amplitude*

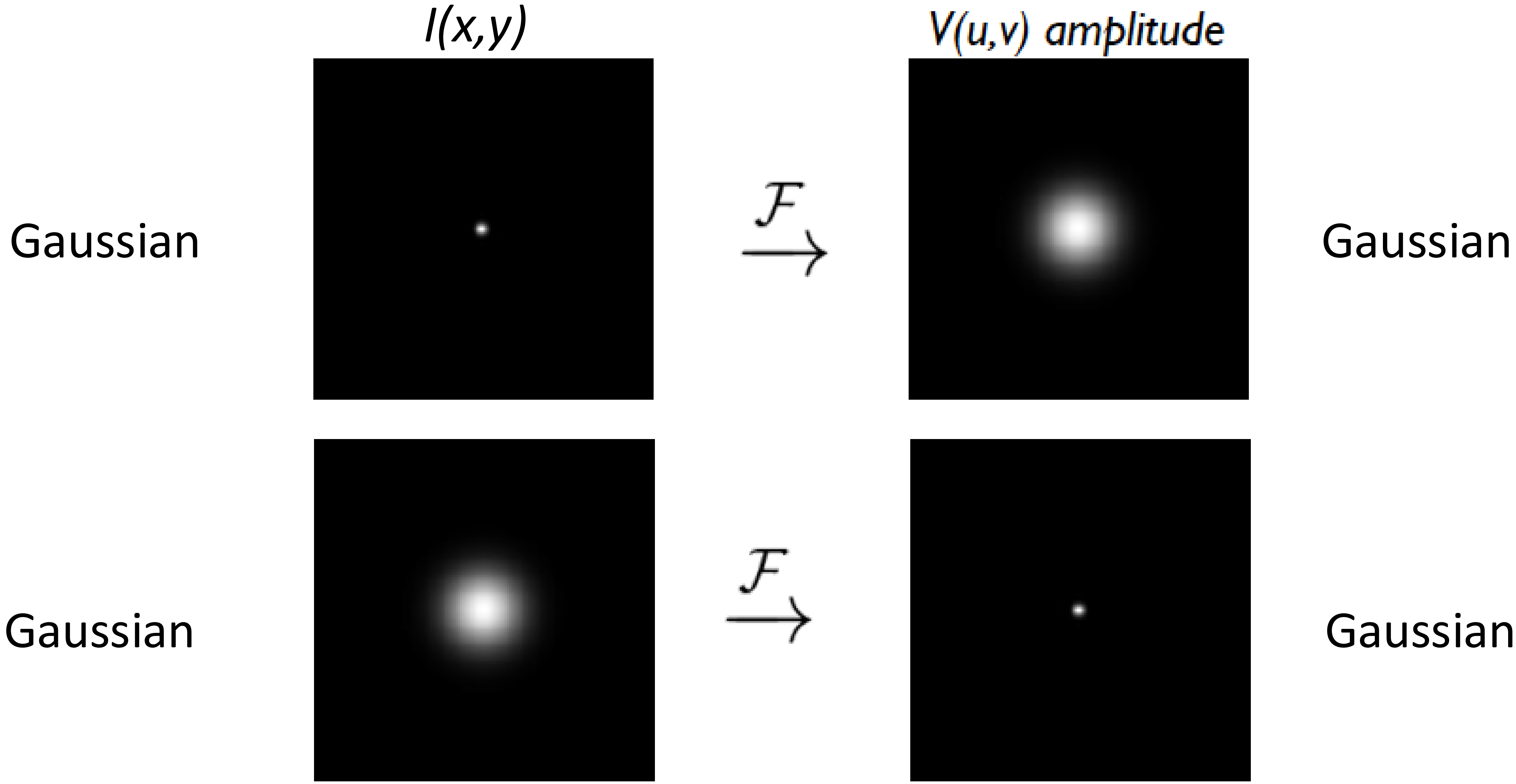


Fourier transform

2D case

$\mathcal{F}$ properties:

- *$\mathcal{F}$(Gaussian) = Gaussian*
- Narrow features transform into wide features and vice versa.

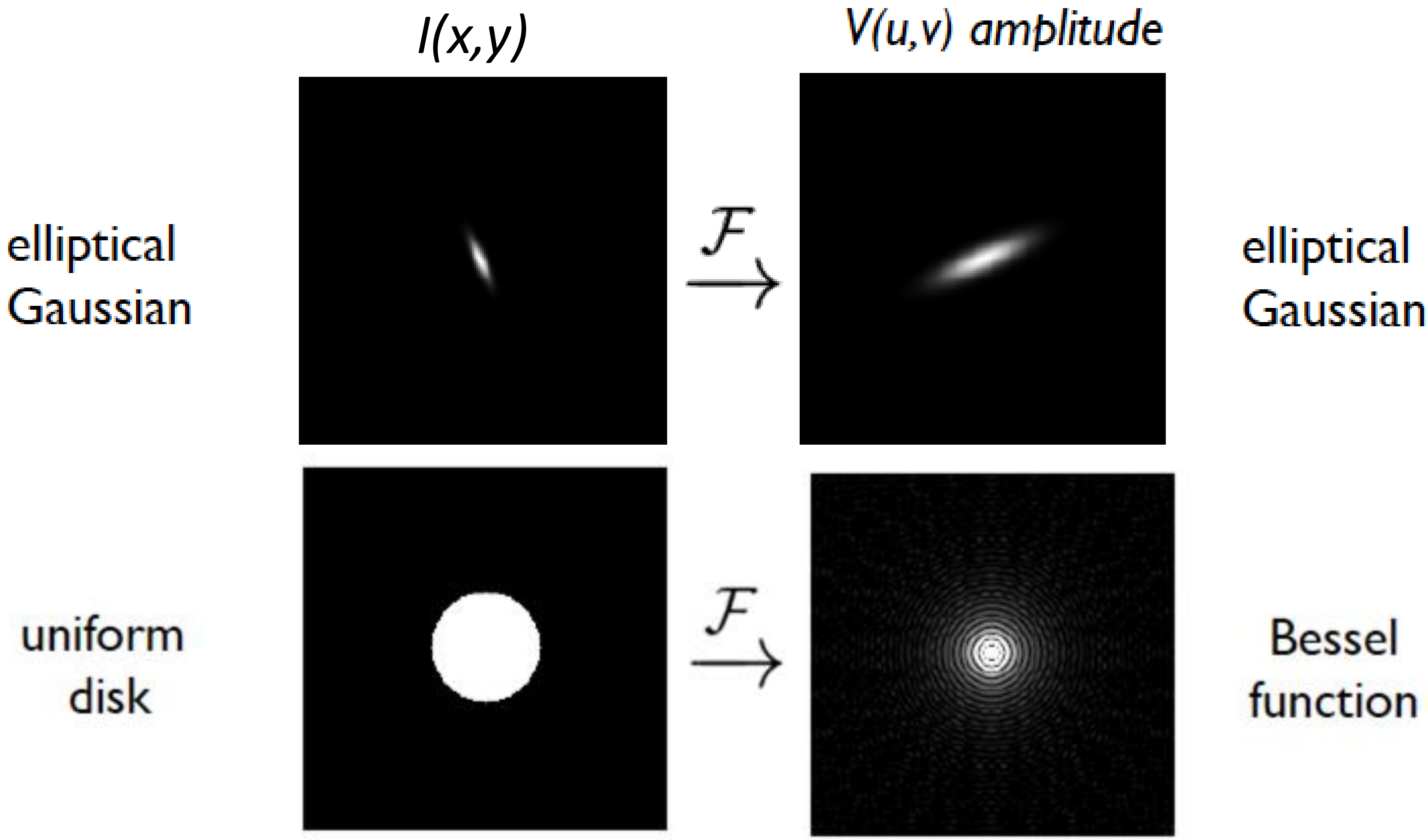


Fourier transform

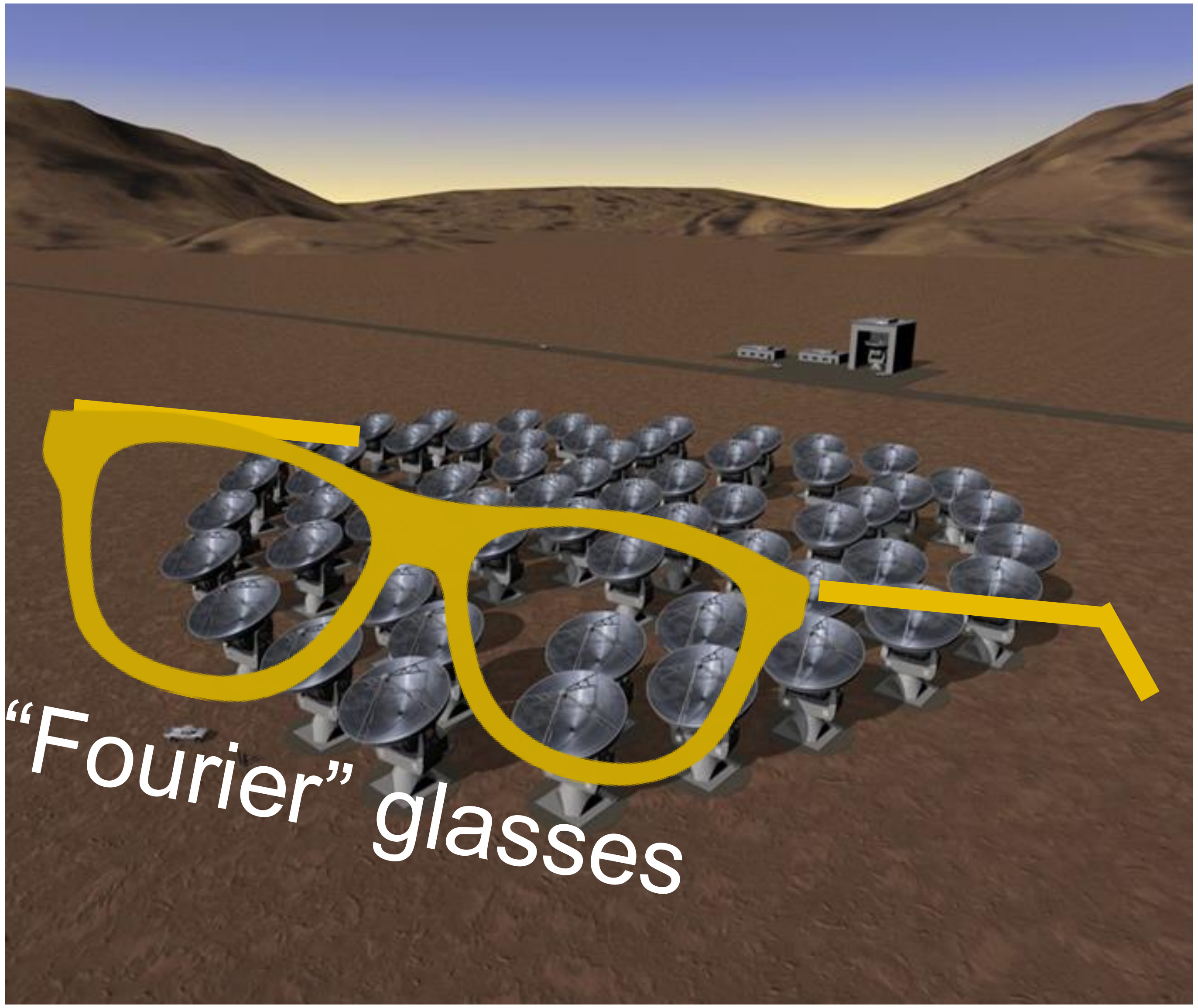
2D case

$\mathcal{F}$ properties:

- Sharp edges produce “ringing” features and vice versa.



Interferometric observations



So we have to think about our data in the Fourier domain!

ALMA array

- The 12-m Array: 55 antennas
- The Atacama Compact Array (ACA):
  - the 7-m array: 12 7-m antennas
  - the total power (TP) array: 4 12-m antennas

#baseline = N(N-1)/2

Credit: ALMA Technical Handbook

Figure 2.1: The ALMA 12-m Array in its compact configuration (left hand side of the image). The ACA 7-m Array with all 7-m antennas (dashed orange circles) and four single-dish 12-m antennas in the TP Array (blue circles) are distributed in the right hand side of the image. A few unoccupied stations can be seen, to which antennas of the 12-m Array can be moved by the transporter as the array is being reconfigured. At its most extended configuration, antennas in the 12-m Array will be about 16 km apart.

ALMA array

Config.	L_{max} / L_{min}	Band / Freq. (GHz)	1 / 40	3 / 100	4 / 150	5 / 185	6 / 230	7 / 345	8 / 460	9 / 650	10 / 870
7-m	45 m	θ_{res} (arcsec)	31.8	12.7	8.47	6.87	5.52	3.68	2.76	1.95	1.46
	9 m	θ_{MRS} (arcsec)	167	66.7	44.5	36.1	29.0	19.3	14.5	10.3	7.67
C-1	161 m	θ_{res} (arcsec)	7.98	3.19	2.13	1.73	1.39	0.93	0.69	0.49	0.37
	15 m	θ_{MRS} (arcsec)	71.2	28.5	19.0	15.4	12.4	8.25	6.19	4.38	3.27
C-2	314 m	θ_{res} (arcsec)	7.63	3.05	2.04	1.65	1.33	0.88	0.66	0.47	0.35
	15 m	θ_{MRS} (arcsec)	56.5	22.6	15.0	12.2	9.81	6.54	4.90	3.47	2.59
C-3	500 m	θ_{res} (arcsec)	5.38	2.15	1.43	1.16	0.94	0.62	0.47	0.33	0.25
	15 m	θ_{MRS} (arcsec)	40.5	16.2	10.8	8.73	7.02	4.68	3.51	2.48	1.86
C-4	784 m	θ_{res} (arcsec)	3.26	1.30	0.87	0.71	0.57	0.38	0.28	0.20	0.15
	15 m	θ_{MRS} (arcsec)	28.0	11.2	7.50	6.08	4.89	3.26	2.44	1.73	1.29
C-5	1.4 km	θ_{res} (arcsec)	2.01	0.80	0.54	0.43	0.35	0.23	0.17	0.12	0.092
	15 m	θ_{MRS} (arcsec)	16.8	6.70	4.47	3.62	2.91	1.94	1.46	1.03	0.77
C-6	2.5 km	θ_{res} (arcsec)	1.16	0.47	0.31	0.25	0.20	0.13	0.10	0.072	0.054
	15 m	θ_{MRS} (arcsec)	10.3	4.11	2.74	2.22	1.78	1.19	0.89	0.63	0.47
C-7	3.6 km	θ_{res} (arcsec)	0.70	0.28	0.19	0.15	0.12	0.081	0.061	0.043	0.032
	64 m	θ_{MRS} (arcsec)	6.45	2.58	1.72	1.40	1.12	0.75	0.56	0.40	0.30
C-8	8.5 km	θ_{res} (arcsec)	0.40	0.16	0.11	0.087	0.070	0.047	0.035	0.025	0.019
	110 m	θ_{MRS} (arcsec)	3.55	1.42	0.95	0.77	0.62	0.41	0.31	0.22	0.16

Table A-1: Angular Resolution (θ_{res}) and maximum recoverable scale (θ_{MRS}) values for different 7-m Array and 12-m Array configurations available in Cycle 12 are shown for one representative frequency in each ALMA

CASA basics
interact with the ipython interface

- **taskhelp**: see all available CASA tasks
- **<task>?**, **help(<task>)**: see the online manual of a task (e.g. listobs?)
- **inp <task>**: to see all parameters of a task (e.g. inp listobs)
- **inp**: check what task is currently activated
- **vis='data.ms'**: to set a parameter via "="
- **par.<parameter>?**: see examples of a parameter (e.g. par.spw?)
- **go**: execute a task
- **tget <task>**: get last input values from file on disk for a specified task (tget listobs)
- **default(<task>)**: reset a task with default parameters
- **execfile('myScript.py')**: *read and execute a **Python script** from a file*

NOTE: if you see warnings about **leap seconds**, try:
in a terminal

```
cd [casa directory]/data
rsync -avz rsync://casa-rsync.nrao.edu/casa-data .
```

Measurement set

as multi-dimensional tables

- Get a summary of a measurement set with listobs and save the summary as a text file:

  > listobs(vis='test.ms', listfile='test.ms.listobs')

- If listfile is not defined, the summary will be shown in the logger.

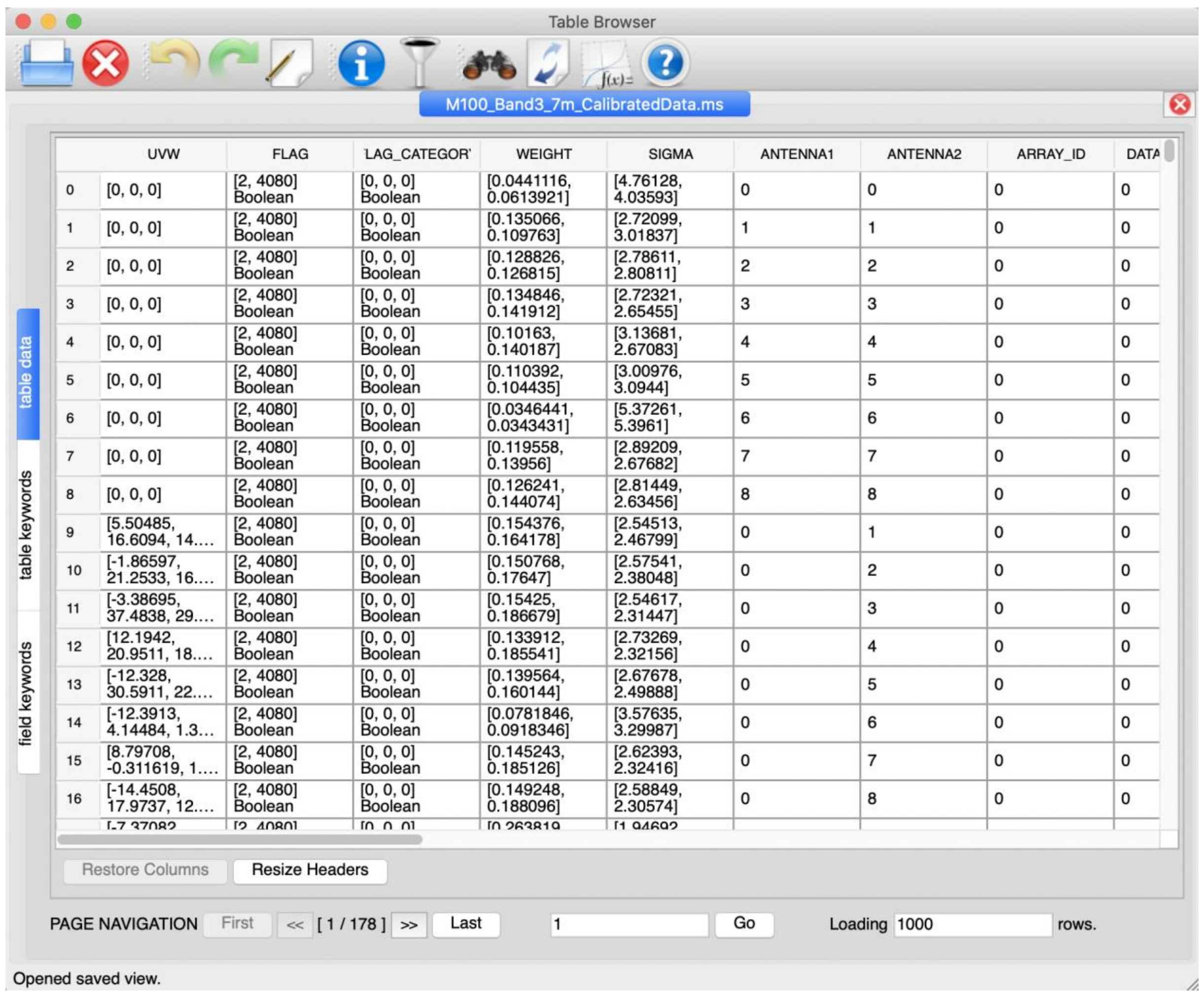
- Open the text file with your text editor (e.g. vi, emacs, etc.)
  - Observation, ObservationID => time
  - Fields => pointing (which parts of skies were observed)
  - Spectral windows => frequency samples
  - Sources => observed targets (calibrators, science targets)
  - Antennas => spatial frequency samples

Visualization of a MS

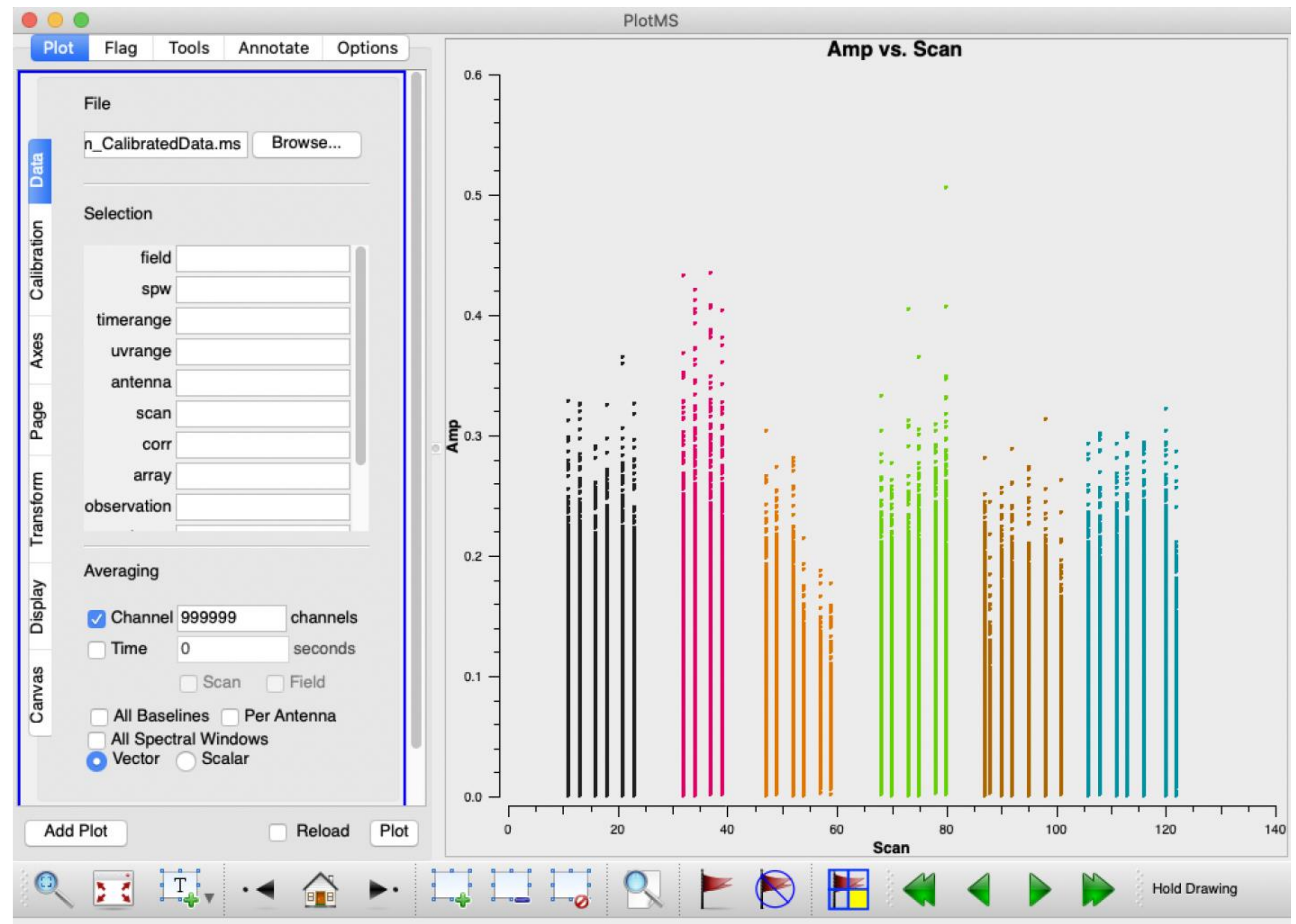
browstable and plotms

- browstable: a GUI to visualize a MS as tables
- plotms: a GUI to visualize a MS as plots

browsetable



plotms



Task 2: ALMA Visibility

Simulation Guide Component Lists CASA 6.6.6
https://casaguides.nrao.edu/index.php?title=Simulation_Guide_Component_Lists_CASA_6.6.6

1. Generate the model images of four cases:
    a. point source,
    b. off-center point source,
    c. Gaussian,
    d. disk.
2. Run each “script_simobserve_*” script to simulate the visibilities (“*.ms” file: Measurement Set) observed by ALMA.

3. Use ”plotms” to check the “*.ms” files:
x-axis=“Uvdist”, y-axis=“phase”

4. (HW) Can you derive the disk radius from the visibility?

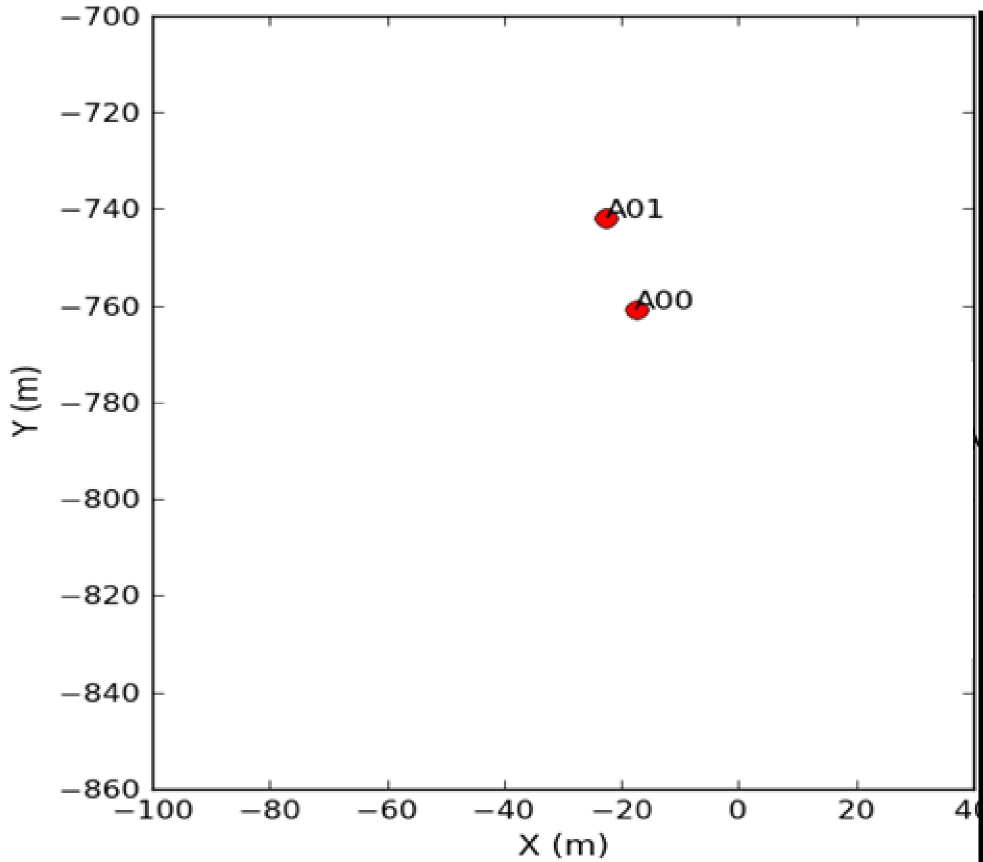
Task 2: ALMA Visibility

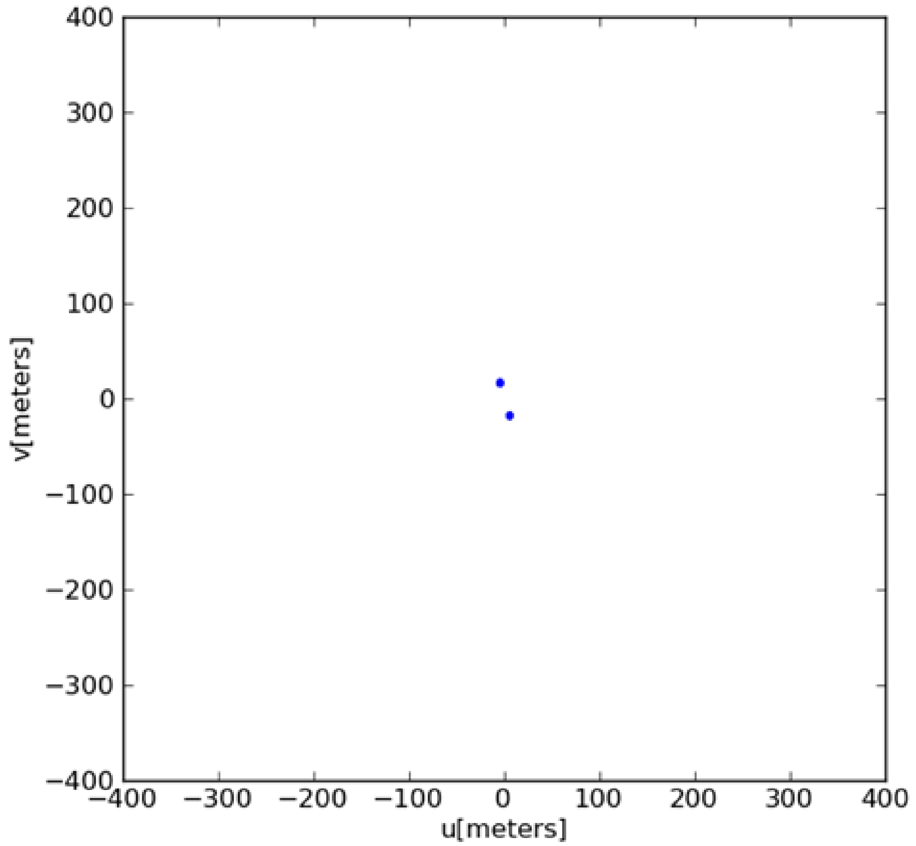
5. What happens if changing
direction = 'J2000 10h00m00.0s -30d00m00.0s’
to
direction = 'J2000 10h00m00.0s -00d00m00.0s’?

6. What happens if changing
totaltime = '28800s’ to ‘60s’?

7. What happens if changing
antennalist = 'alma.cycle12.1.cfg’ to 'alma.cycle12.10.cfg’?
Ref: https://casaguides.nrao.edu/index.php/Antenna_Configurations_Models_in_CASA

Example: Fringe pattern with 2 Antennas (one baseline)





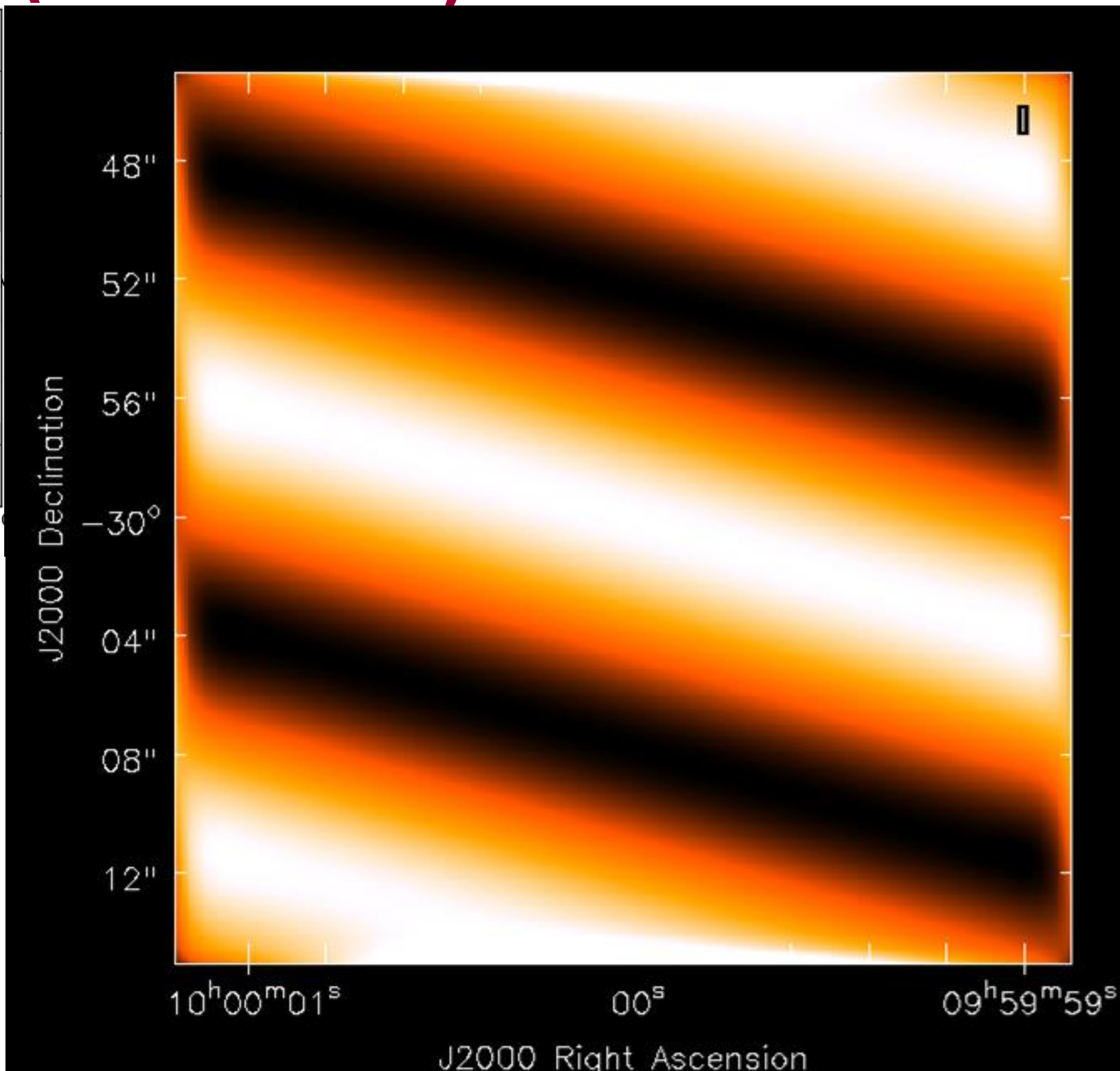


Image Credit: Ian Czekala (NRAO)

Example: Fringe pattern with 3 Antennas (3 baselines)

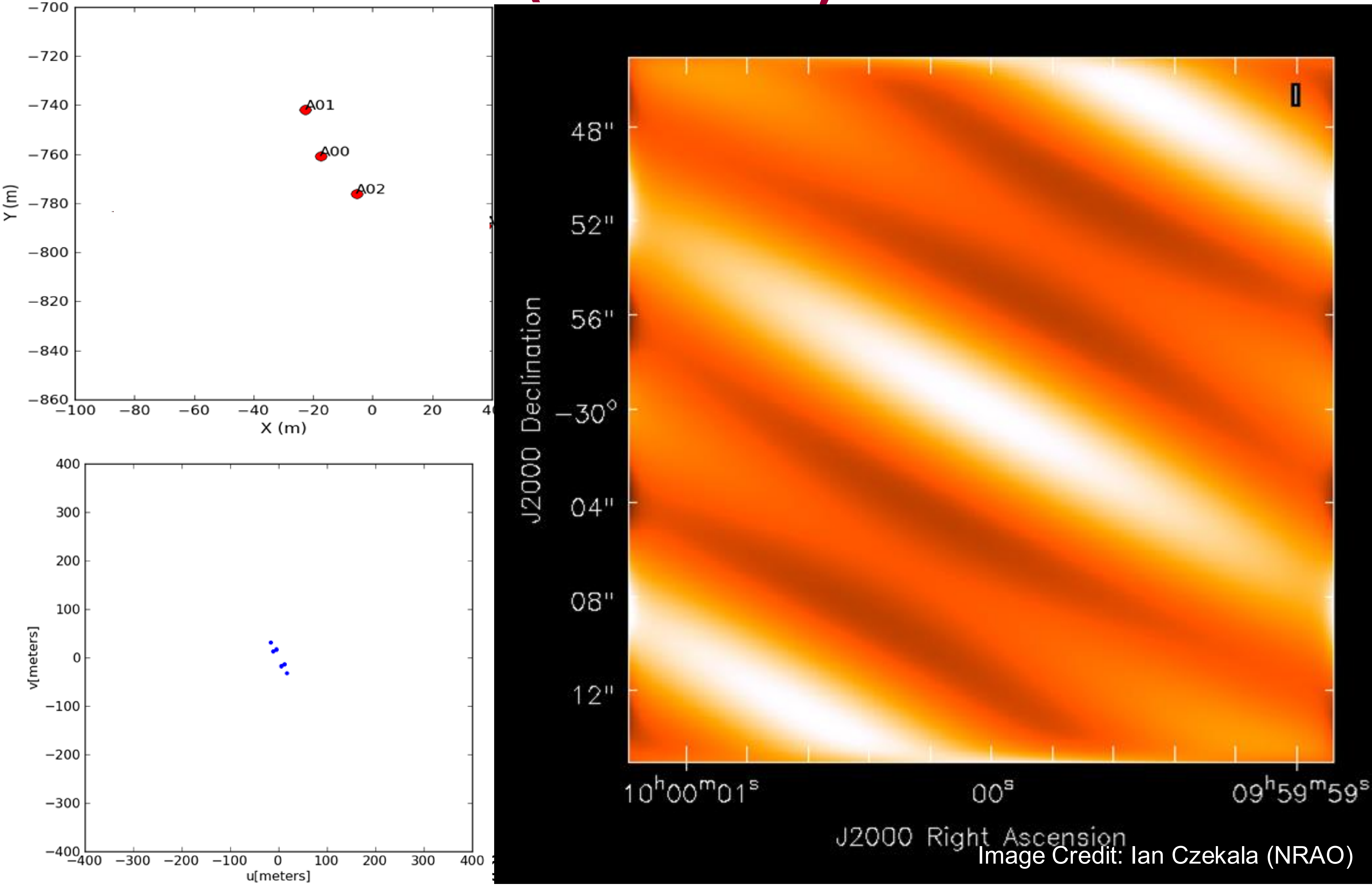
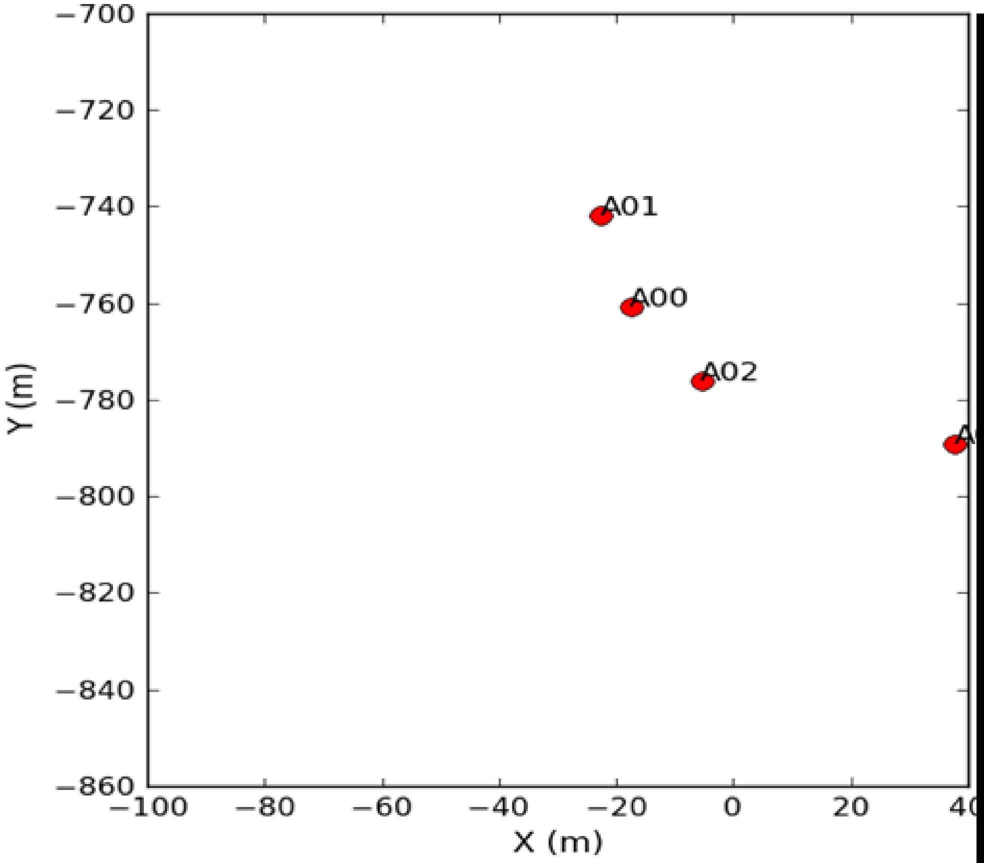
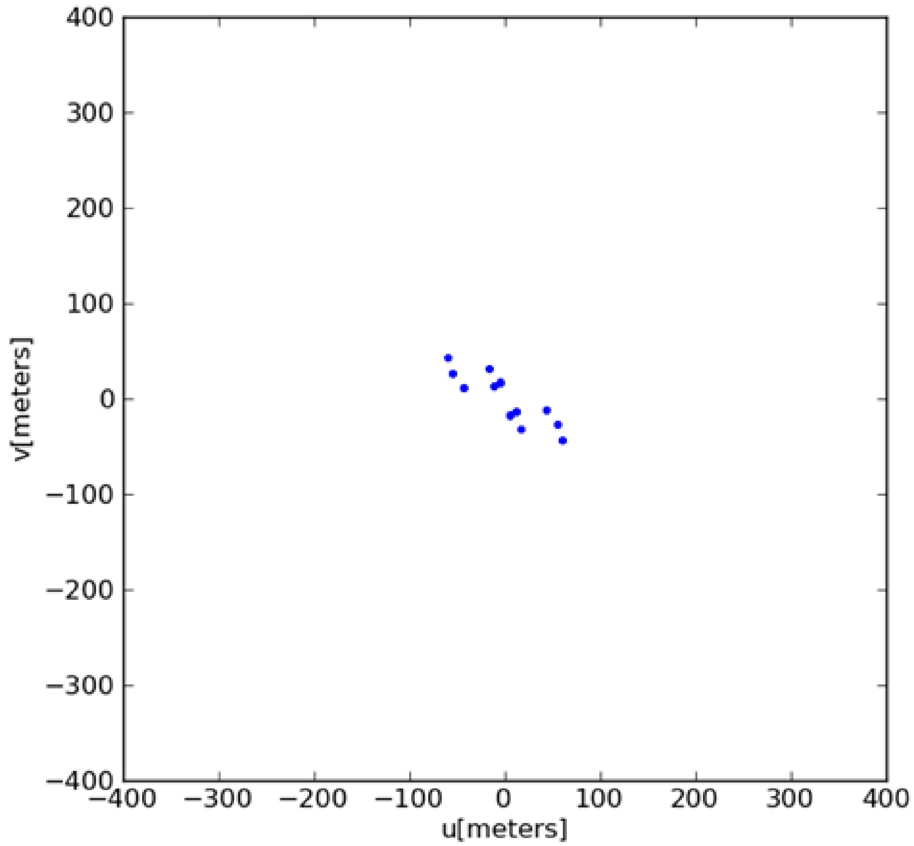


Image Credit: Ian Czekala (NRAO)

Example: Fringe pattern with 4 Antennas (6 baselines)





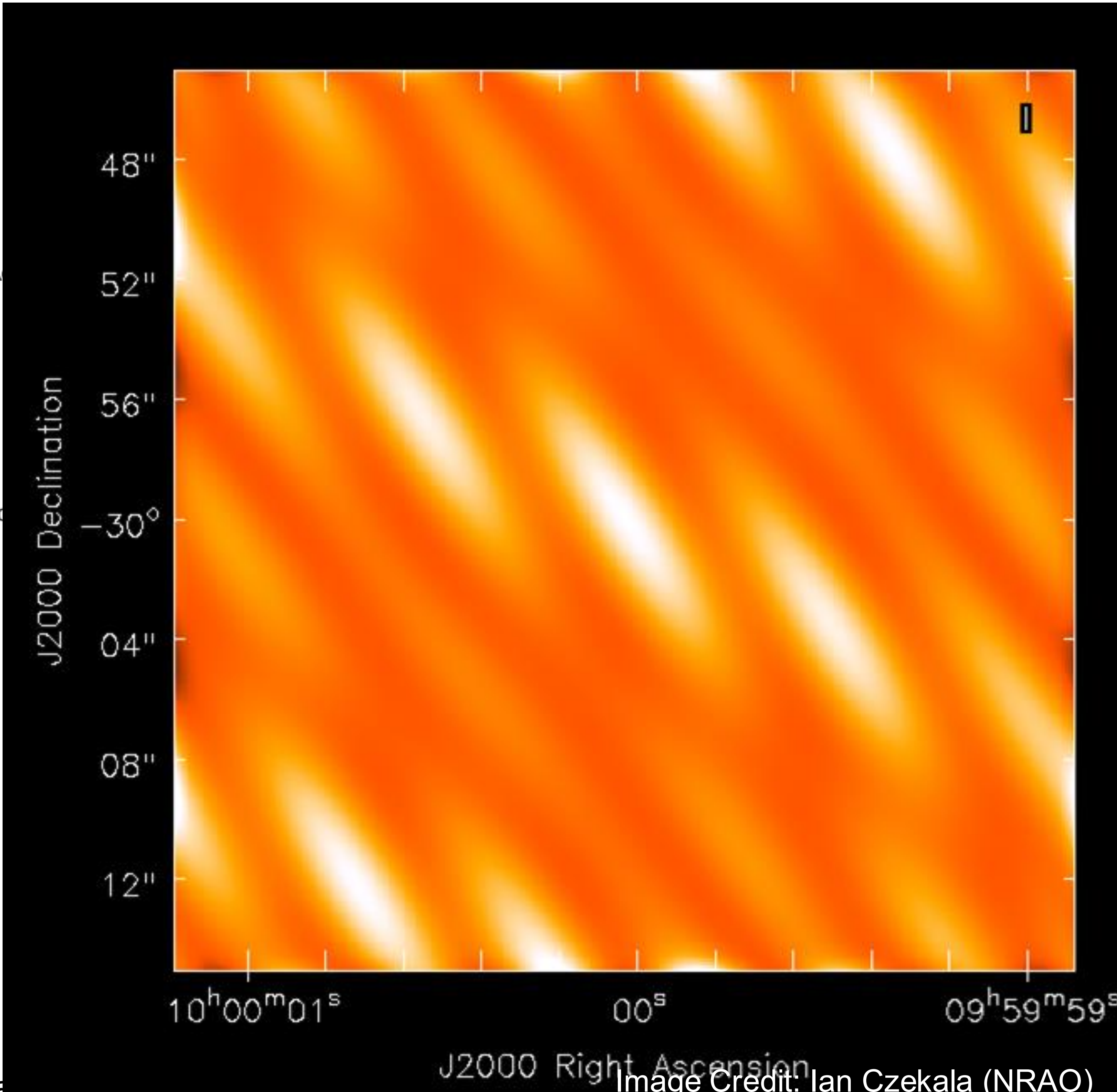
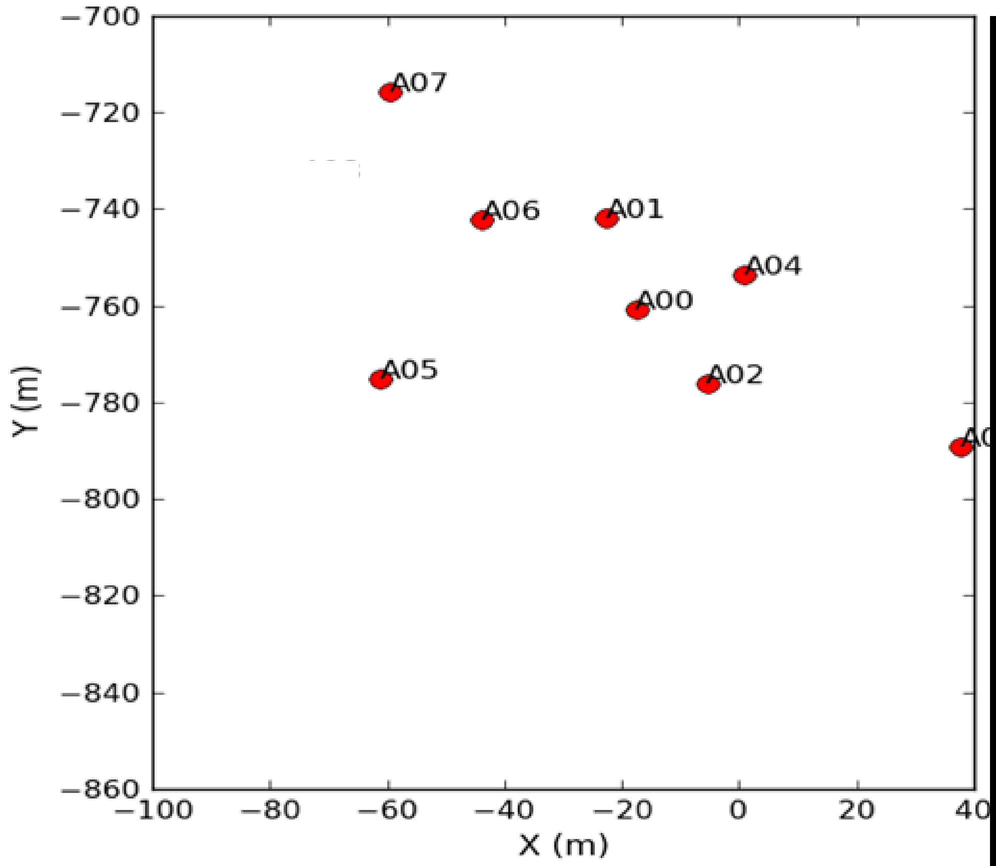
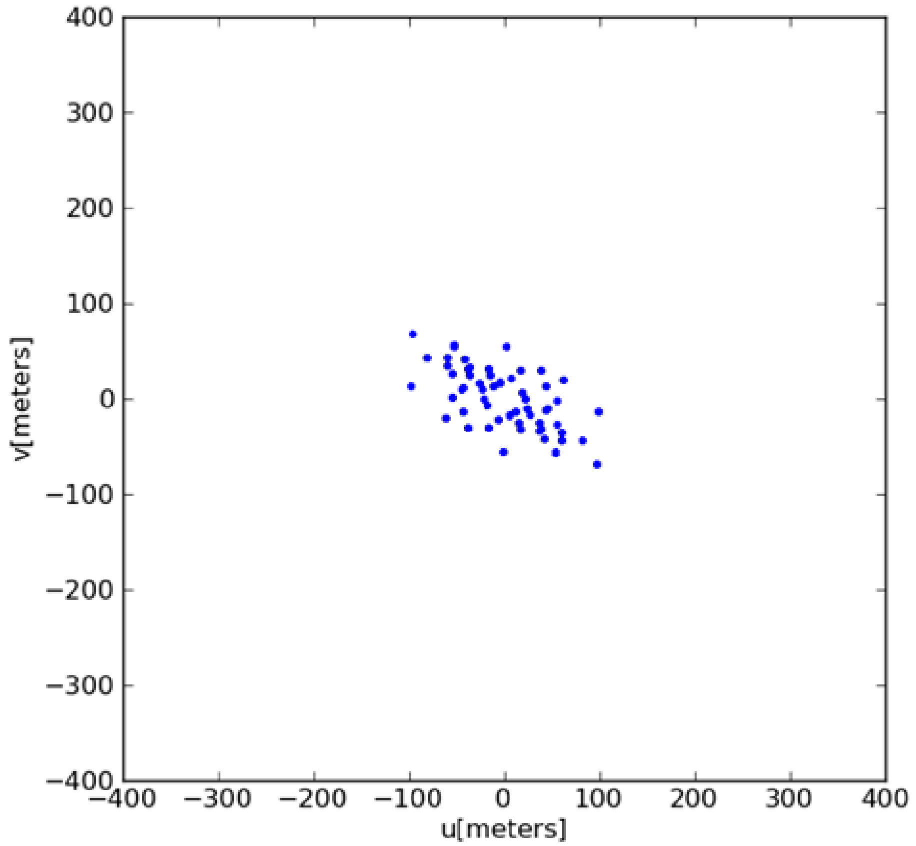


Image Credit: Ian Czekala (NRAO)

Example: Fringe pattern with 8 Antennas (28 baselines)





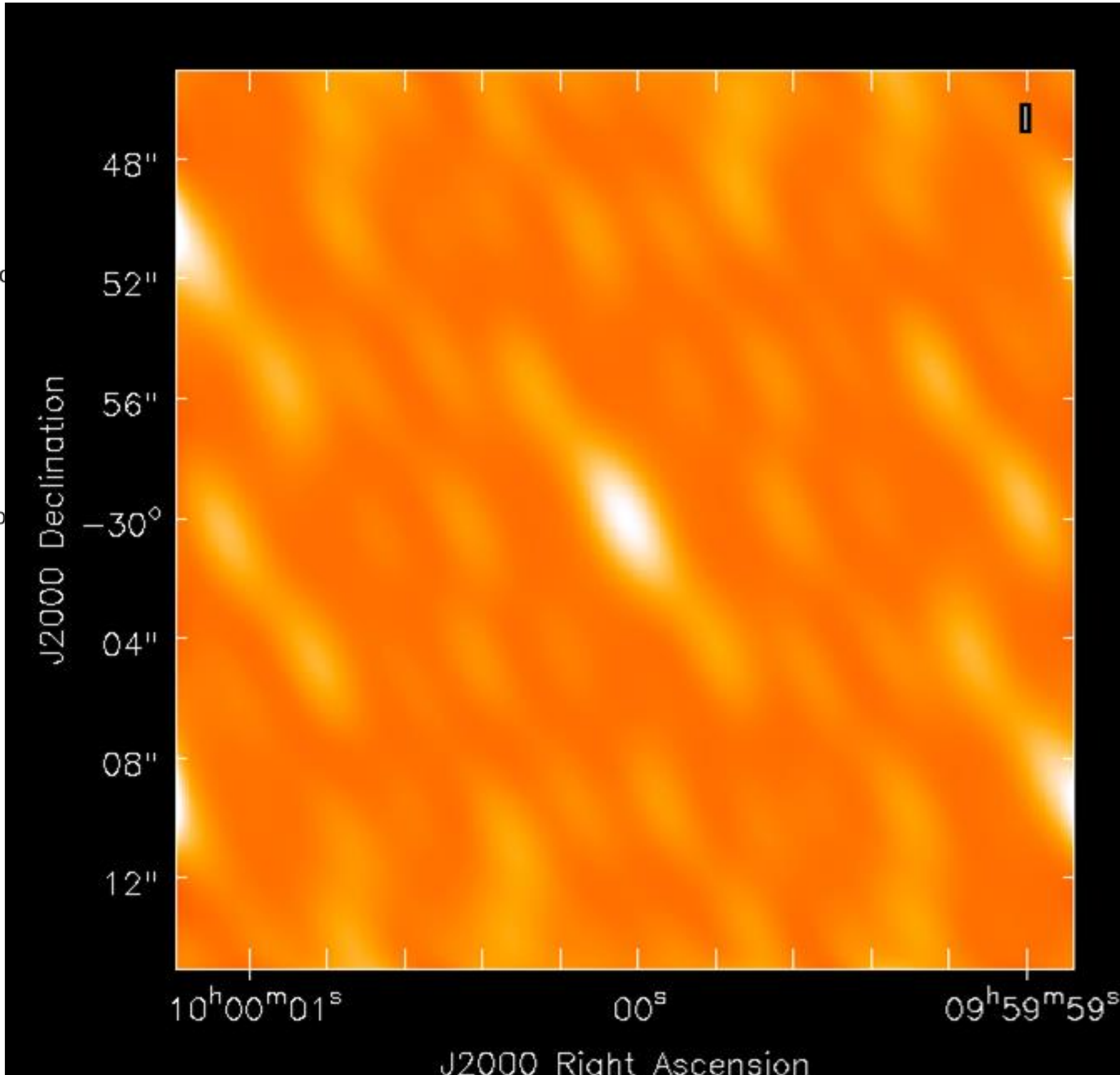
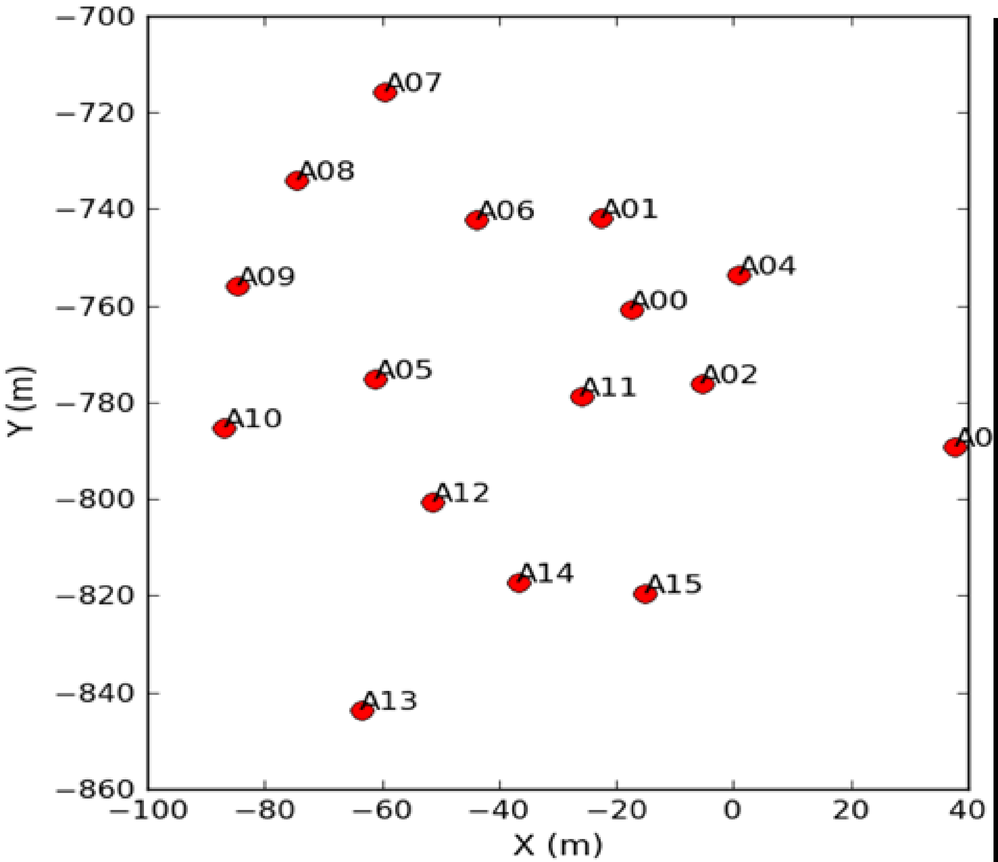
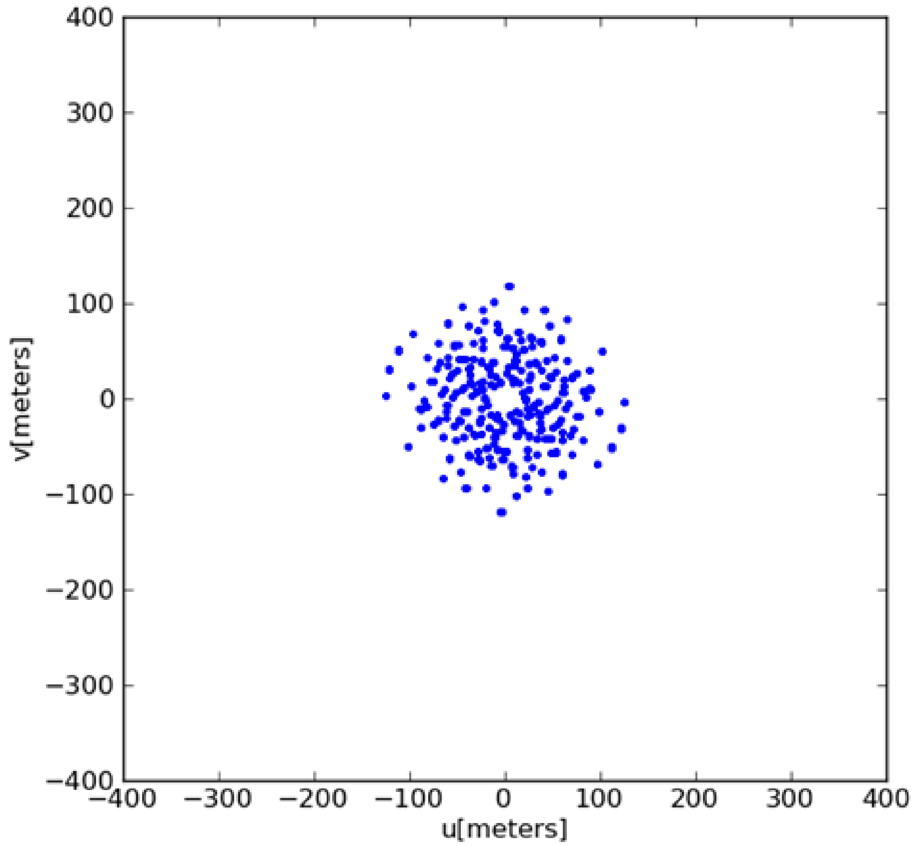


Image Credit: Ian Czekala (NRAO)

16 Antennas – Compact Configuration





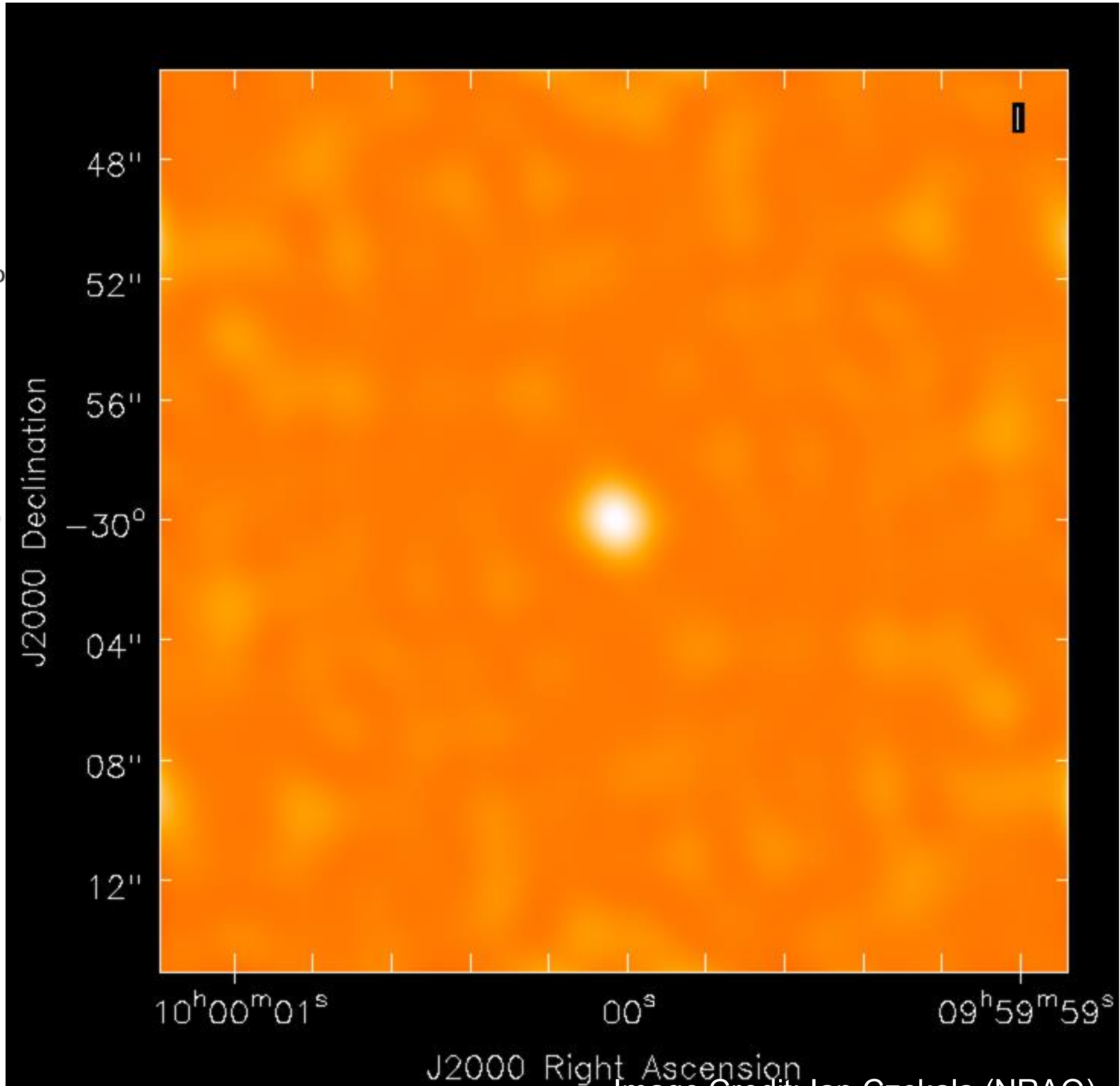
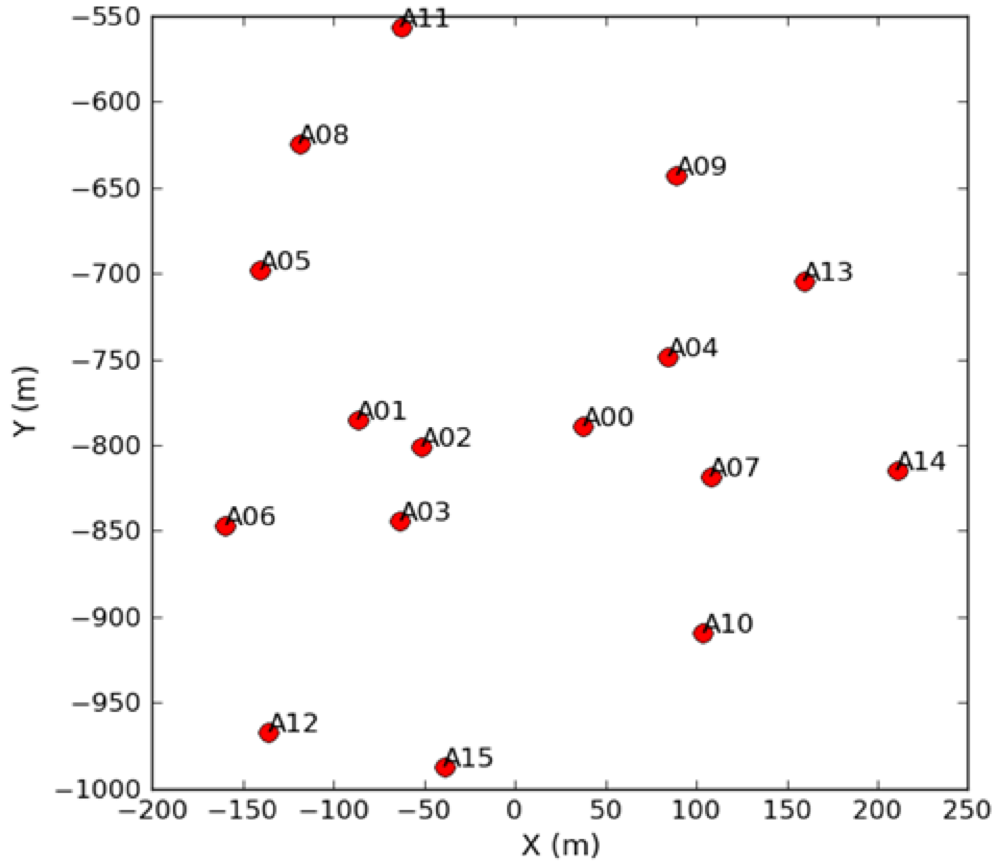
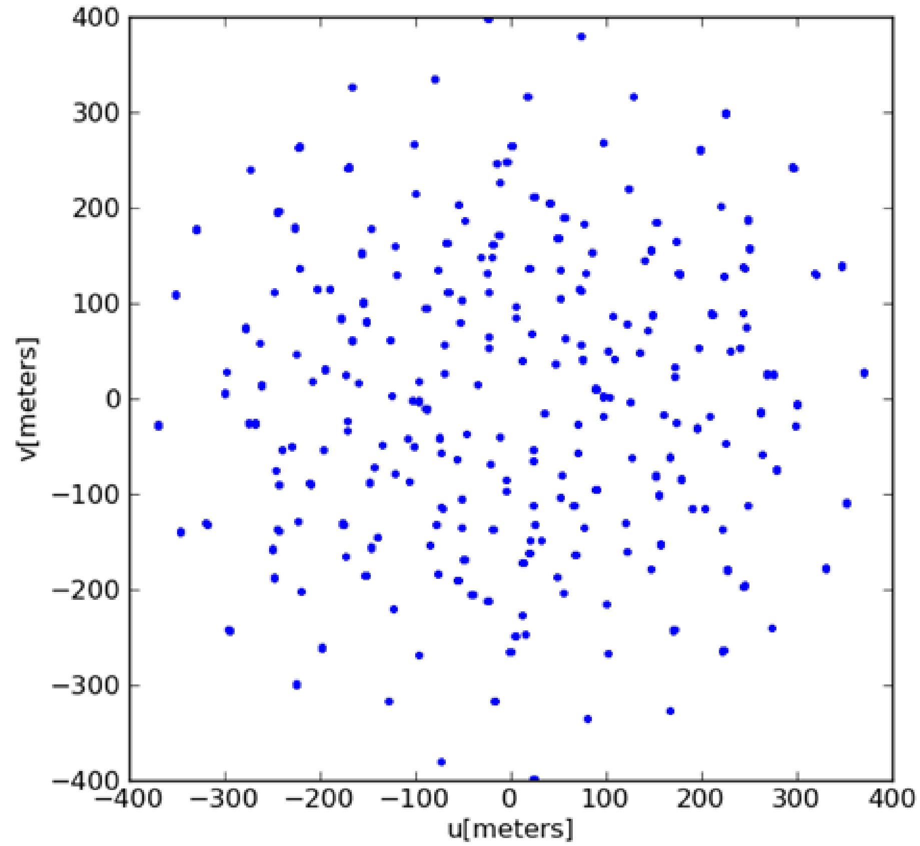


Image Credit: Ian Czekala (NRAO)

16 Antennas – Extended Configuration





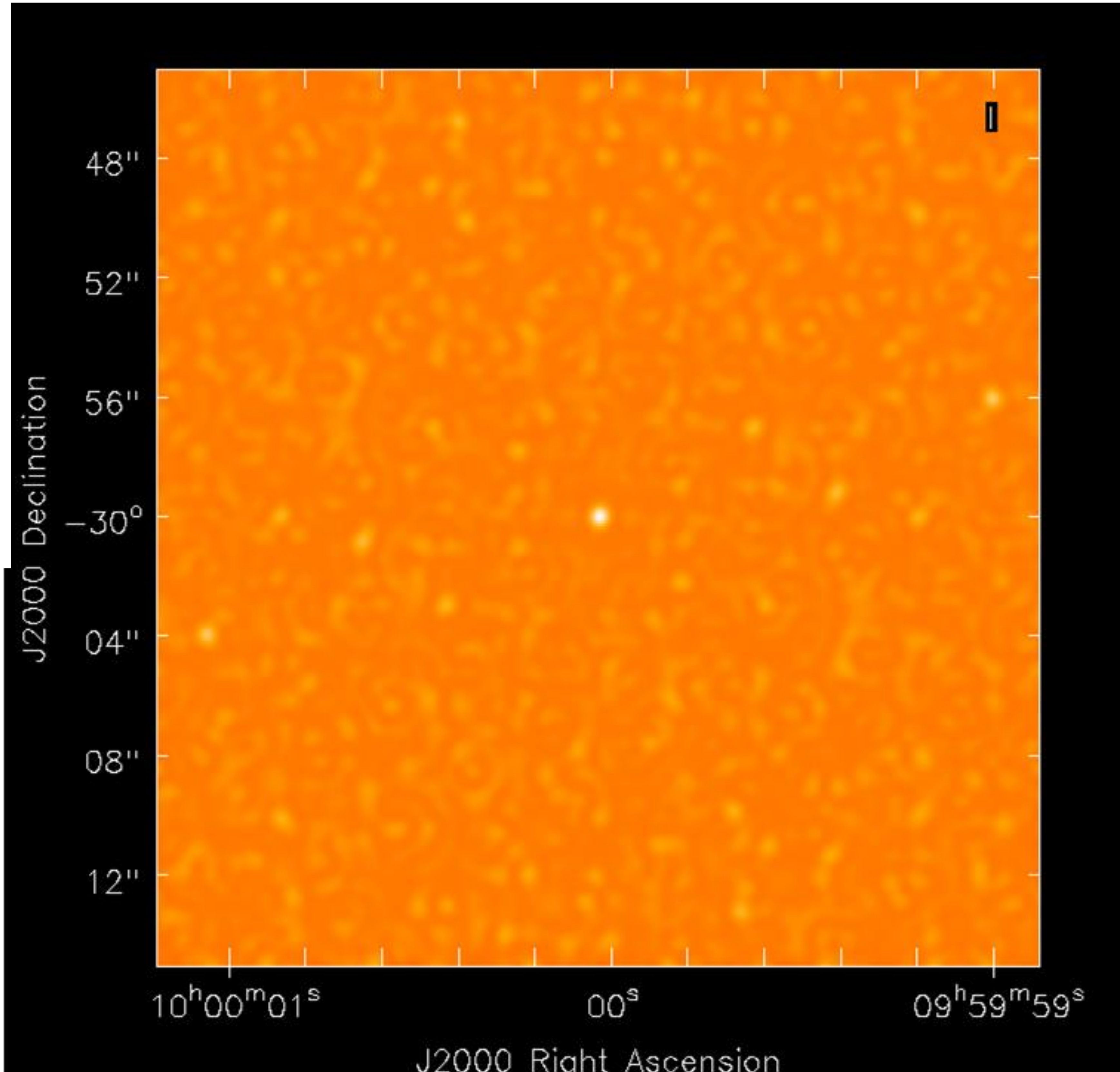
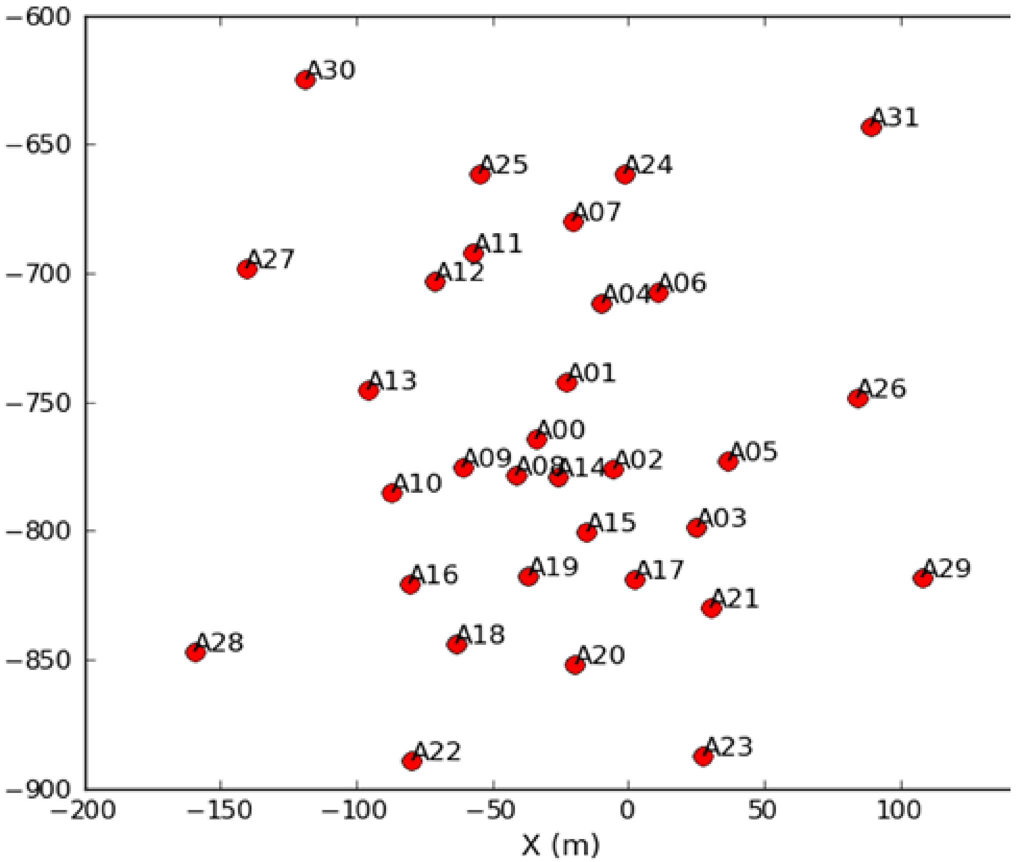
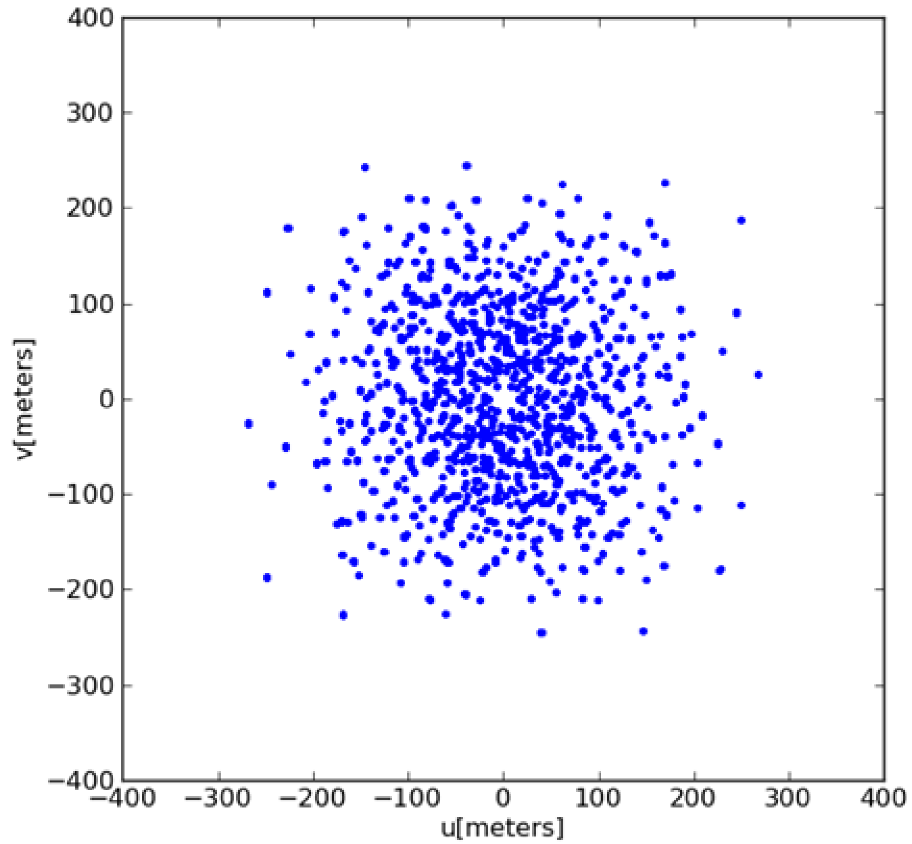


Image Credit: Ian Czekala (NRAO)

32 Antennas – Instantaneous





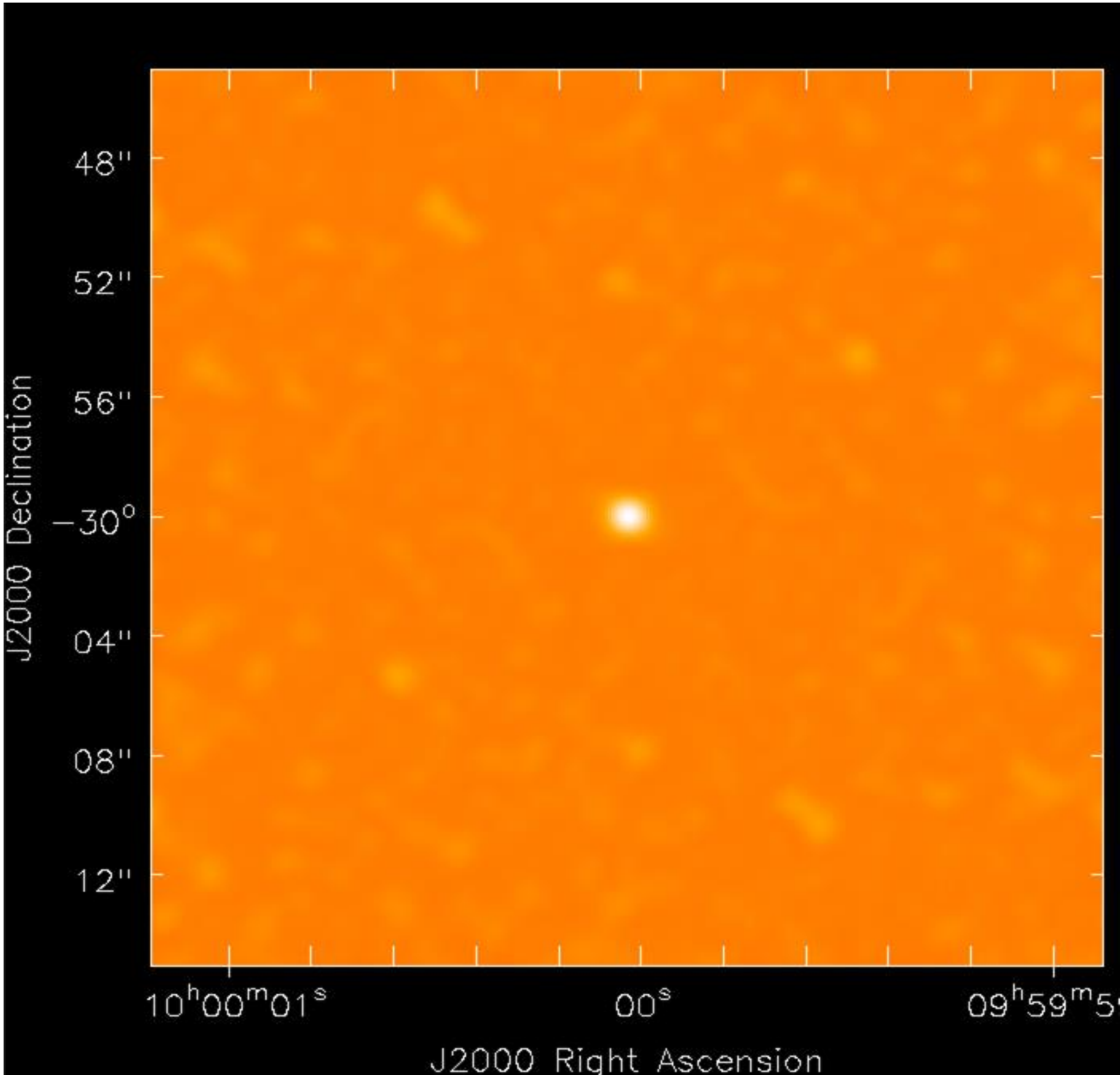
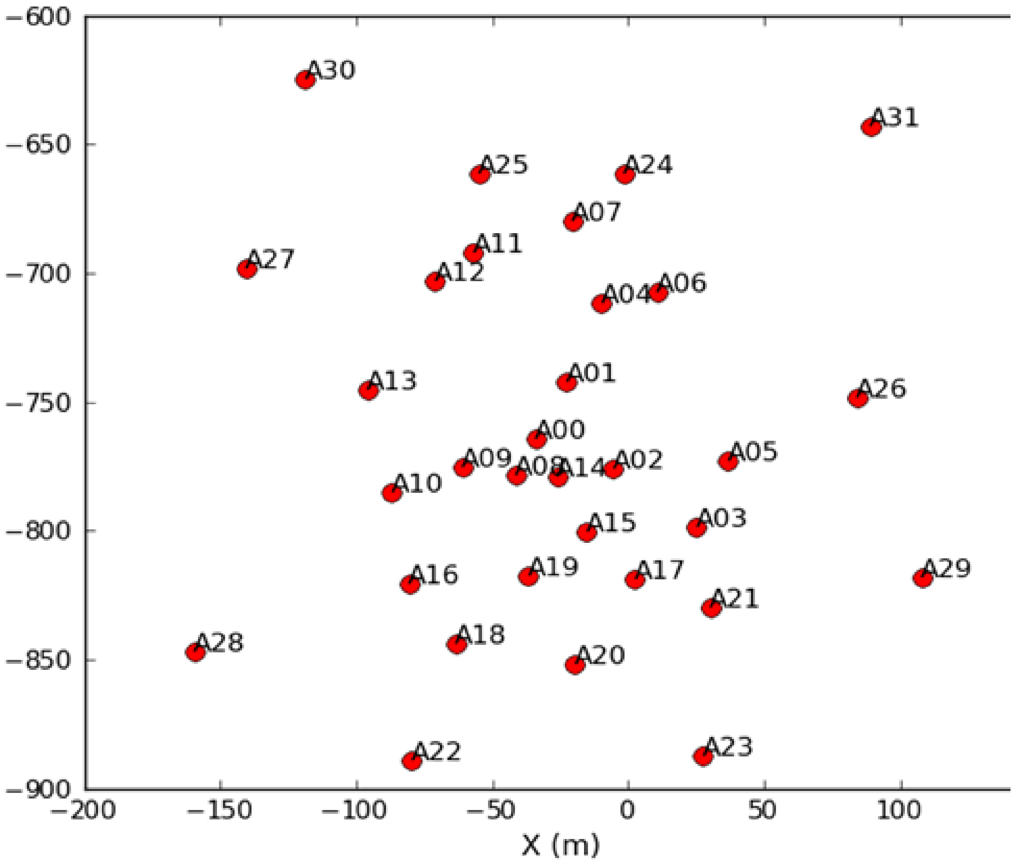
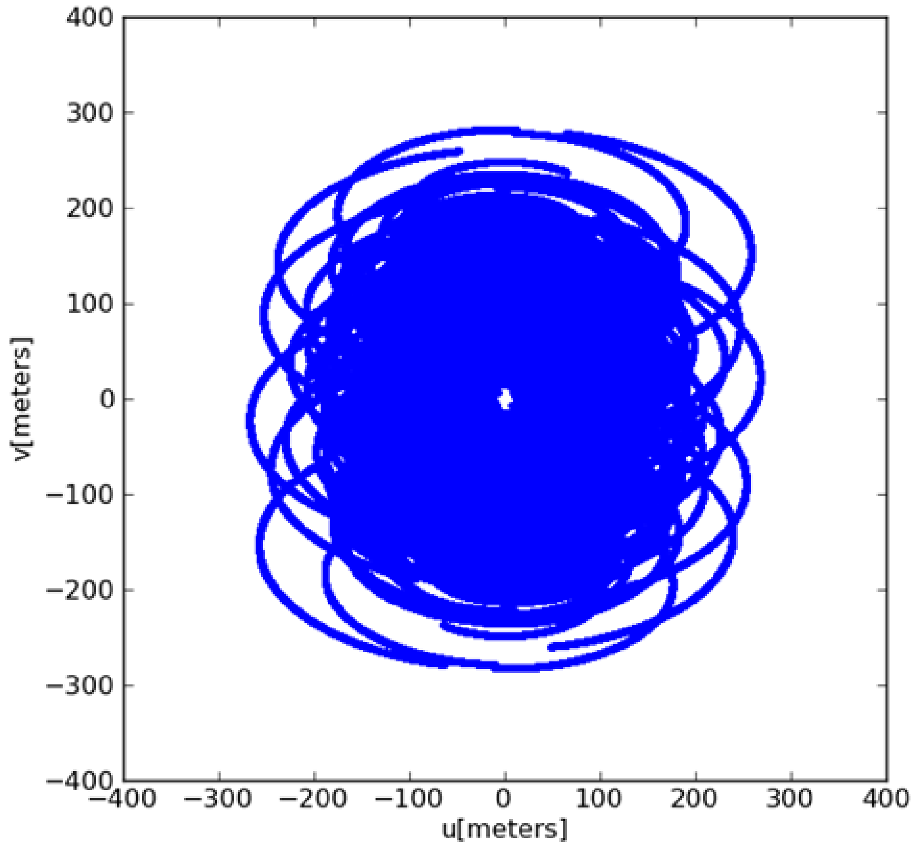


Image Credit: Ian Czekala (NRAO)

32 Antennas – 8 hours





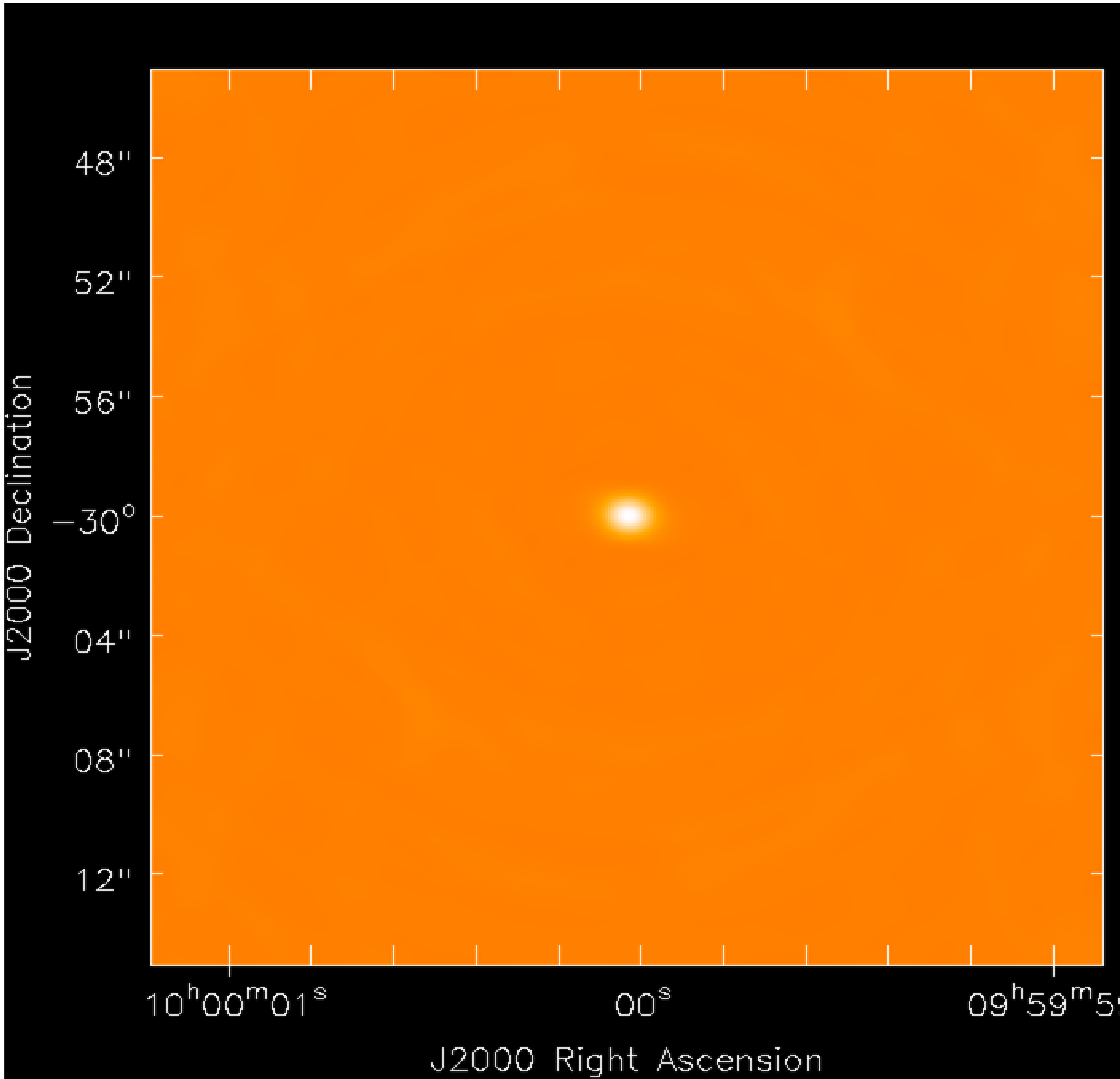


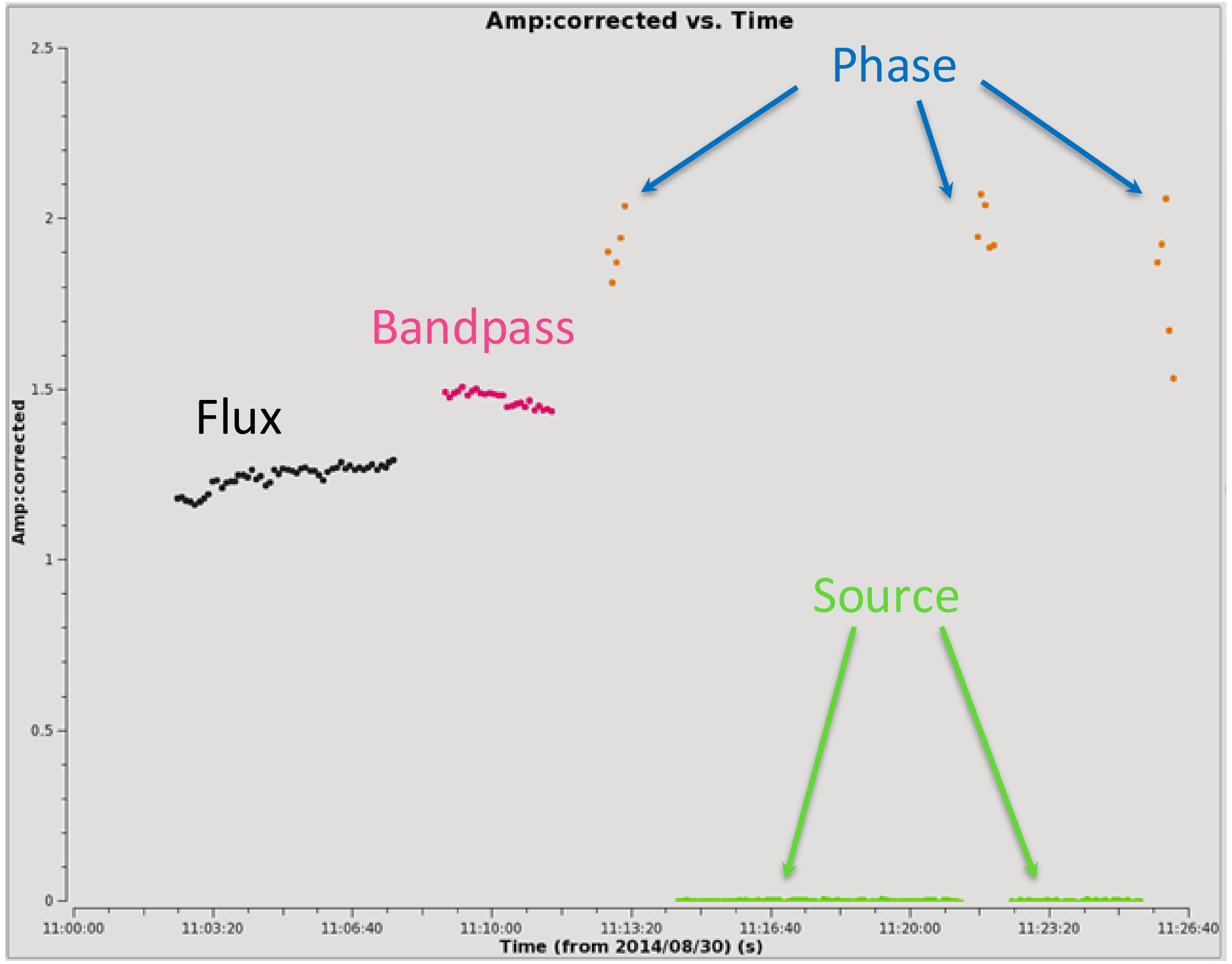
Image Credit: Ian Czekala (NRAO)

Interferometric observations

Calibration

- Calibrators are typically point-like sources

Uncalibrated Data



Calibrated Data