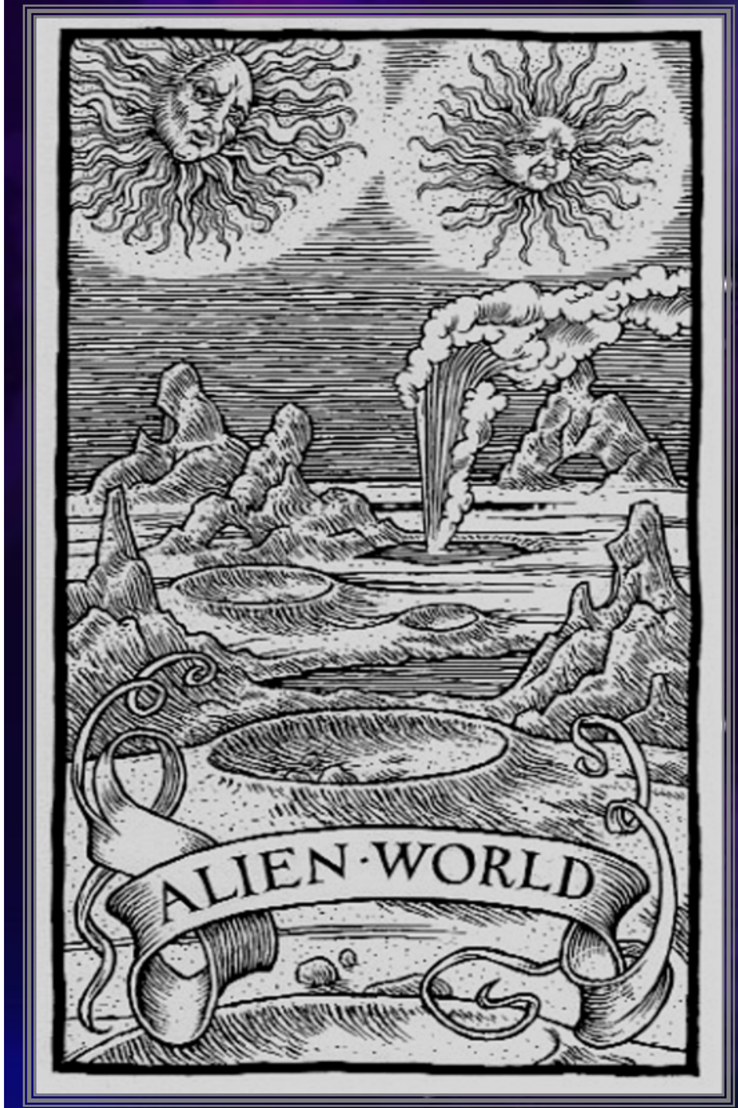


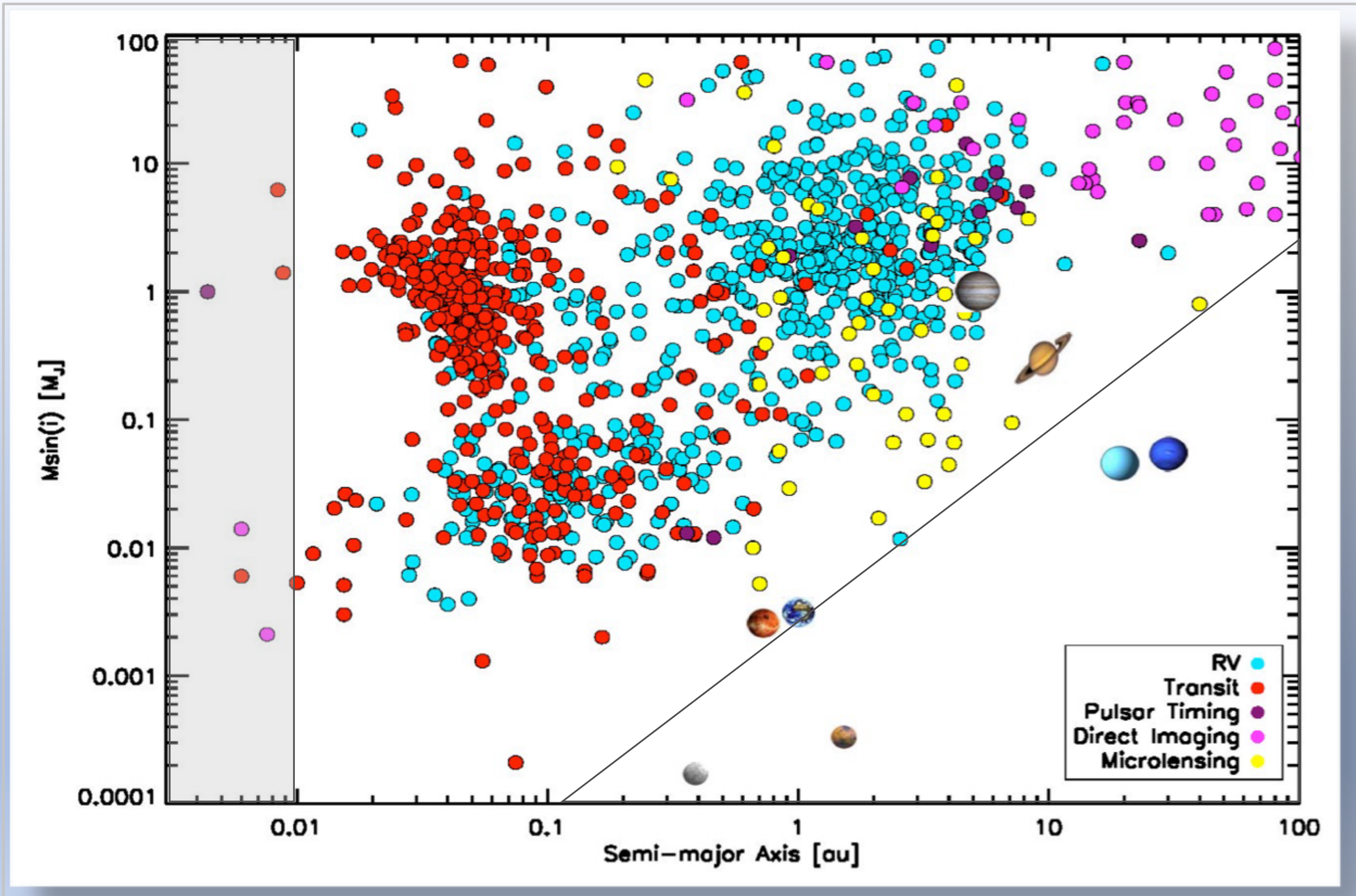
ASIAA Summer Student Program 2026

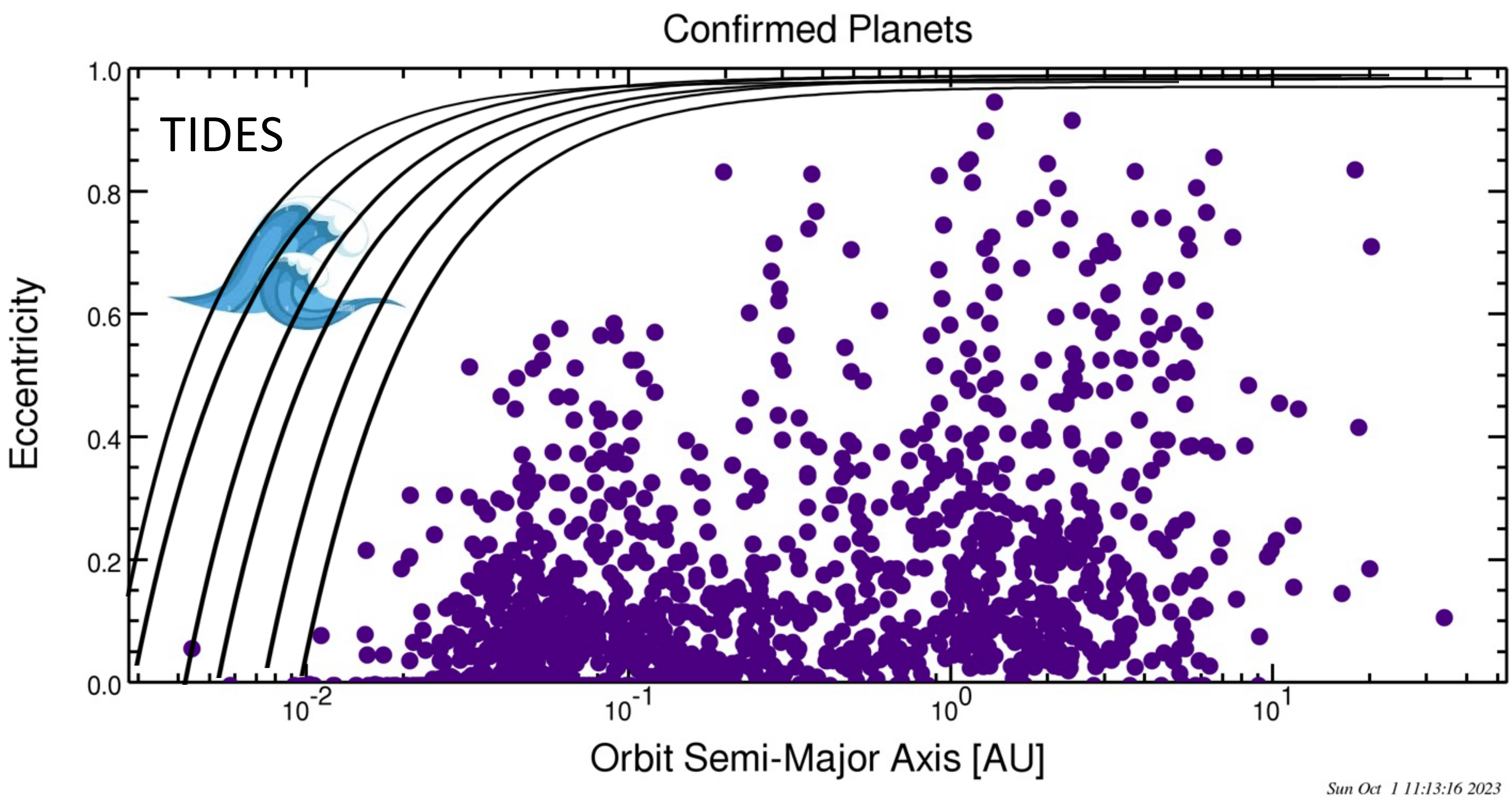
Lecture Two: Planet Formation

Fred C Adams
University of Michigan

Outline

- Overview of the Observational Landscape of Exoplanets
- Review of Circumstellar Disk properties → Initial Conditions
- Two mechanisms: Disk Instability vs Core Accretion
- Disk Instability produces Brown Dwarfs at 100 AU
- Core Accretion produces the Majority of Planets (if rocky planets are considered failed attempts of the paradigm)
- The Extreme Importance of SuperEarths
- Late stages of Core Accretion
- The end of accretion and the Planetary Mass Function
- Organizational Considerations for Planetary Systems


100
10
1
0.1
0.01
0.001
0.0001
Msin(i) [MJ]
0.01
0.1
1
10
100
Semi-major Axis [au]
RV
Transit
Pulsar Timing
Direct Imaging
Microlensing


Confirmed Planets
TIDES
1.0
0.8
0.6
0.4
0.2
0.0
Eccentricity
10^-2
10^-1
10^0
10^1
Orbit Semi-Major Axis [AU]
Sun Oct 1 11:13:16 2023

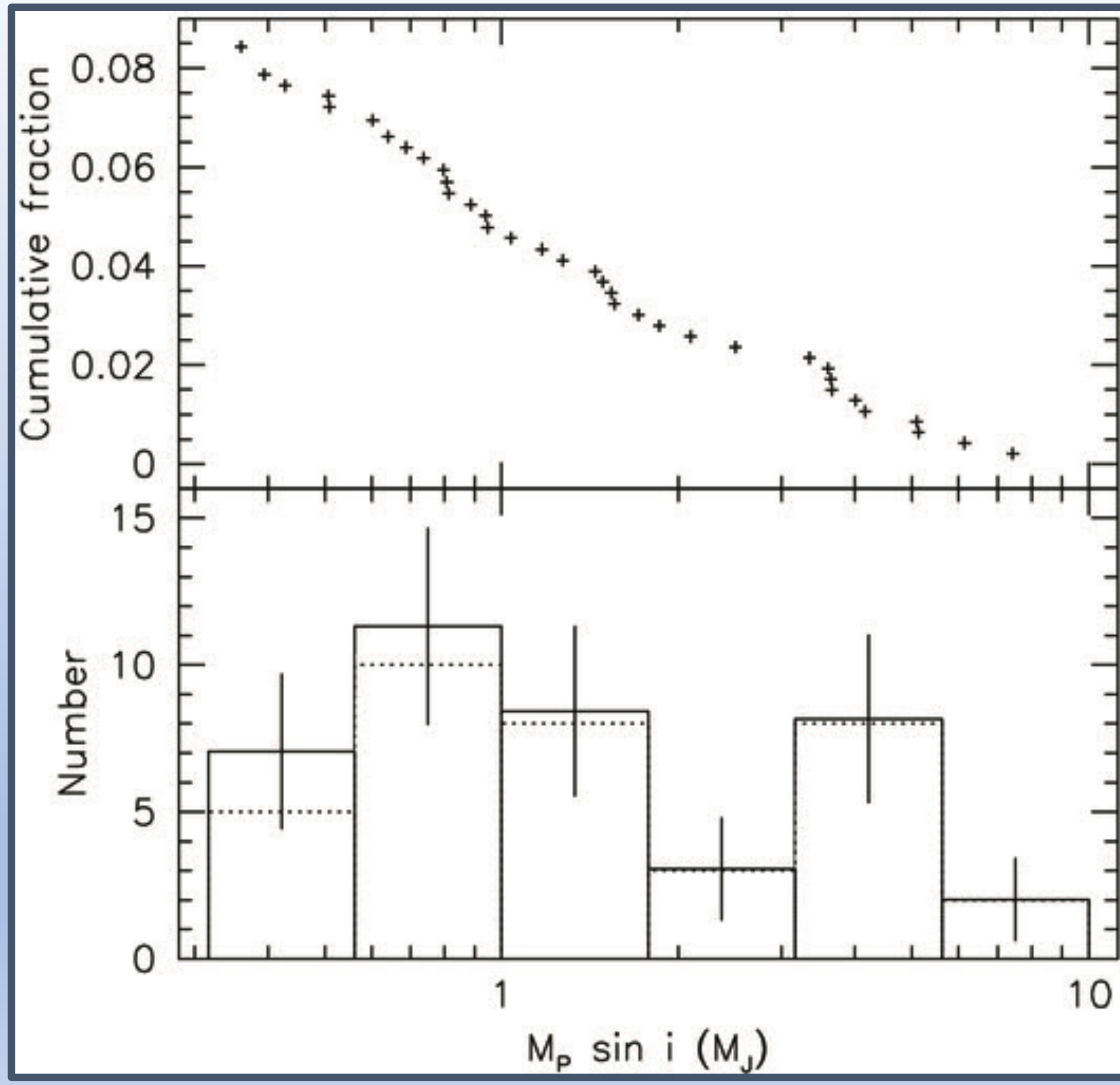
PMF Observed

$$\frac{dP}{dM} = \frac{A}{M^p}$$

$$p = 1.1 - 1.5$$

$$M = 0.3 - 10M_J$$

A. Cumming et al. 2008, PASP, 120, 531



PMF Observed

$$\frac{dP}{dM} = \frac{A}{M^p}$$

$$p = 1.1 - 1.5$$

$$M = 0.3 - 10M_J$$

B. Fulton et al. 2021

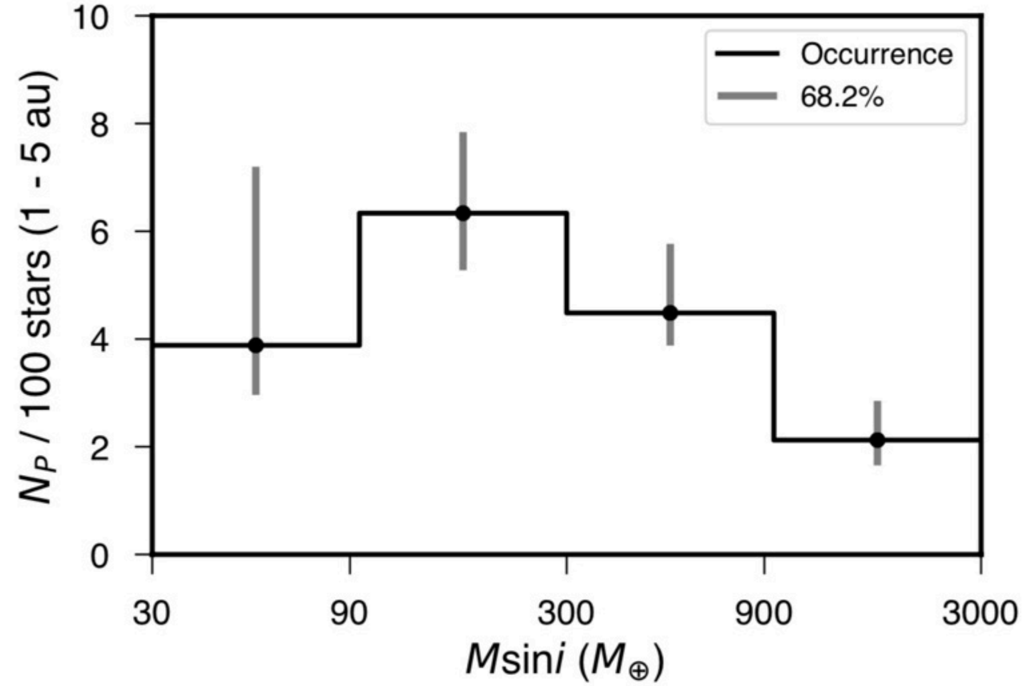
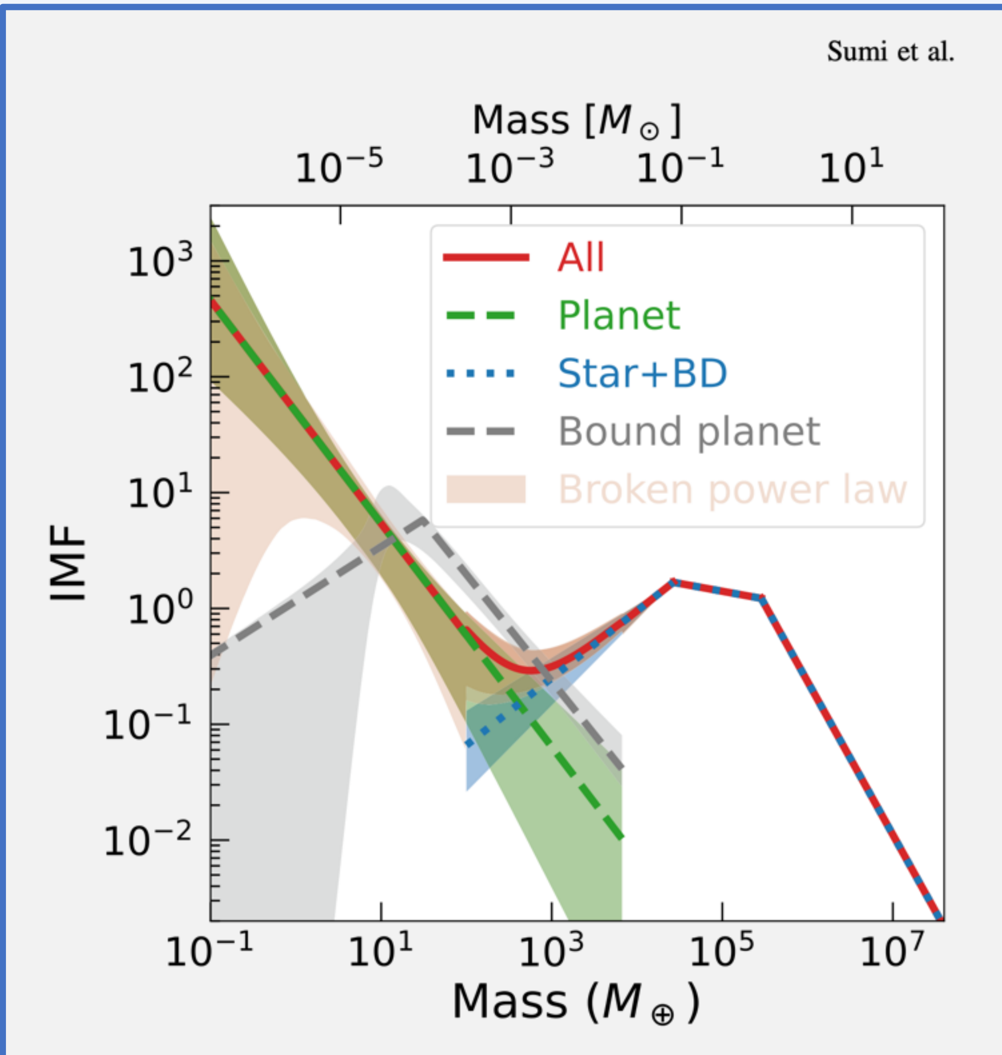


Figure 6. Planet occurrence within 1–5 au with respect to $M \sin i$. Steps and dots show maximum posterior values, and vertical lines show 15.9–84.1% confidence intervals. The mass function is constant within 30–900 $M_\oplus$, and falls off beyond 900 $M_\oplus$.



Star + Planet Mass Function

- Stars and Planets separate distributions
- More planets than stars
- PMF may or may not turn over
- Brown dwarfs are somewhat rarer

MASS SCALES:

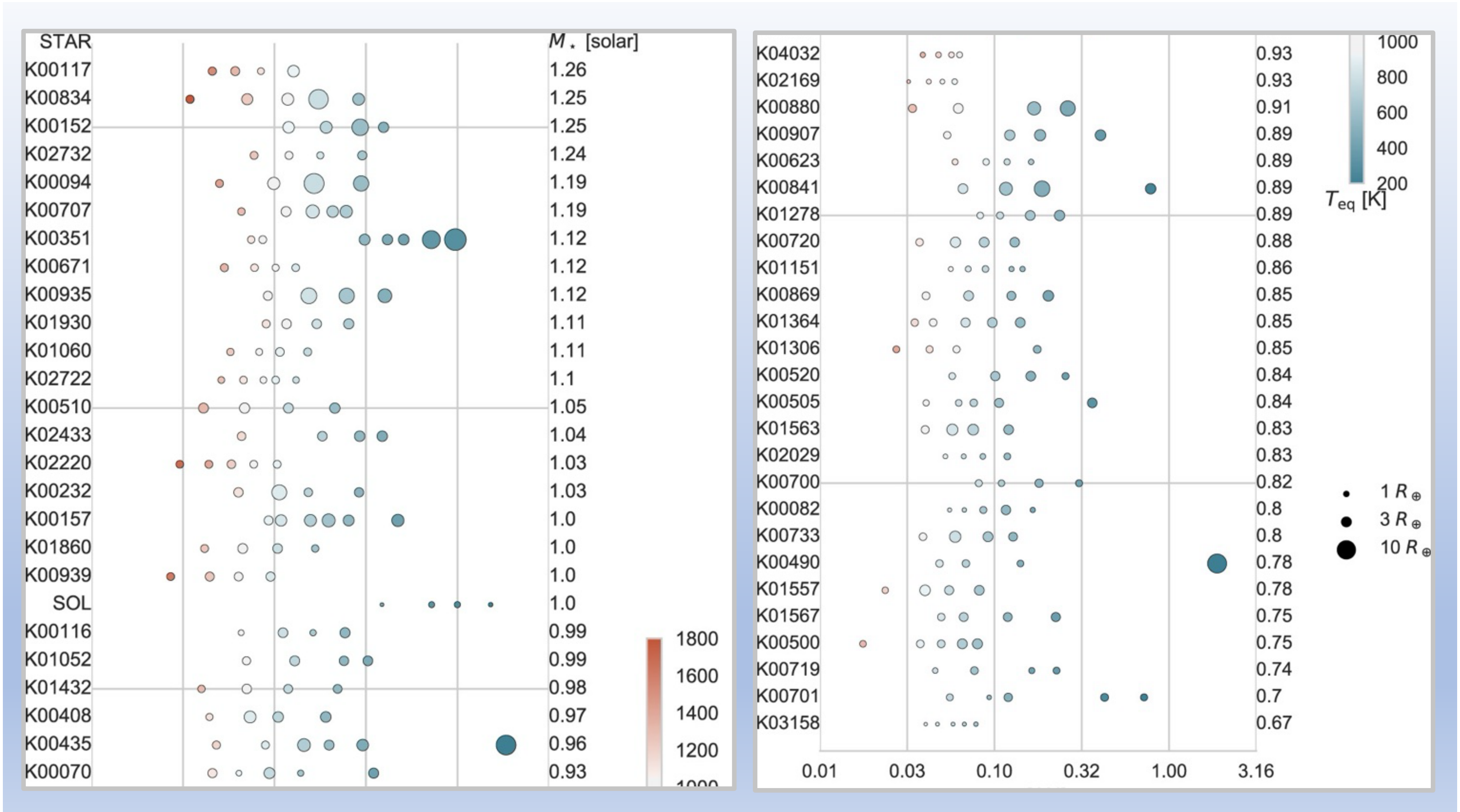
Stars: peak at 0.3 M_{Sun}

Brown Dwarfs: minimum at 10 M_{jup}

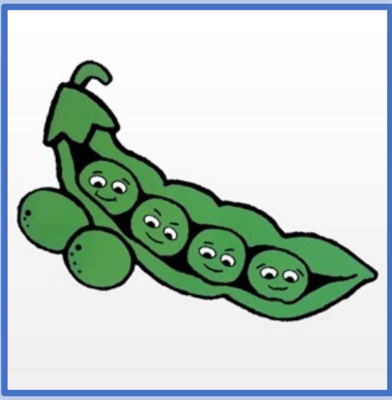
Power-law shape for planet masses

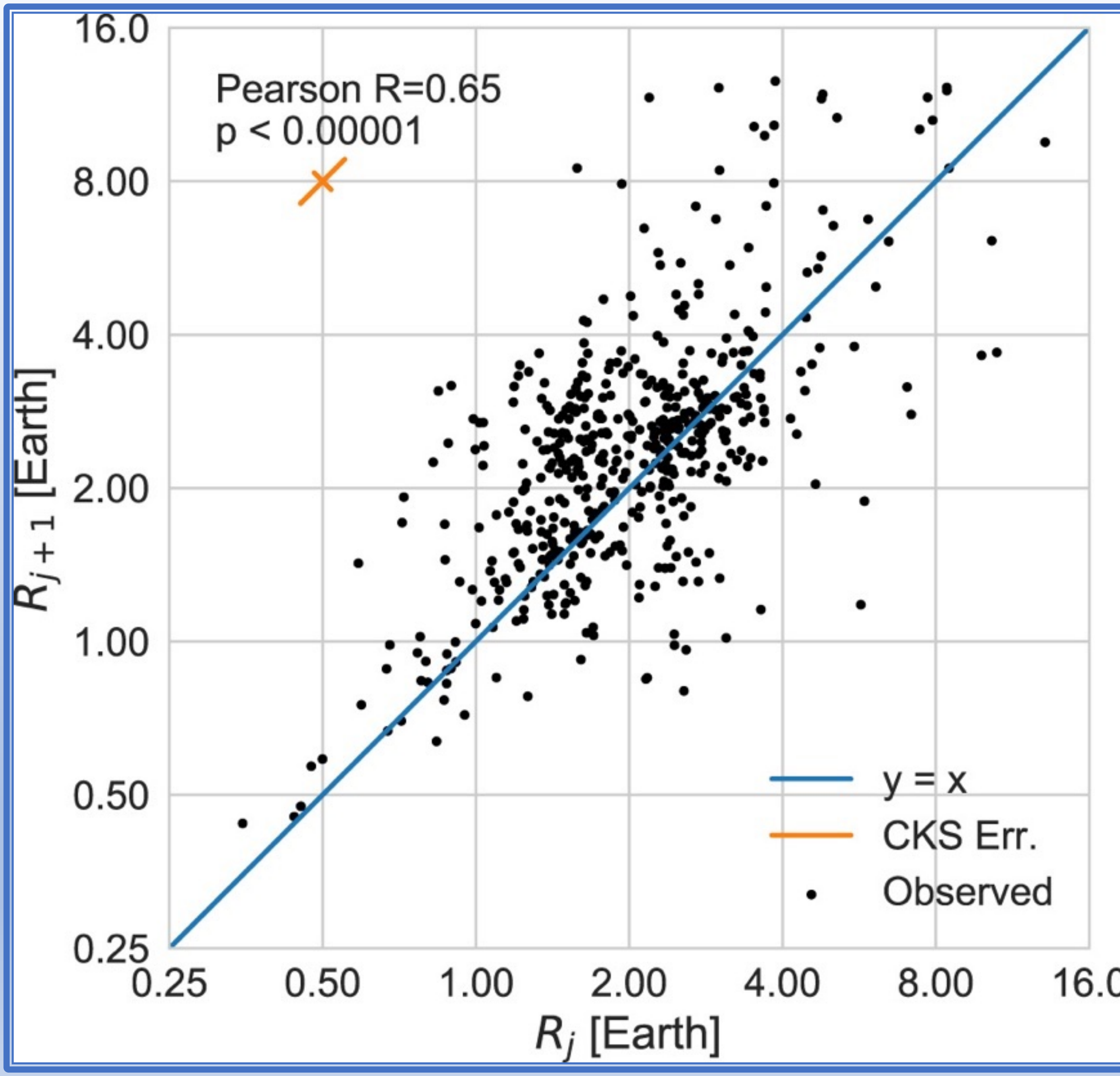
Planets: possible scale at 10 M_{earth}

(much uncertainty)

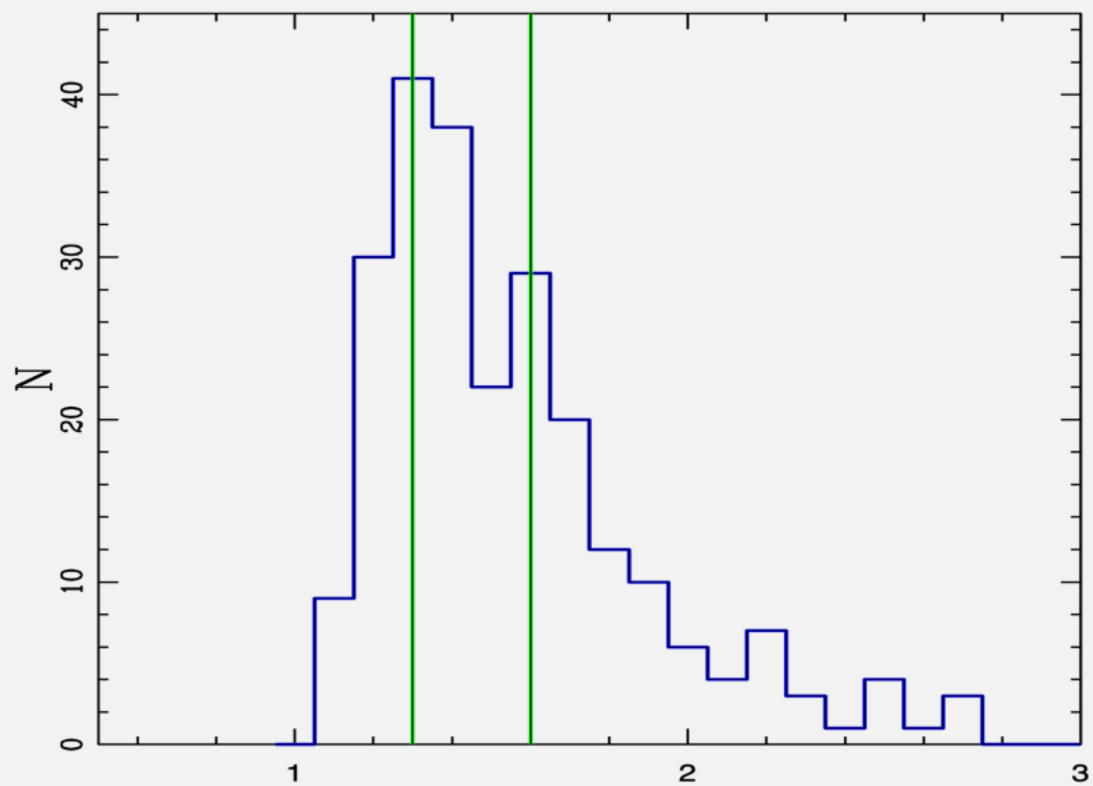

STAR
$M_\star$ [solar]
K00117 1.26
K00834 1.25
K00152 1.25
K02732 1.24
K00094 1.19
K00707 1.19
K00351 1.12
K00671 1.12
K00935 1.12
K01930 1.11
K01060 1.11
K02722 1.1
K00510 1.05
K02433 1.04
K02220 1.03
K00232 1.03
K00157 1.0
K01860 1.0
K00939 1.0
SOL 1.0
K00116 0.99
K01052 0.99
K01432 0.98
K00408 0.97
K00435 0.96
K00070 0.93
1800
1600
1400
1200
1000
K04032 0.93
K02169 0.93
K00880 0.91
K00907 0.89
K00623 0.89
K00841 0.89
K01278 0.89
K00720 0.88
K01151 0.86
K00869 0.85
K01364 0.85
K01306 0.85
K00520 0.84
K00505 0.84
K01563 0.83
K02029 0.83
K00700 0.82
K00082 0.8
K00733 0.8
K00490 0.78
K01557 0.78
K01567 0.75
K00500 0.75
K00719 0.74
K00701 0.7
K03158 0.67
0.01 0.03 0.10 0.32 1.00 3.16
1000
800
600
400
200
T_{eq} [K]
1 $R_\oplus$
3 $R_\oplus$
10 $R_\oplus$

Planetary pairs in N> 3 systems tend to have similar radii (Weiss et al. 2018) and similar masses (Wang 2017)



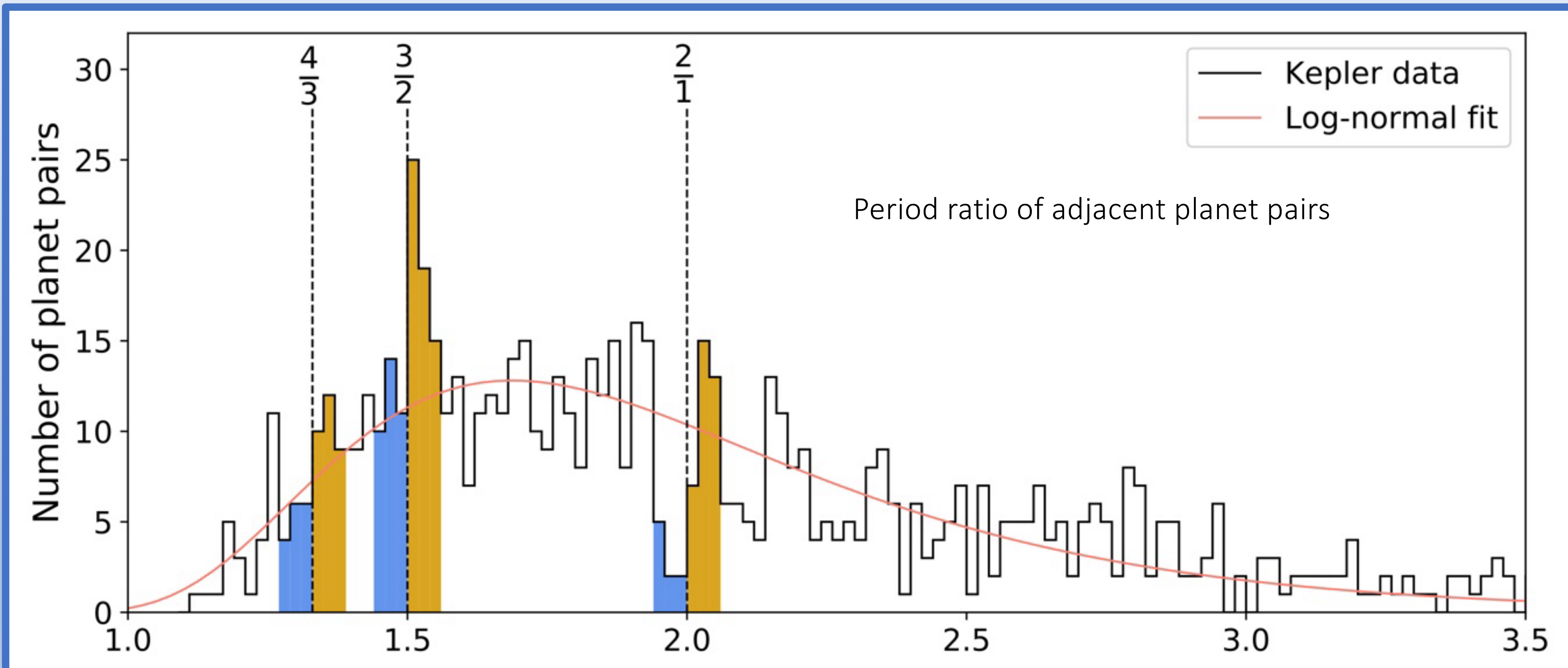


Distribution of Spacing Parameters

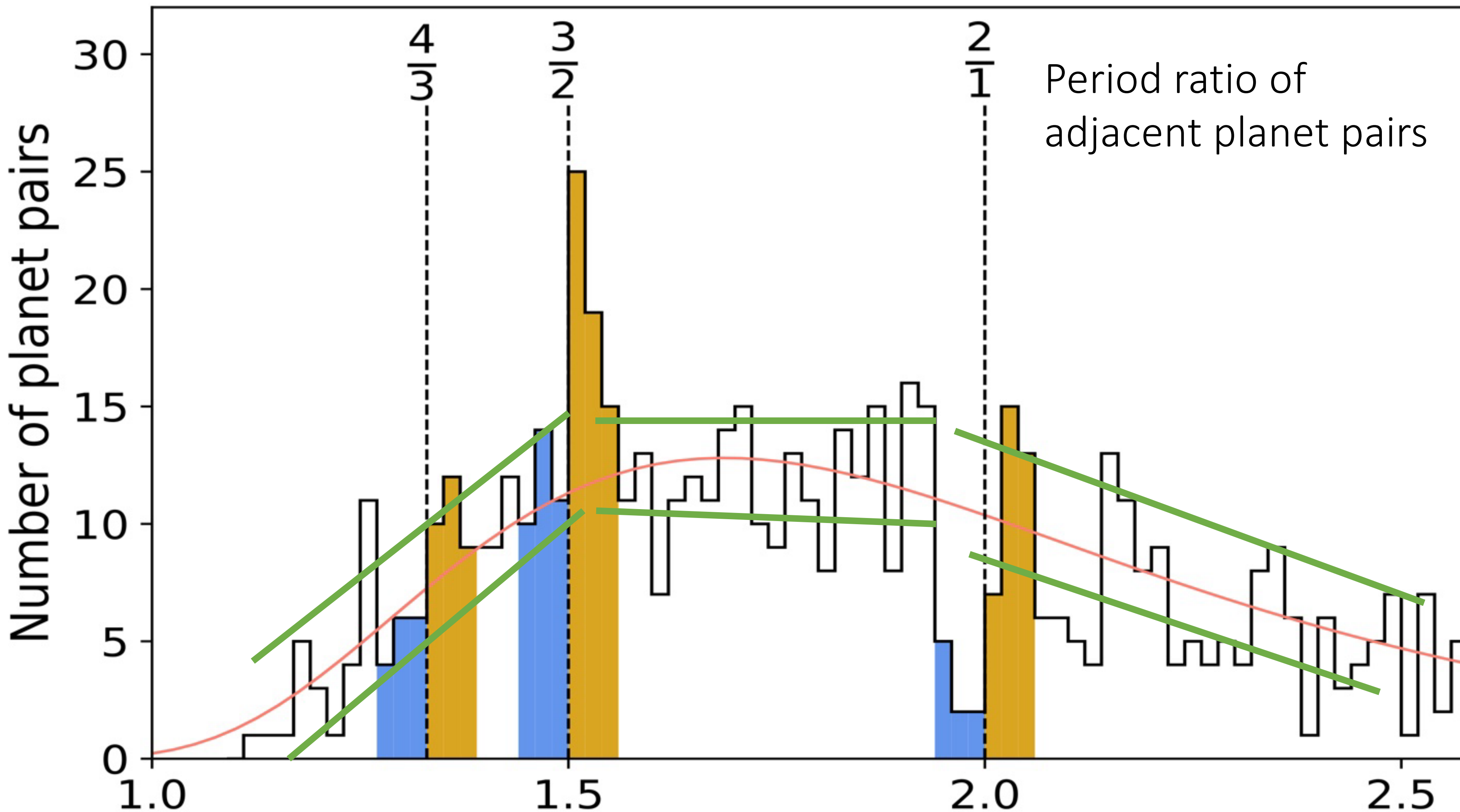


$$\Lambda = a_2/a_1$$

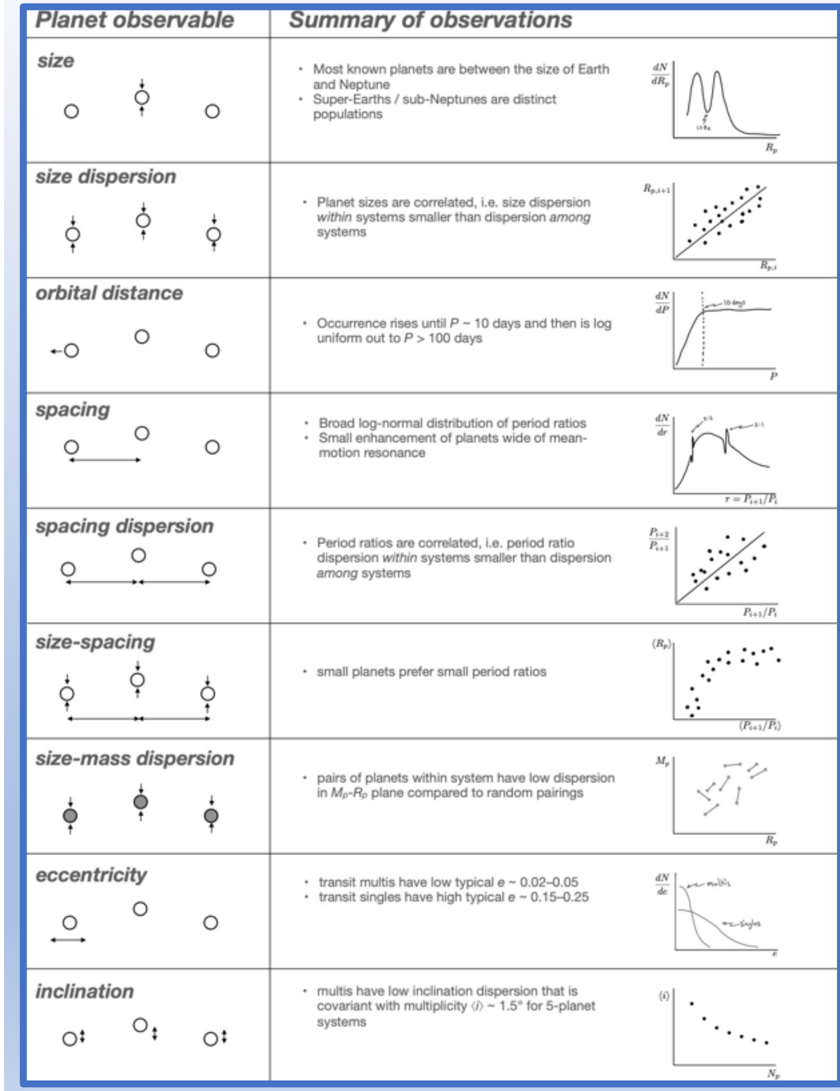
Distribution of Period Ratios



Predominantly non-resonant, but slight overabundance wide-of-resonance



Number of planet pairs
30
25
20
15
10
5
0
1.0
1.5
2.0
2.5
4/3
3/2
2/1
Period ratio of adjacent planet pairs

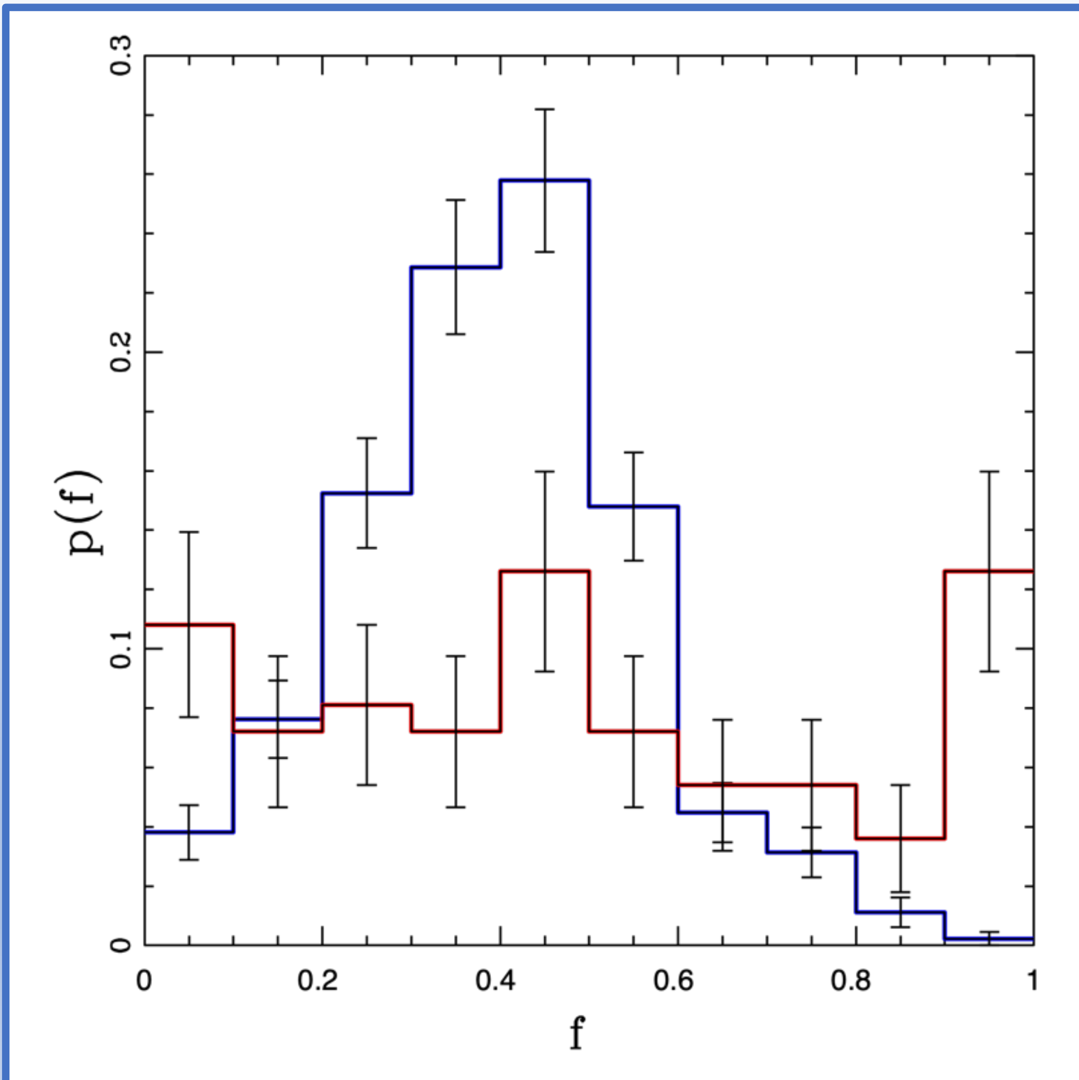


Planet observable	Summary of observations
size	• Most known planets are between the size of Earth and Neptune • Super-Earths / sub-Neptunes are distinct populations
size dispersion	• Planet sizes are correlated, i.e. size dispersion *within* systems smaller than dispersion *among* systems
orbital distance	• Occurrence rises until $P \sim 10$ days and then is log uniform out to $P > 100$ days
spacing	• Broad log-normal distribution of period ratios • Small enhancement of planets wide of mean-motion resonance
spacing dispersion	• Period ratios are correlated, i.e. period ratio dispersion *within* systems smaller than dispersion *among* systems
size-spacing	• small planets prefer small period ratios
size-mass dispersion	• pairs of planets within system have low dispersion in M_p–R_p plane compared to random pairings
eccentricity	• transit multis have low typical $e \sim 0.02$–0.05 • transit singles have high typical $e \sim 0.15$–0.25
inclination	• multis have low inclination dispersion that is covariant with multiplicity $\langle i \rangle \sim 1.5°$ for 5-planet systems

Peas in a Pod Properties

- Equal mass (radius) planets
- Circular orbits (low eccentricity)
- Planets orbit in the same plane (i = 0)
- Regular orbital spacing (a_2/a_1 = const)
- Not in mean motion resonance
- Composite surface density: $\sum = 1/r^2$
- Planet masses: Earth → Neptune

(Weiss et al. 2023, Protostars & Planets VII)



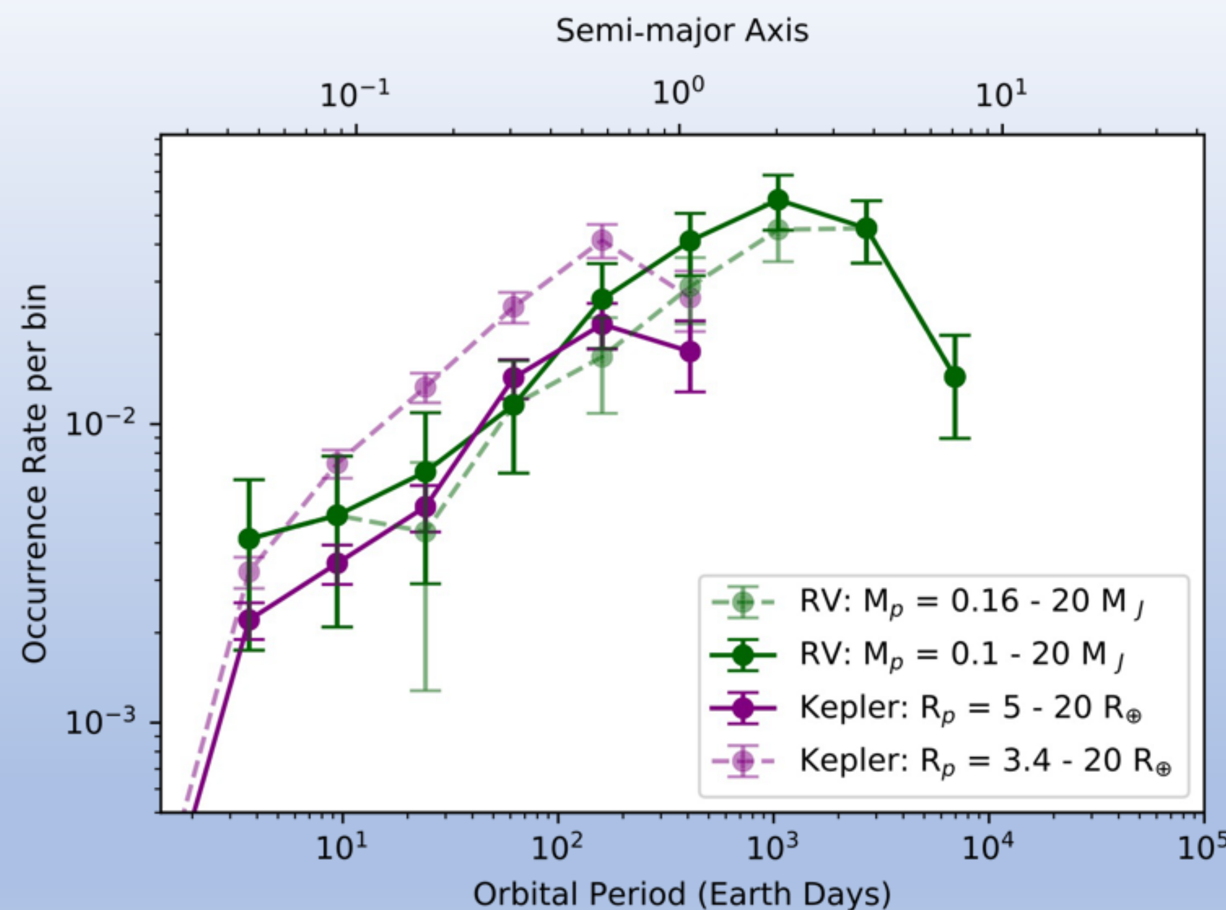
Distributions of Observed Mass Fractions

Blue: $m_T < 40m_\oplus$

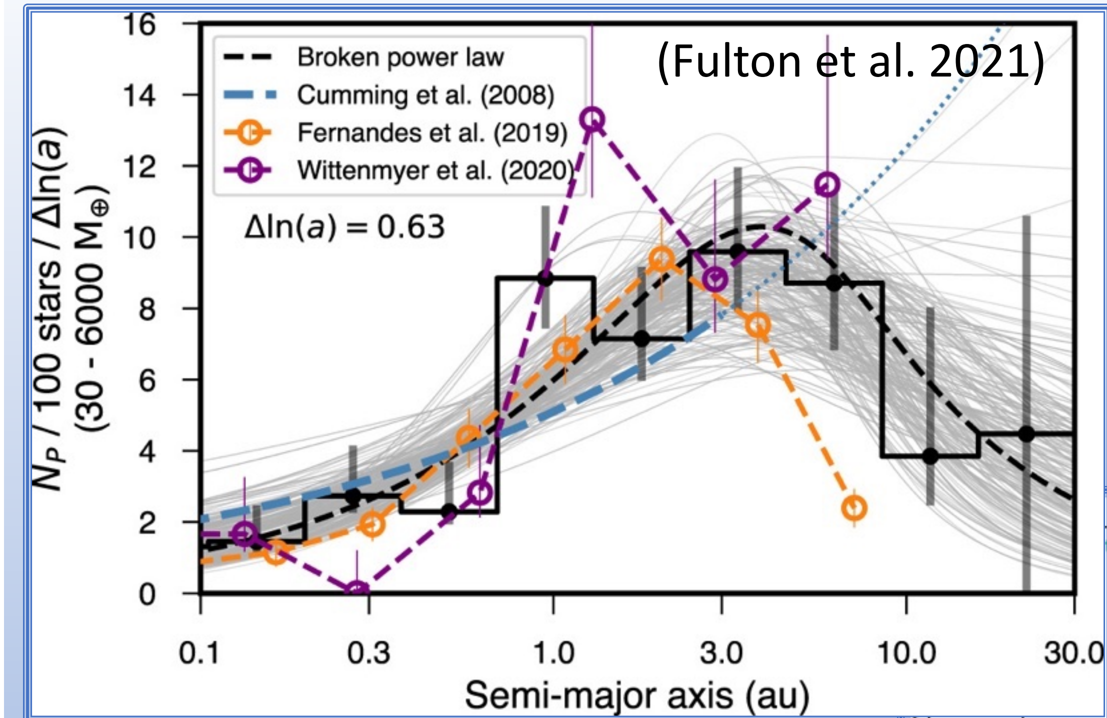
Red: $m_T > 40m_\oplus$

Giant Planet Occurrence Rates

Current data suggest that the Occurrence Rate for Giant Planets peaks around 3-5 AU.
→ Consistent with old idea that planets tend to form near ice line.



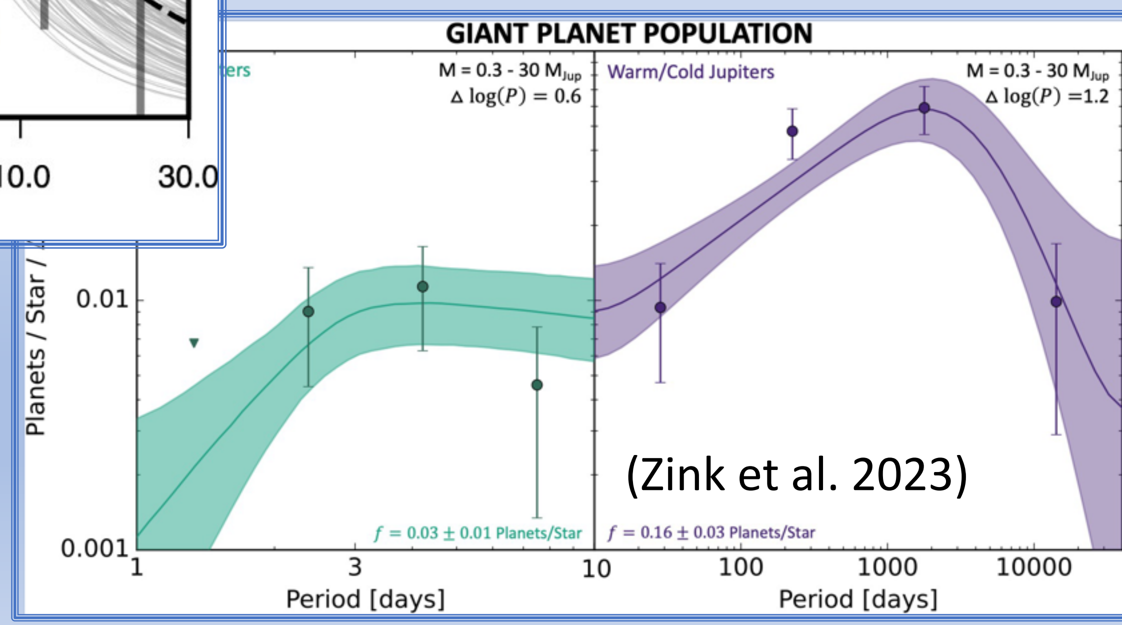
(Fernandes et al. 2019)

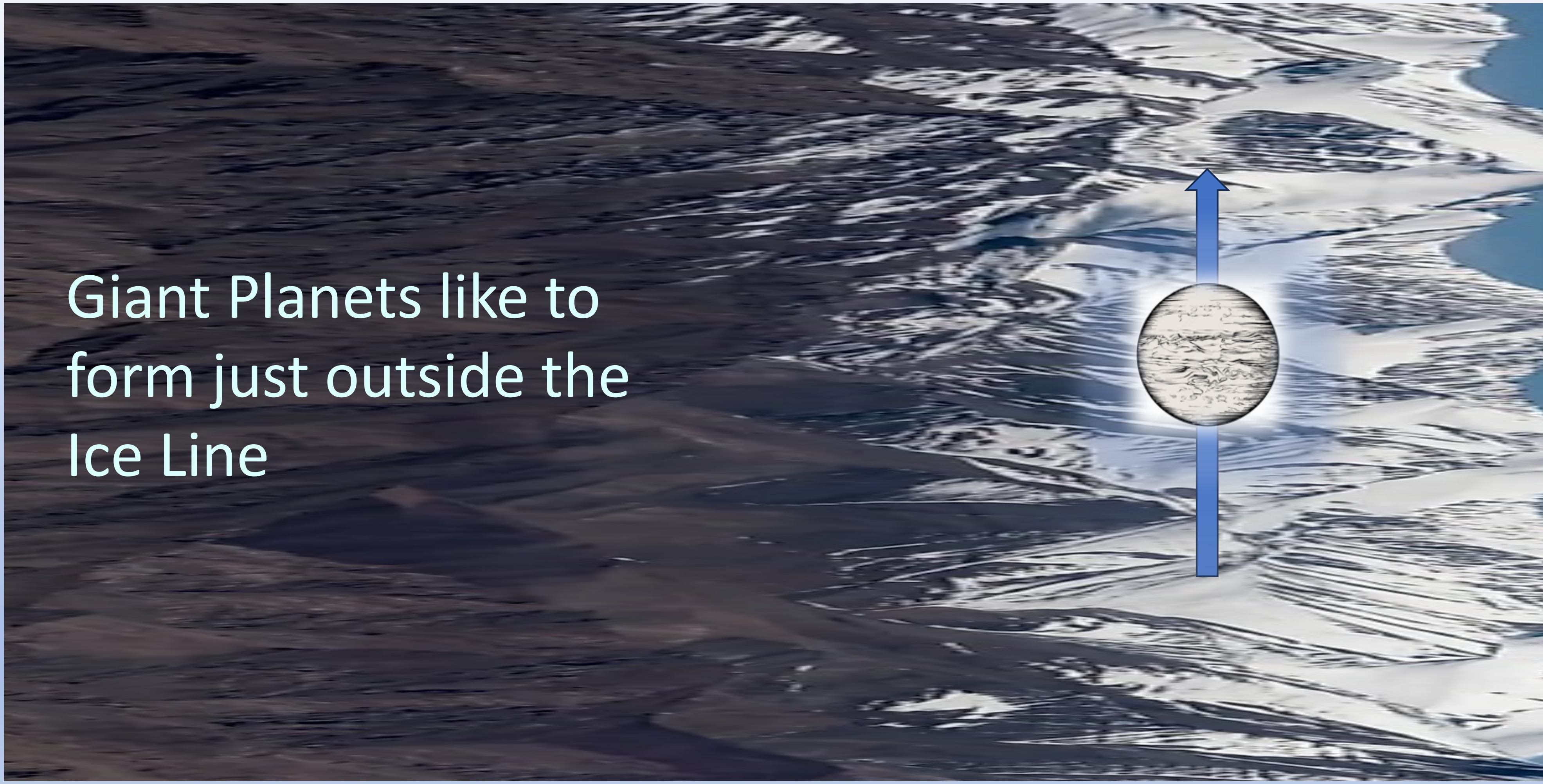


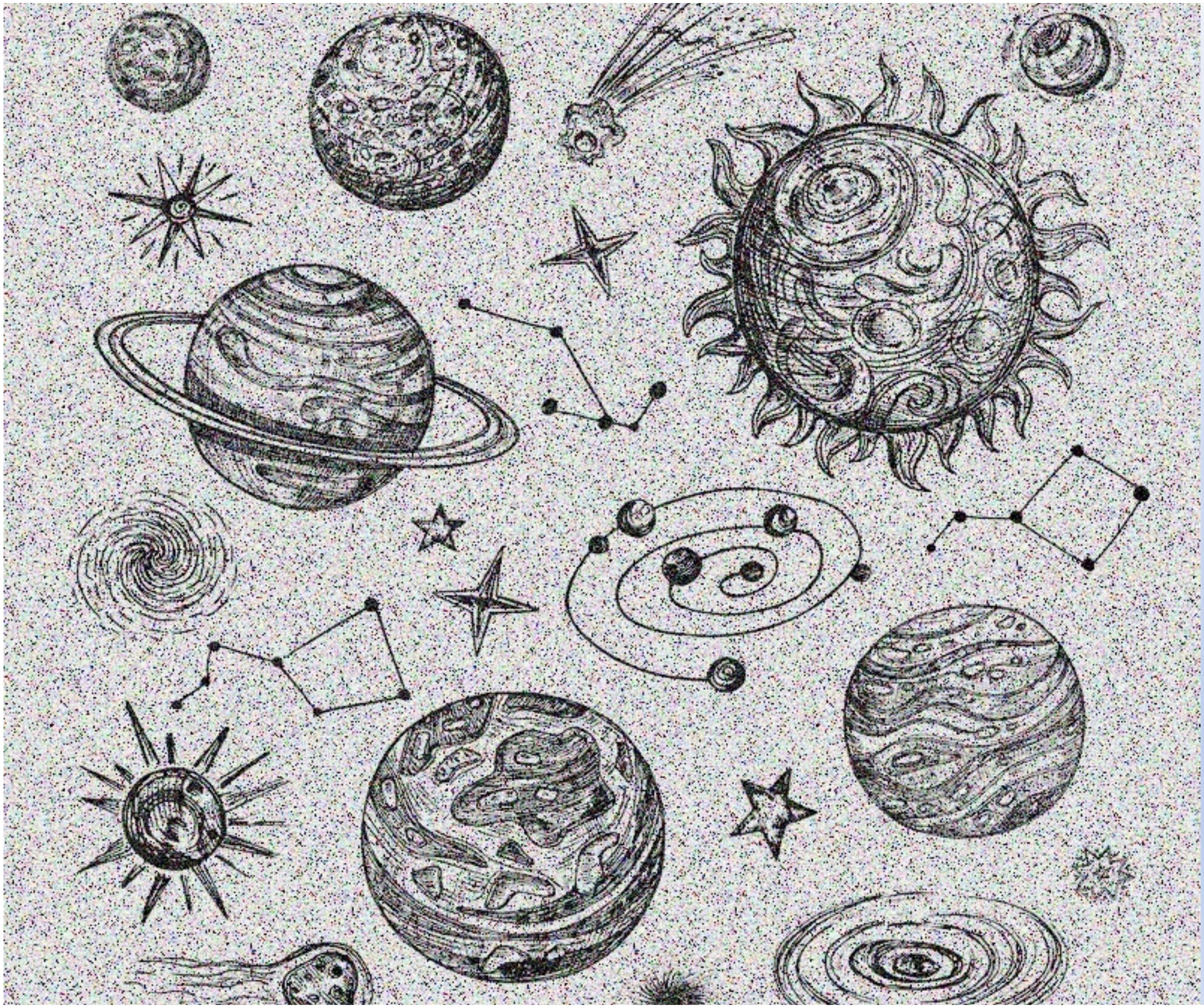
Current data suggest that the Occurrence Rate for Giant Planets peaks around 3-5 AU.

→ Consistent with old idea that planets tend to form near ice line.

Giant Planet Occurrence Rates




Giant Planets like to form just outside the Ice Line



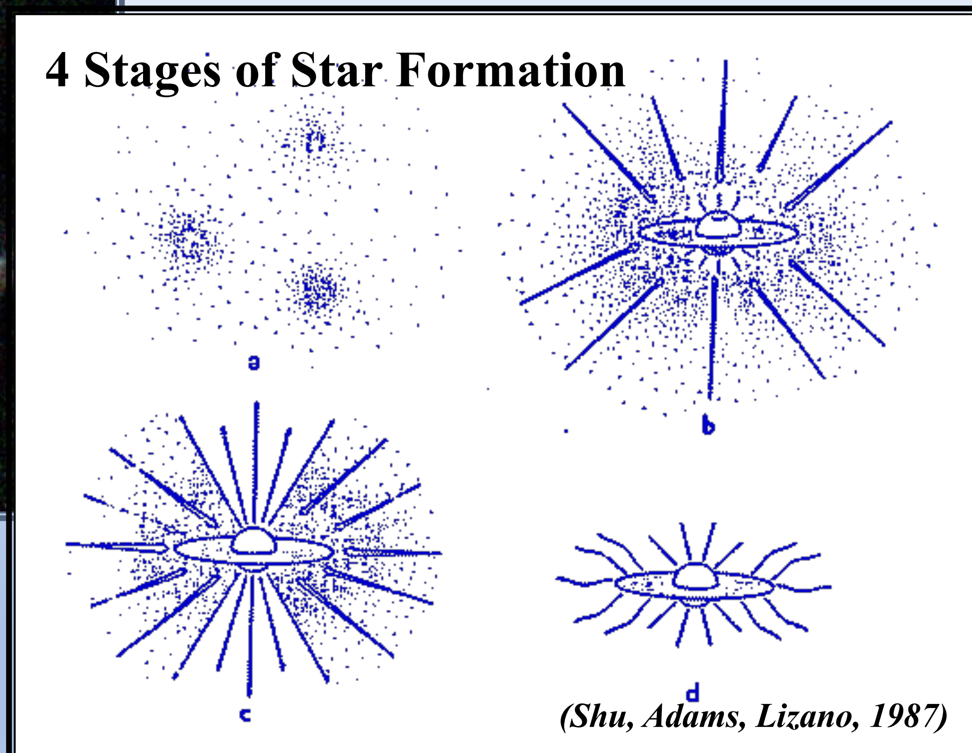
How does the Galaxy produce this diverse collection of Planetary Systems with its observed Distributions of properties?

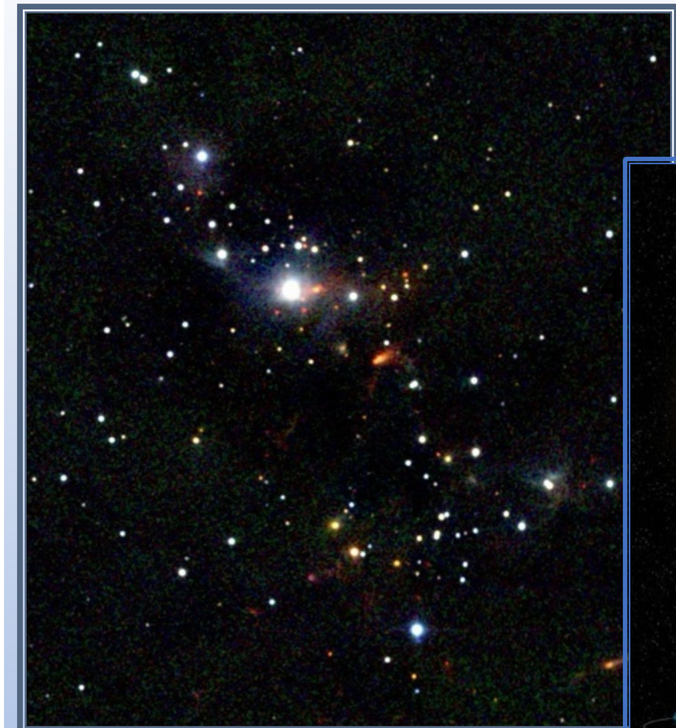
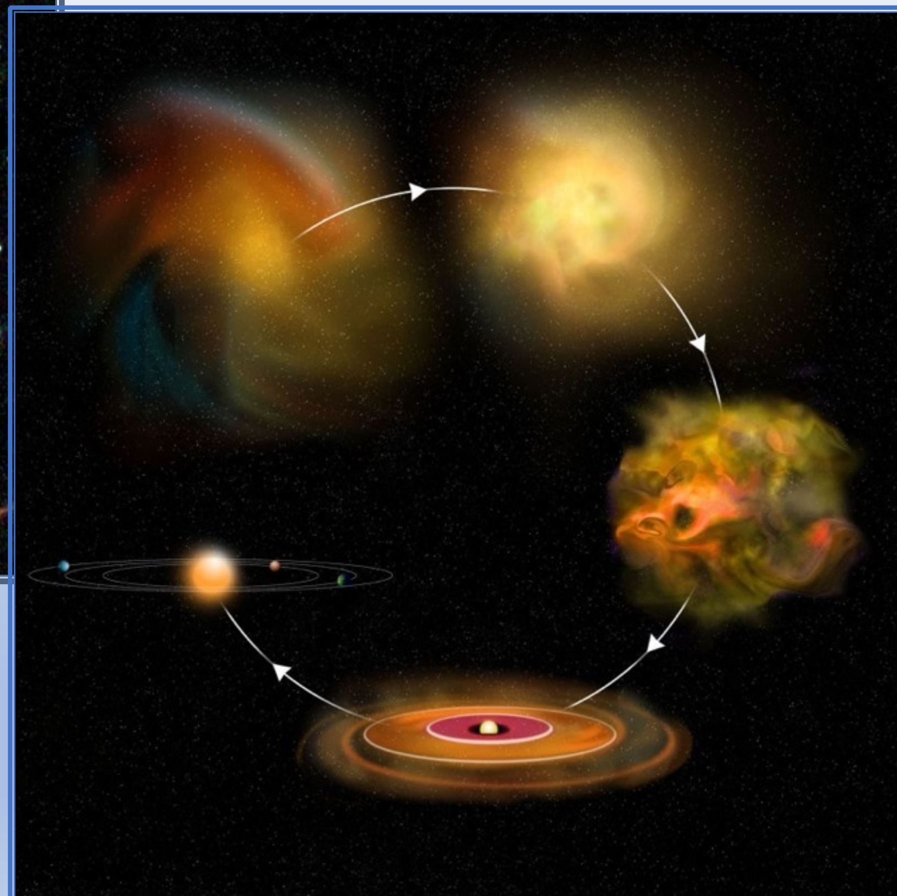
Distribution of masses

Orbital elements (a,e,i)

System architectures

Initial Conditions

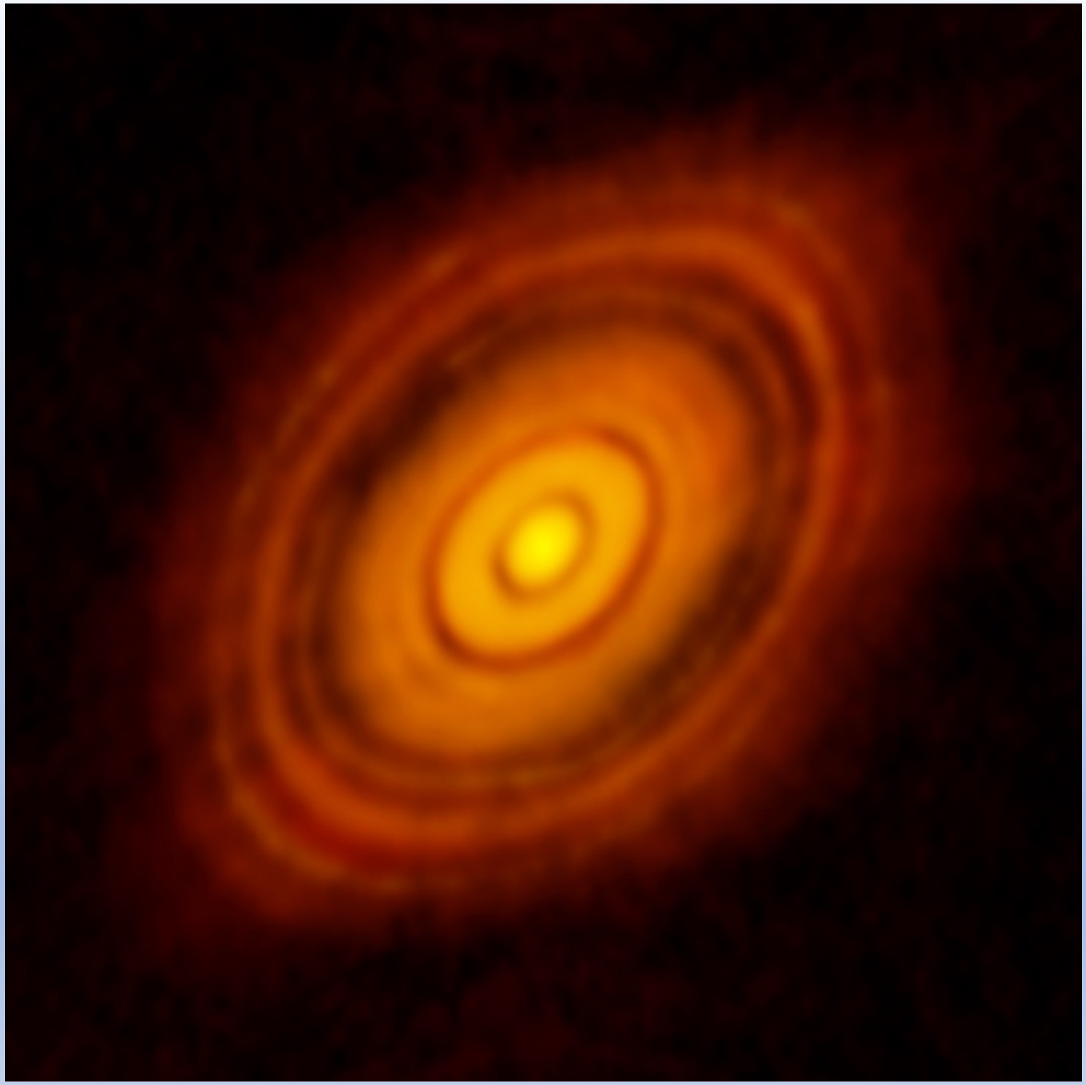



Star Formation Process → Circumstellar Disks w. Radii R = 10-100 AU

- Conservation of angular momentum: Cores have velocity gradients of 1 km/s/pc → disk size 10 – 100 AU
- Spectral Energy Distribution of protostars imply modest column density, which imply disk-like regions of size 10 – 100 AU
- Our Solar System has disk size 10 – 100 AU
- Spectral Energy Distributions of T Tauri disks systems have turnovers which imply disk edge at size scale 10 – 100 AU
- We can now measure disk sizes directly (ALMA) find 10 – 100 AU

Circumstellar Disks Observed: HL Tau

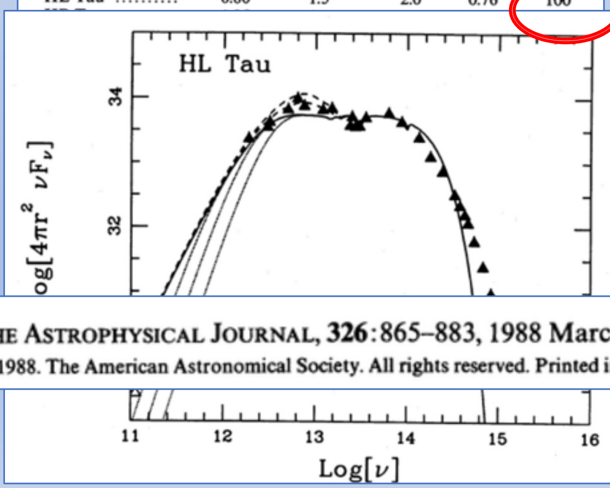


Observation of Circumstellar Disk: HL Tau

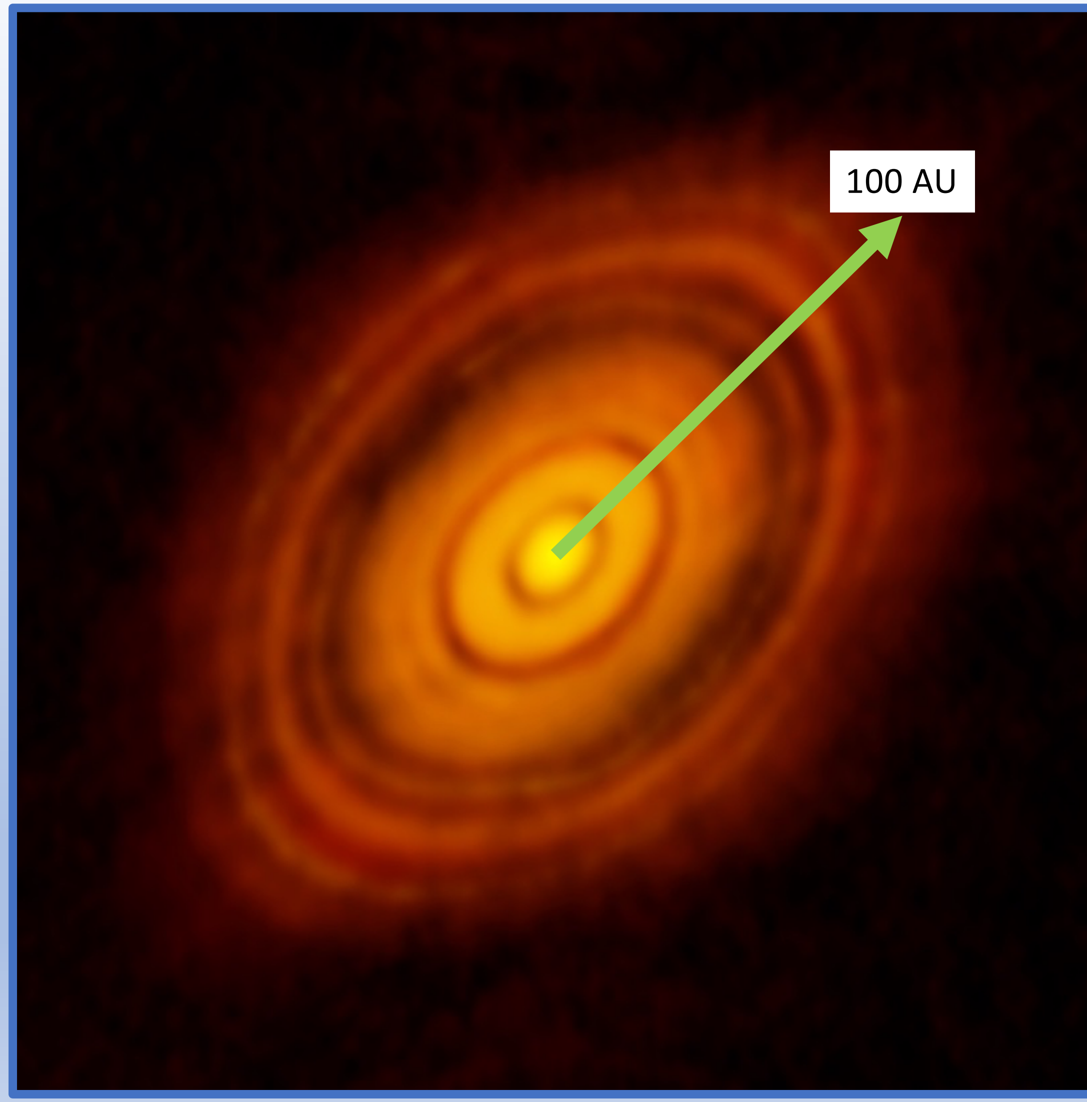
TABLE 2

Output Parameters for Star/Disk Systems

Source	M_* ($M_\odot$)	R_* (10^{11} cm)	M_D ($M_\odot$)	M_Q ($M_\odot$)	$R_{D\min}$ (AU)
T Tau	1.80	2.1	0.2	1.3	120
DG Tau	0.95	0.62	1.0	0.78	75
DG Tau[a]	0.40	1.5	1.0	0.50	75
HL Tau	0.80	1.5	2.0	0.76	100



The Astrophysical Journal, 326:865–883, 1988 March 15



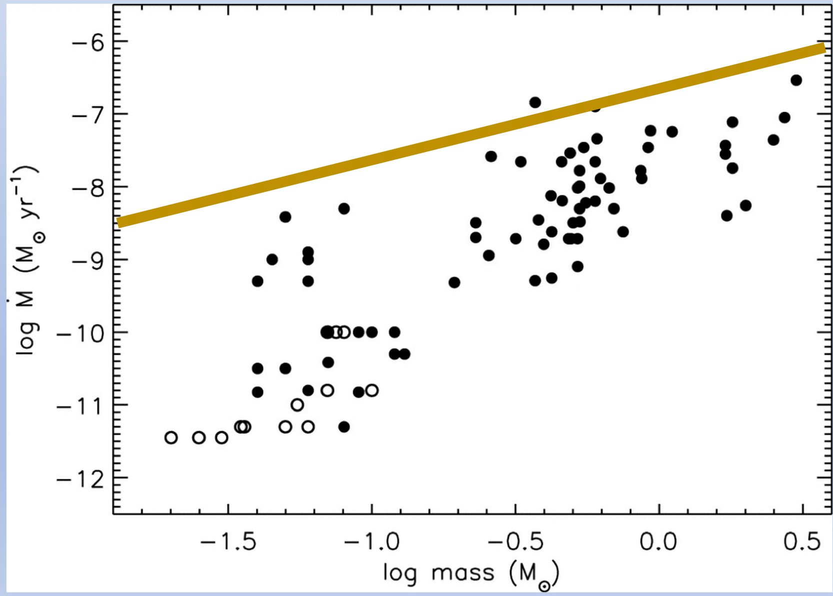
Disk Masses

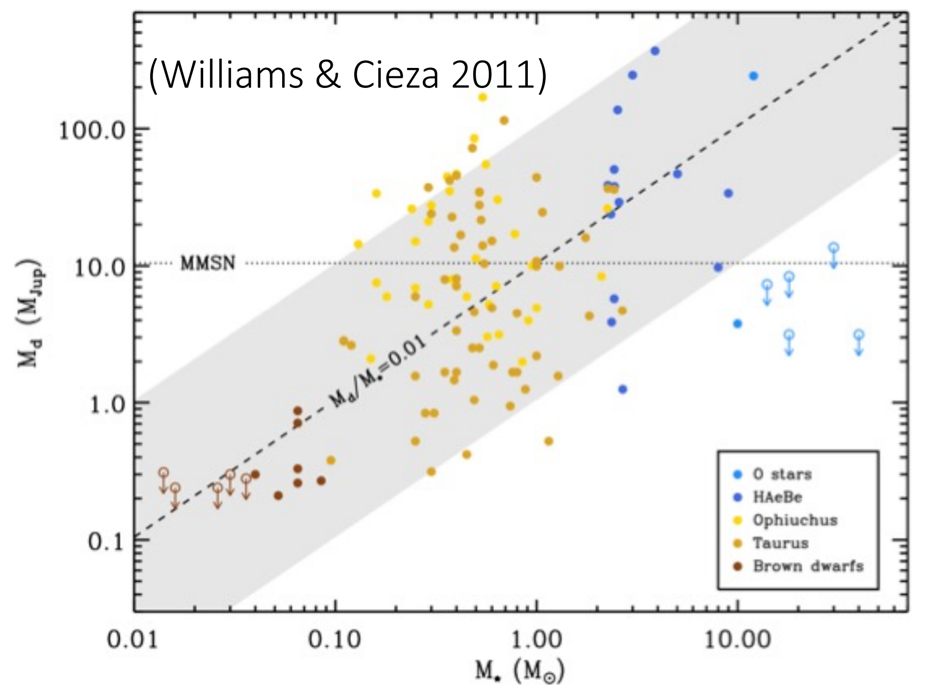
Maximum Mass Nebula (Shu+1990):
$M_{disk} / (M_{disk} + M_{star}) < 3/(4\pi)$

Disk masses inferred to fall below gravitational instability limit $M_{disk} = 0.1\ M_{star}$

Minimum Mass Solar Nebula: $M_{min} = 0.03\ M_{sun}$

Fig. 5. from Measuring Accretion in Young Substellar Objects: Approaching the Planetary Mass Regime
Muzerolle et al. 2005 ApJ 625 906 doi:10.1086/429483
https://dx.doi.org/10.1086/429483

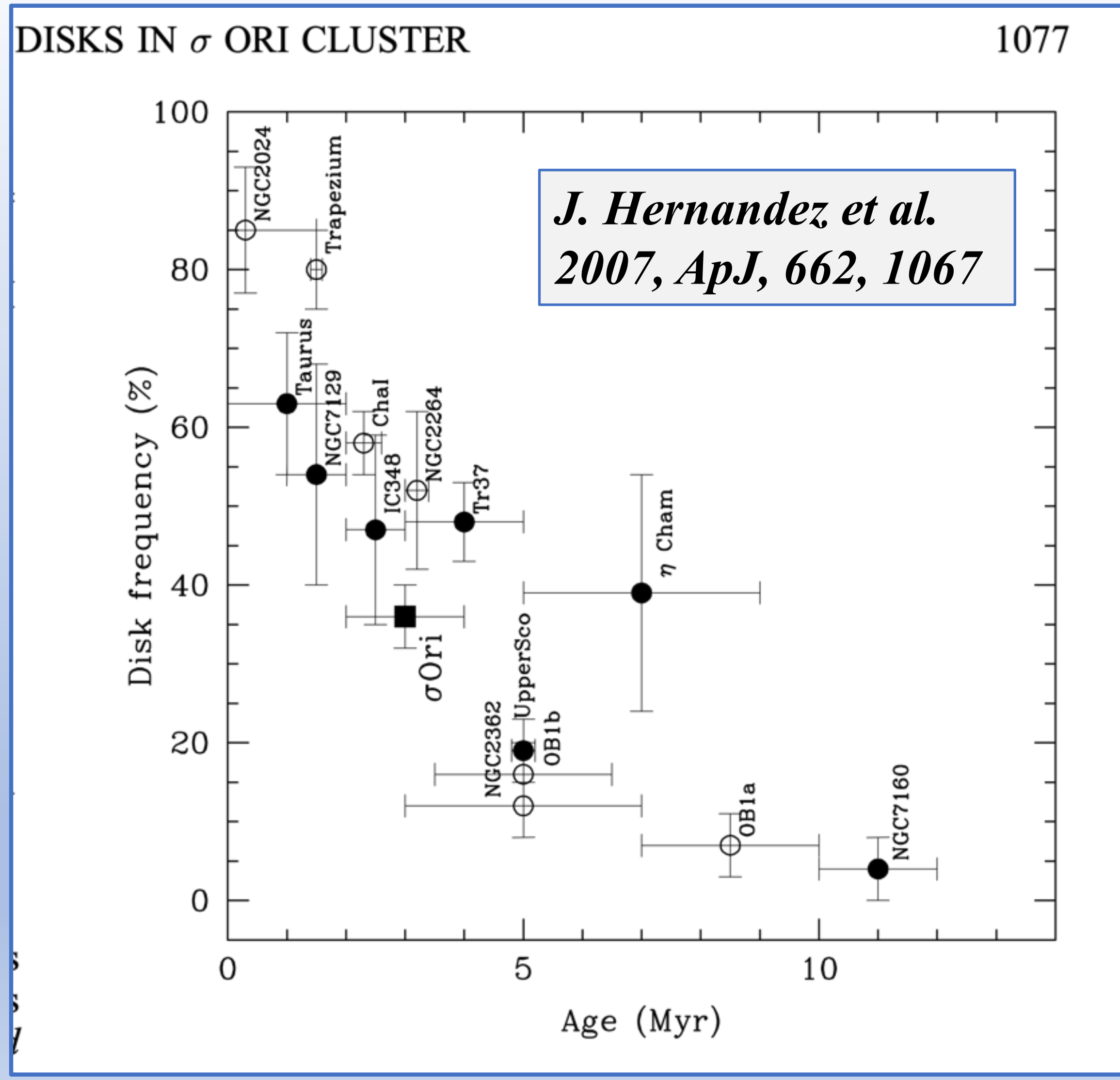


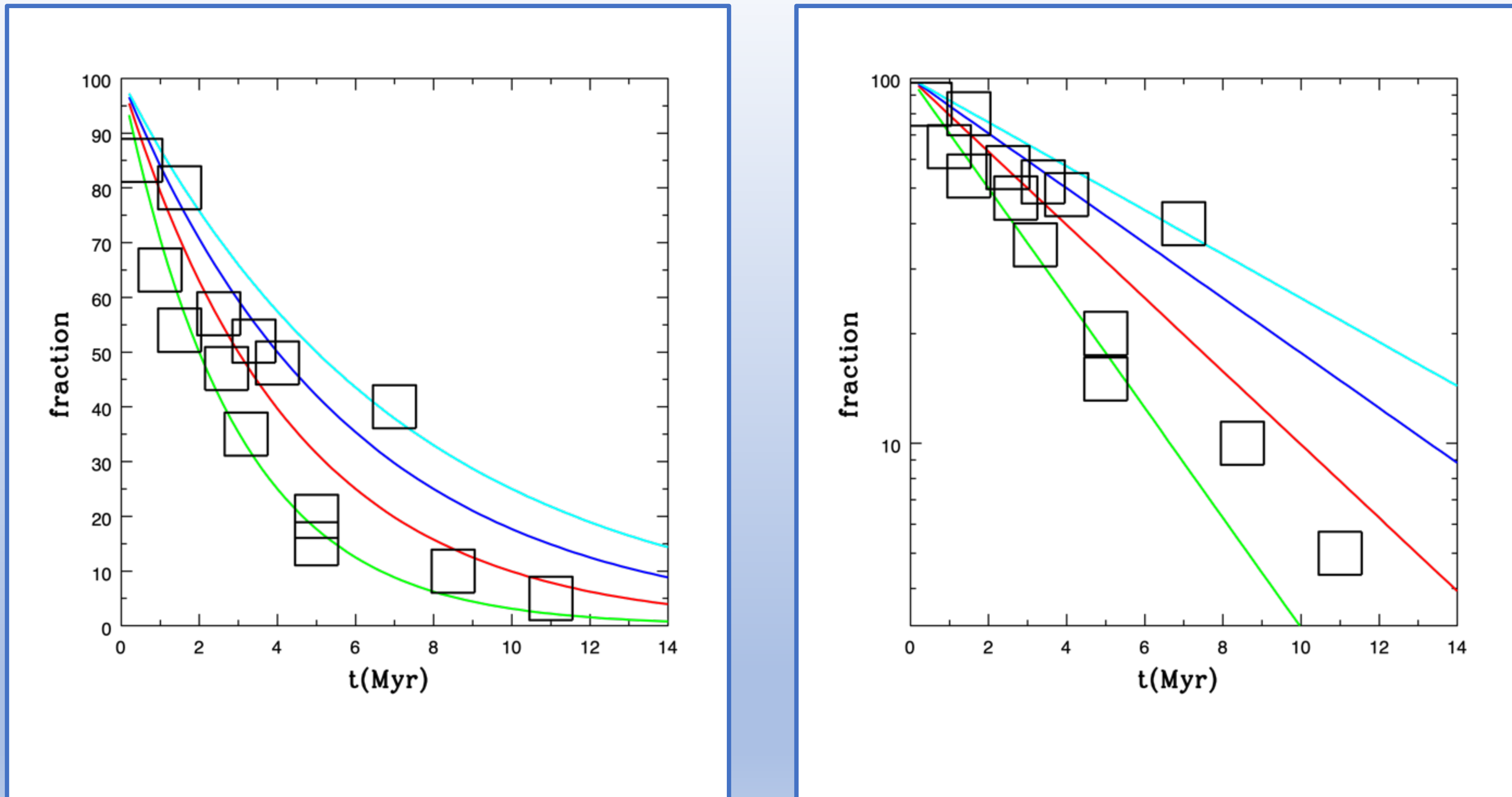


Disk Lifetimes

Observations suggest that disk lifetimes follow a nearly exponential distribution.

$$\frac{dF}{dt} = \frac{1}{\tau} e^{-t/\tau}$$





Exponential half-life of 2 Myr (green), 3 Myr (red), 4 Myr (blue), 5 Myr (cyan)

Disk Surface Density

$$\Sigma \sim r^{-p}$$

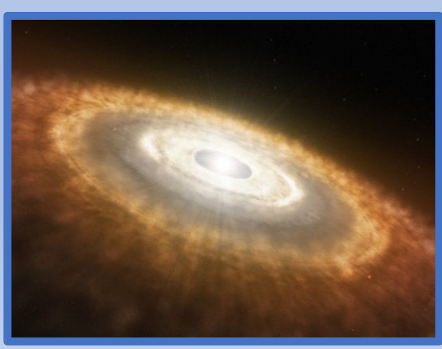
Often find Power-law Surface Density Profiles

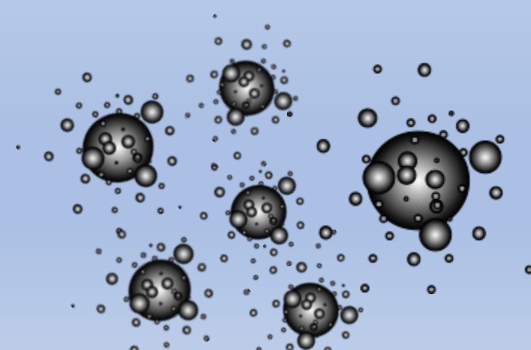
- Conservation of angular momentum p = 3/2
- Conservation + energy dissipation p = 7/4
- Reconstructing minimum Solar Nebula p = 3/2
- Reconstructing all exoplanetary systems p = 8/5
- Equal mass and spacing (peas-in-a-pod) p = 2
- Energy optimized masses, equal spacing p = 11/6
- ALMA and SMA observational maps p = 1 (gas)

Disk Properties

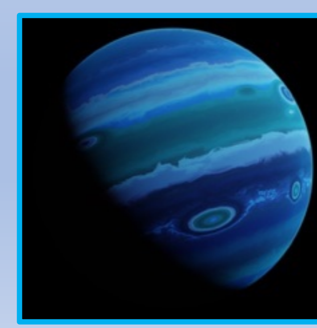
- The Star Formation Process naturally produces circumstellar disks that provide the initial conditions for Planet Formation
- Disks have radial sizes $R_{disk} = 10 – 100$ AU
- Disks have (starting, maximum) masses $M_{disk} = 0.1M_{star}$
- Disks have power-law surface densities with index p = 1 –2
- Disks have exponential distribution of lifetimes $\tau = 3$ Myr

100 AU





1 cm = 0.0000000000001 AU

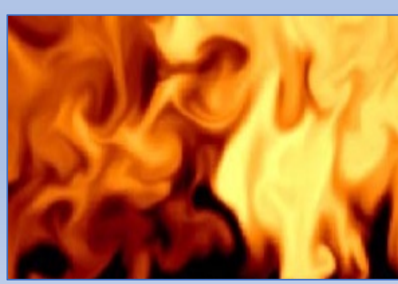


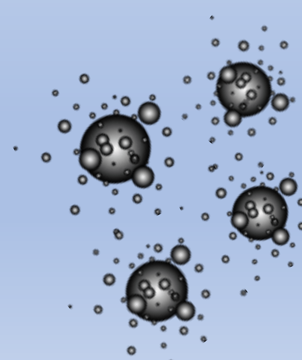
0.001 AU

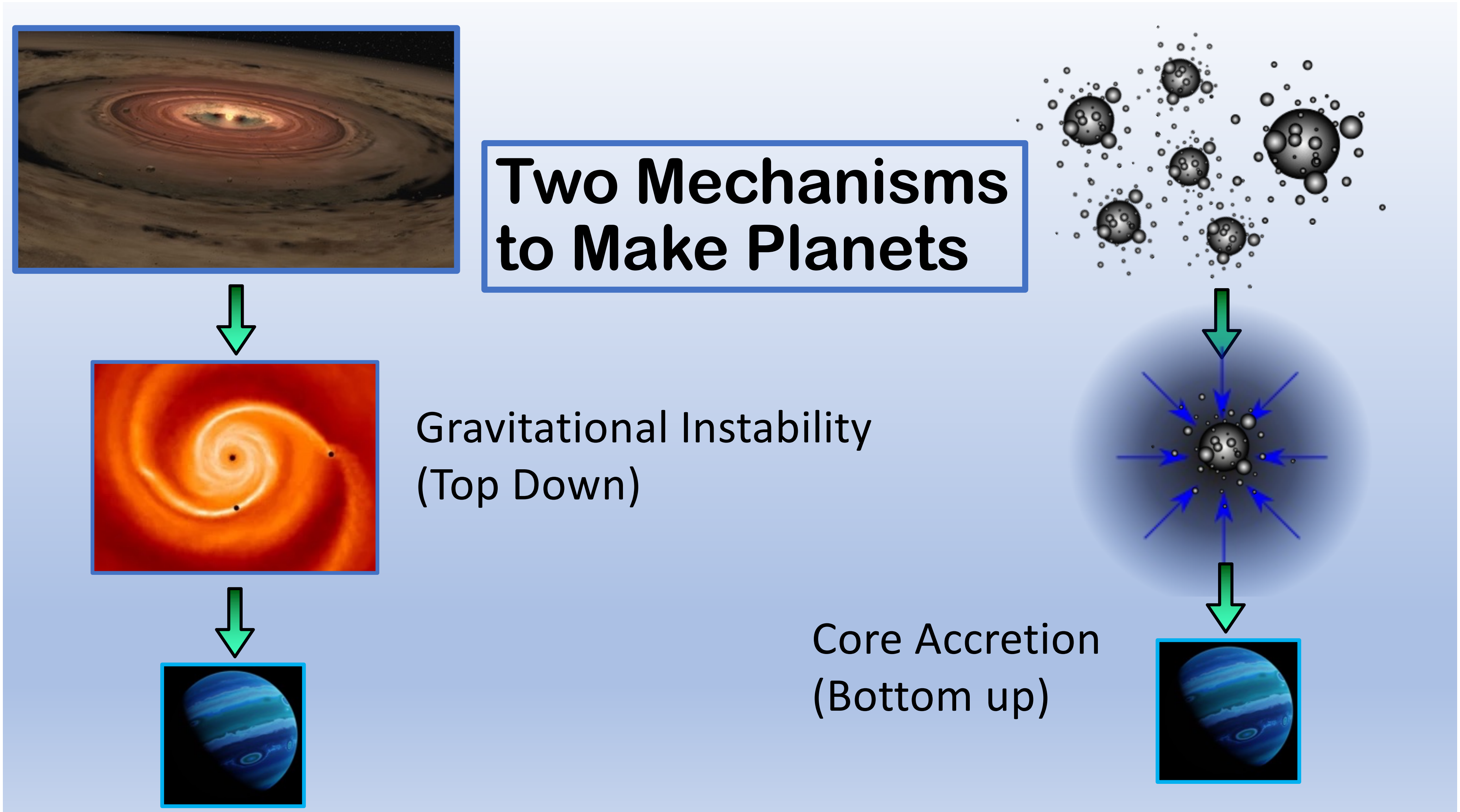
Disk Evolutionary Trends

- Disks have power-law surface densities with index p = 1 –2
- Formation considerations → $p \approx \frac{3}{2}$
- Rocky planet reconstruction → $p \approx 2$
- Observations of gaseous disks → $p \approx 1$

➔ Gas spreads *outward* and rocky material moves *inward*, while the total disk mass steadily *decreases* with time





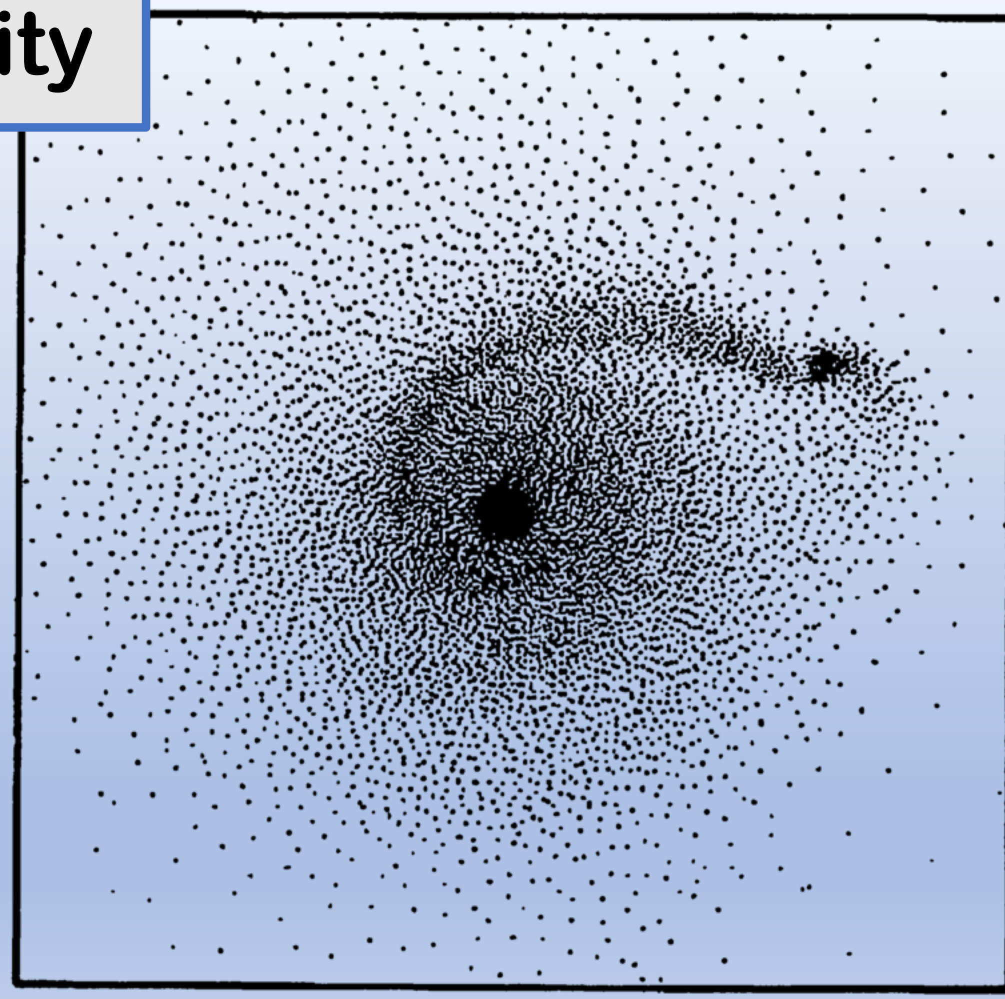

Two Mechanisms
to Make Planets
Gravitational Instability
(Top Down)
Core Accretion
(Bottom up)

Gravitational Instability

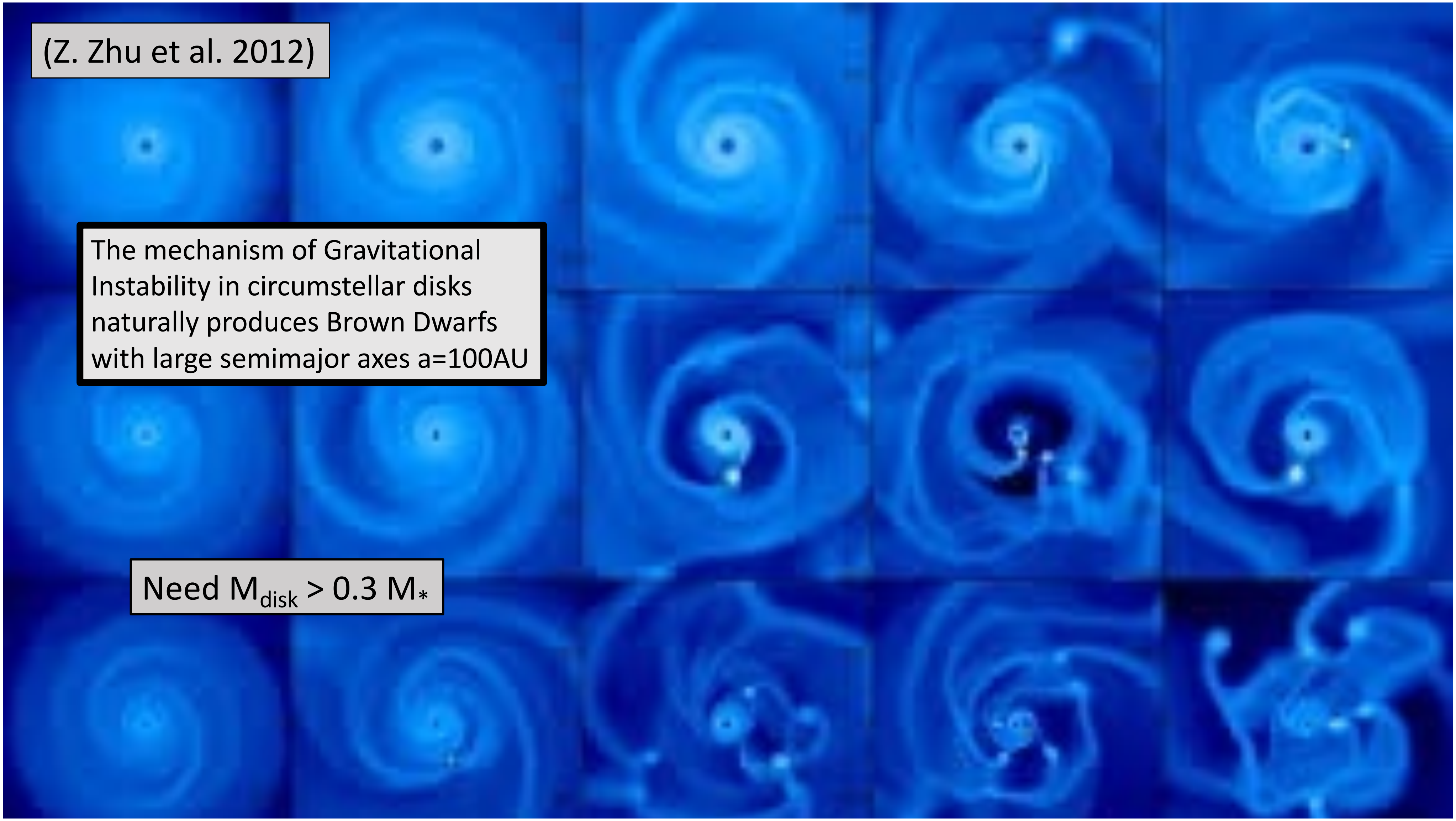
Need large mass, efficient cooling
Occurs in outer disks (at 100 AU)
Produces objects w. $M = 10 - 30\ M_{Jup}$

The mechanism of Gravitational Instability in circumstellar disks naturally produces Brown Dwarfs with large semimajor axes: a = 100AU

Need mass $M_{disk} > 0.1 - 0.5\ M_*$

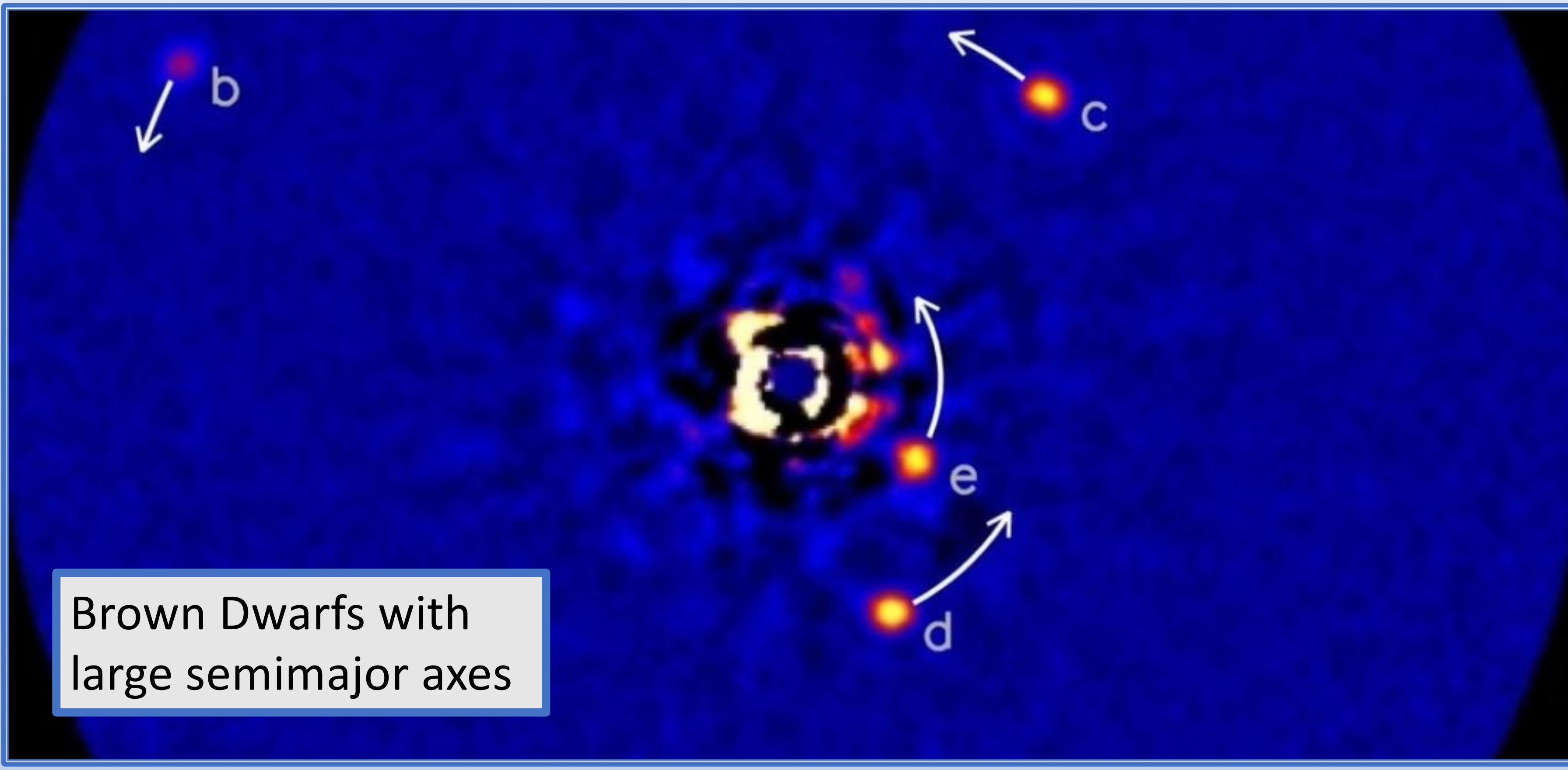


(Adams & Benz 1992)


(Z. Zhu et al. 2012)
The mechanism of Gravitational Instability in circumstellar disks naturally produces Brown Dwarfs with large semimajor axes a=100AU
Need $M_{disk} > 0.3\ M_*$

HR 8799 Planetary System

$M = 6 - 9\ M_{Jup}$
$a = 16 - 72$ AU



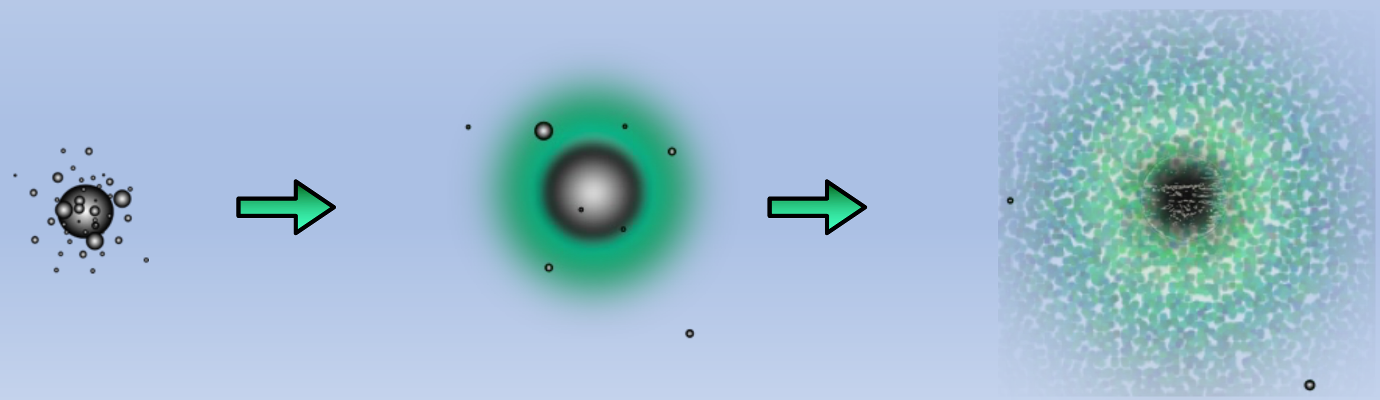
Brown Dwarfs with large semimajor axes

Gravitational Instability Summary

- Disk instabilities produce brown dwarfs in wide orbits (100AU)
- Need massive disks, efficient cooling (+magnetic resistivity)

→ This mechanism works, does occur, has low duty cycle

→ Most planets are produced with alternate mechanism, i.e., planets mostly form via the core accretion scenario



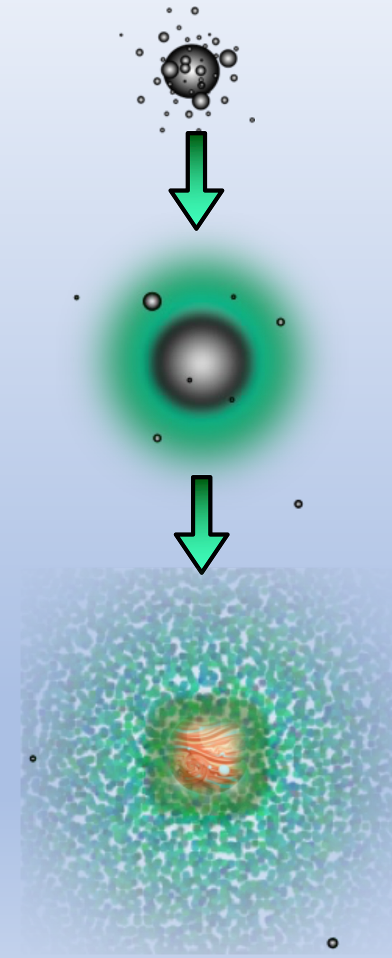
Core Accretion Paradigm

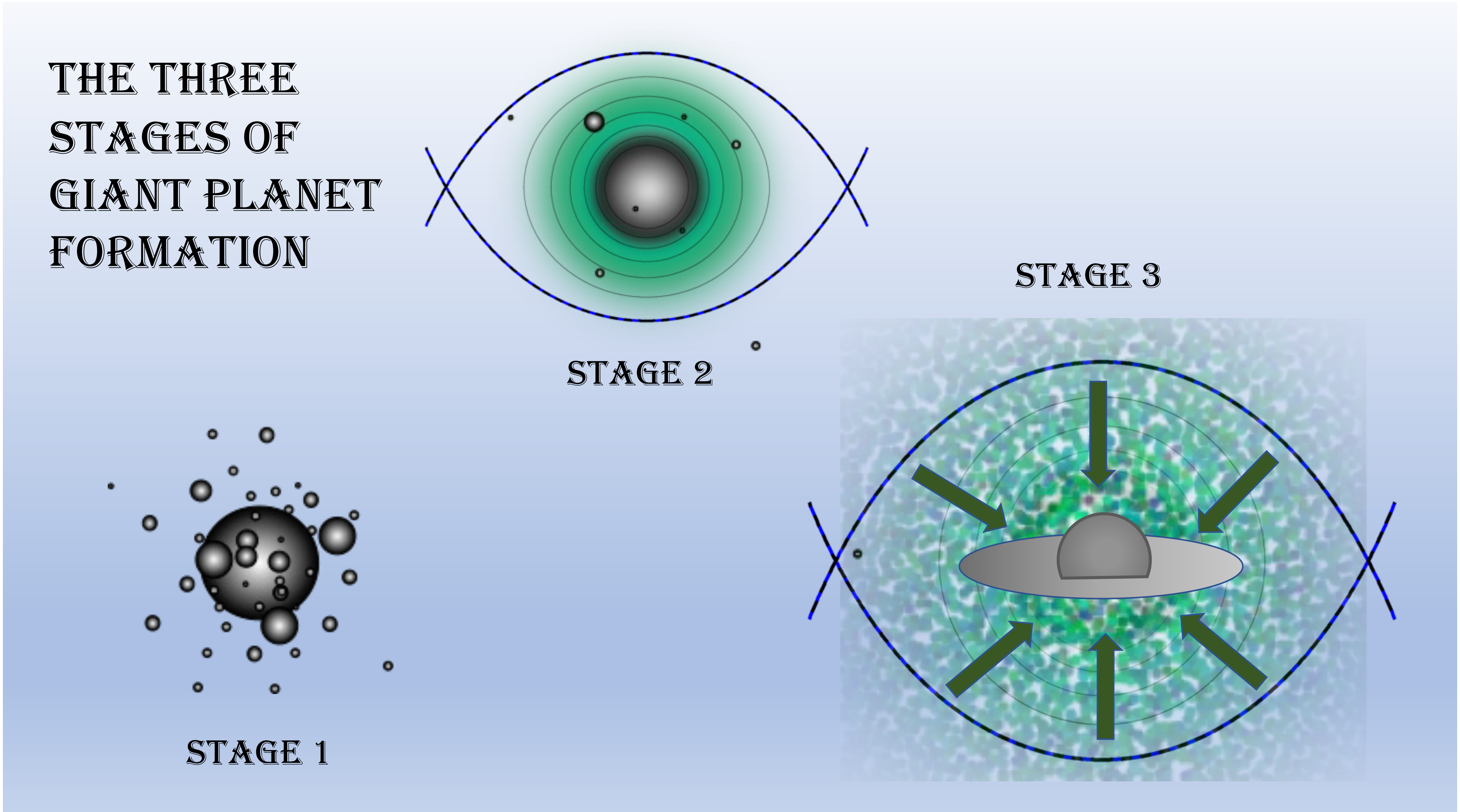
Perri & Cameron 1974, Mizuno et al 1978, Mizuno 1980, Bodenheimer & Pollack 1986, Pollack et al 1996

Phase 1: Growing planet consists mostly of solid material. Planet experiences runaway accretion until the feeding zone is depleted. Solid accretion occurs much faster than gas accretion during this phase.

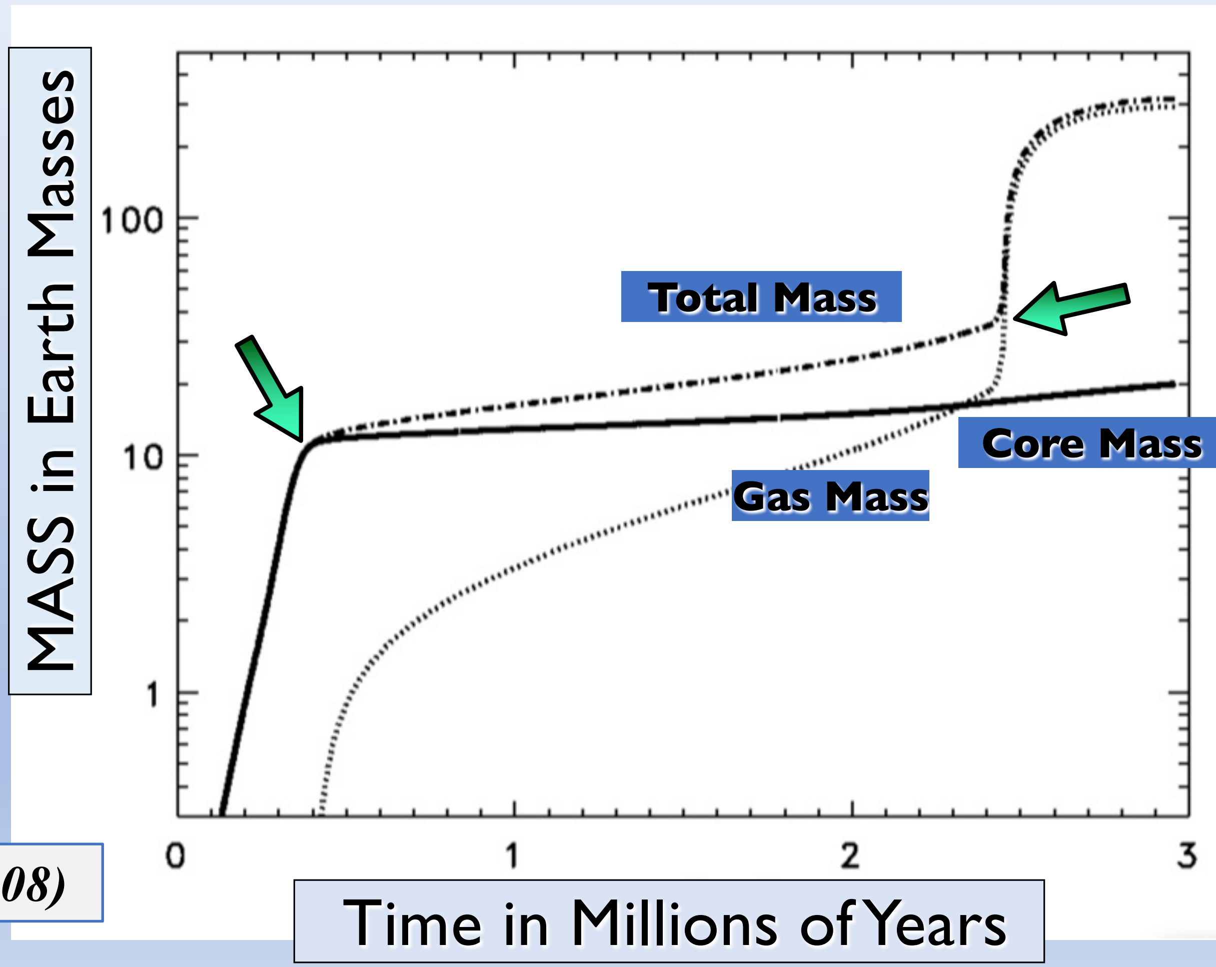
Phase 2: Solid and gas accretion rates are both small and nearly independent of time. This phase dictates the overall time-scale.

Phase 3: Runaway gas accretion occurs after the solid and gas masses are roughly equal.




THE THREE STAGES OF GIANT PLANET FORMATION
STAGE 2
STAGE 3
STAGE 1

Formation Model for Jupiter(s)

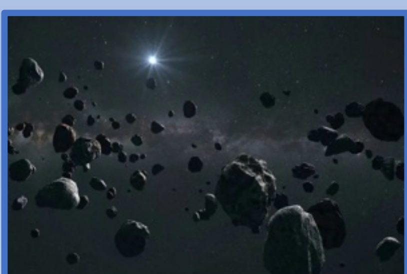


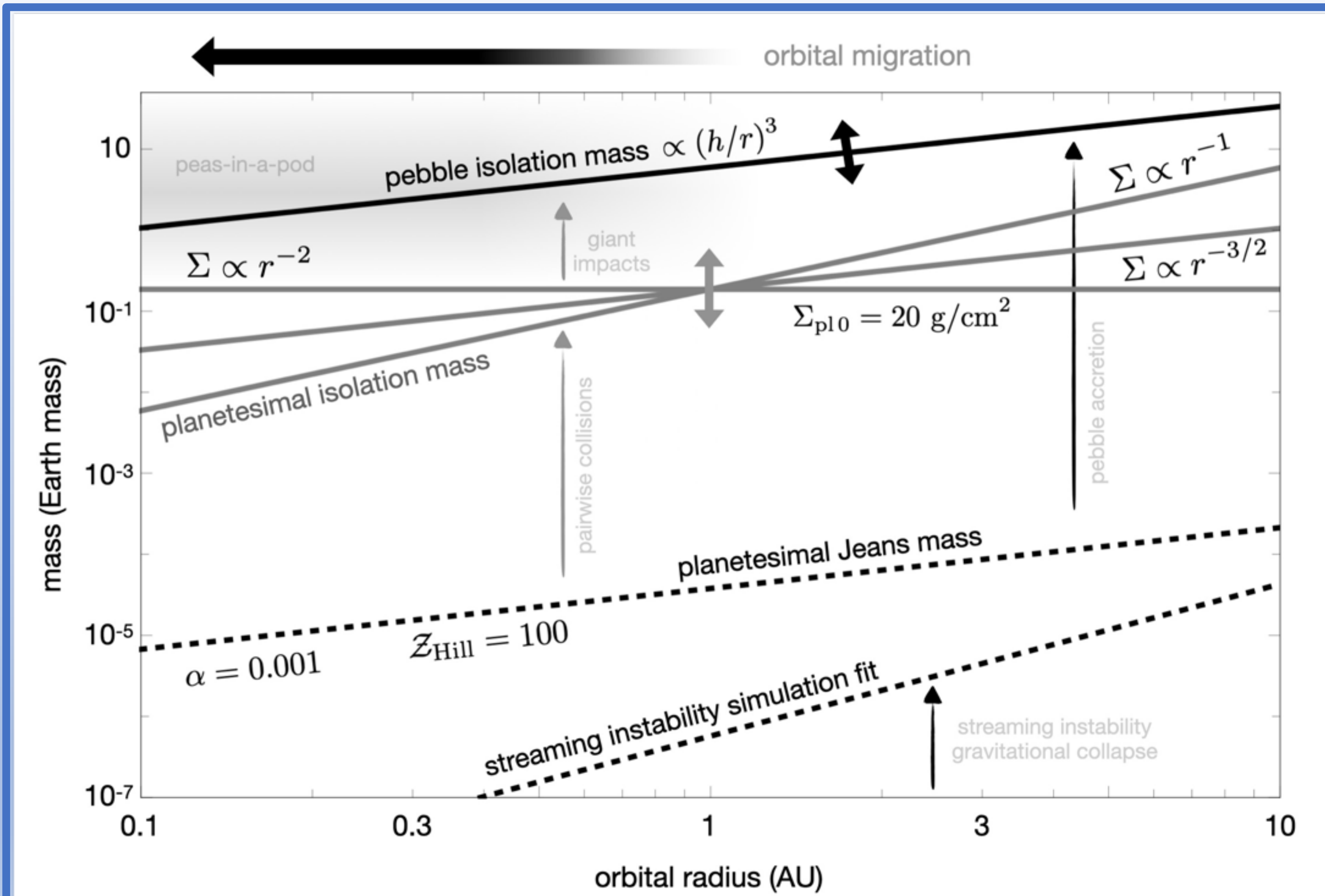
(Estrada et al. 2008)


STAGE 1: ROCKS IN SPACE

Rocks

→ Streaming Instability allows the breaking of the meter barrier.
→ Competition between two body collisions and pebble accumulation.
→ Pebble Isolation Mass produces bodies with mass ~10 M_{Earth}

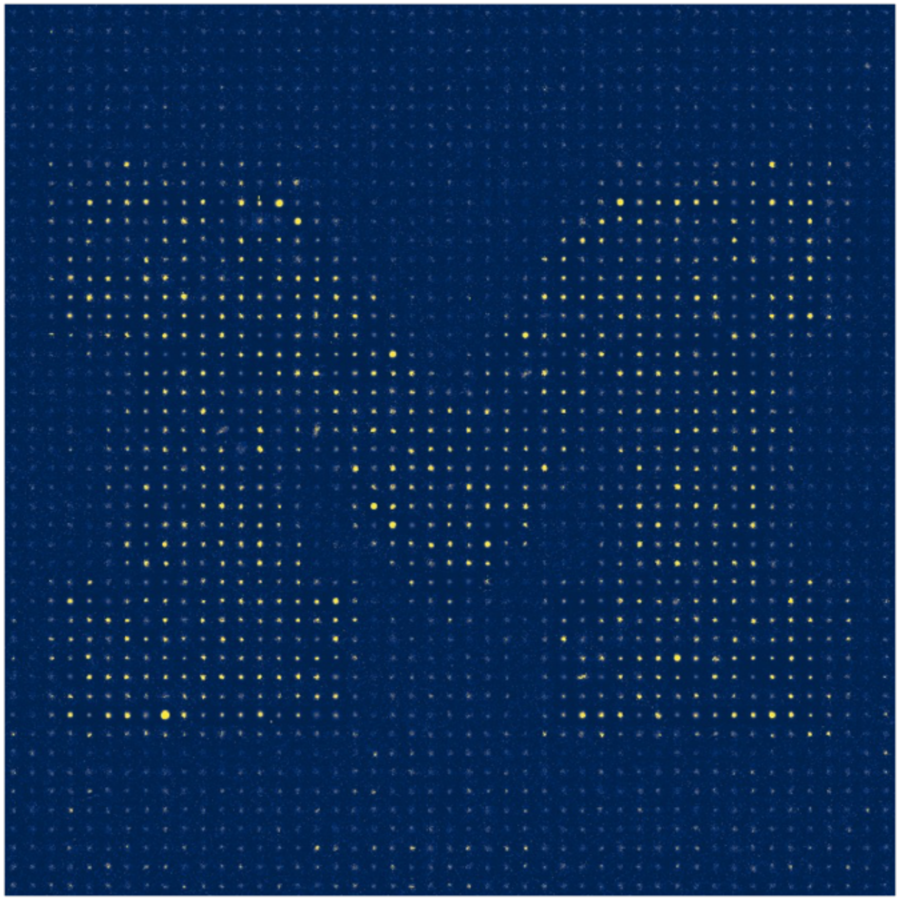




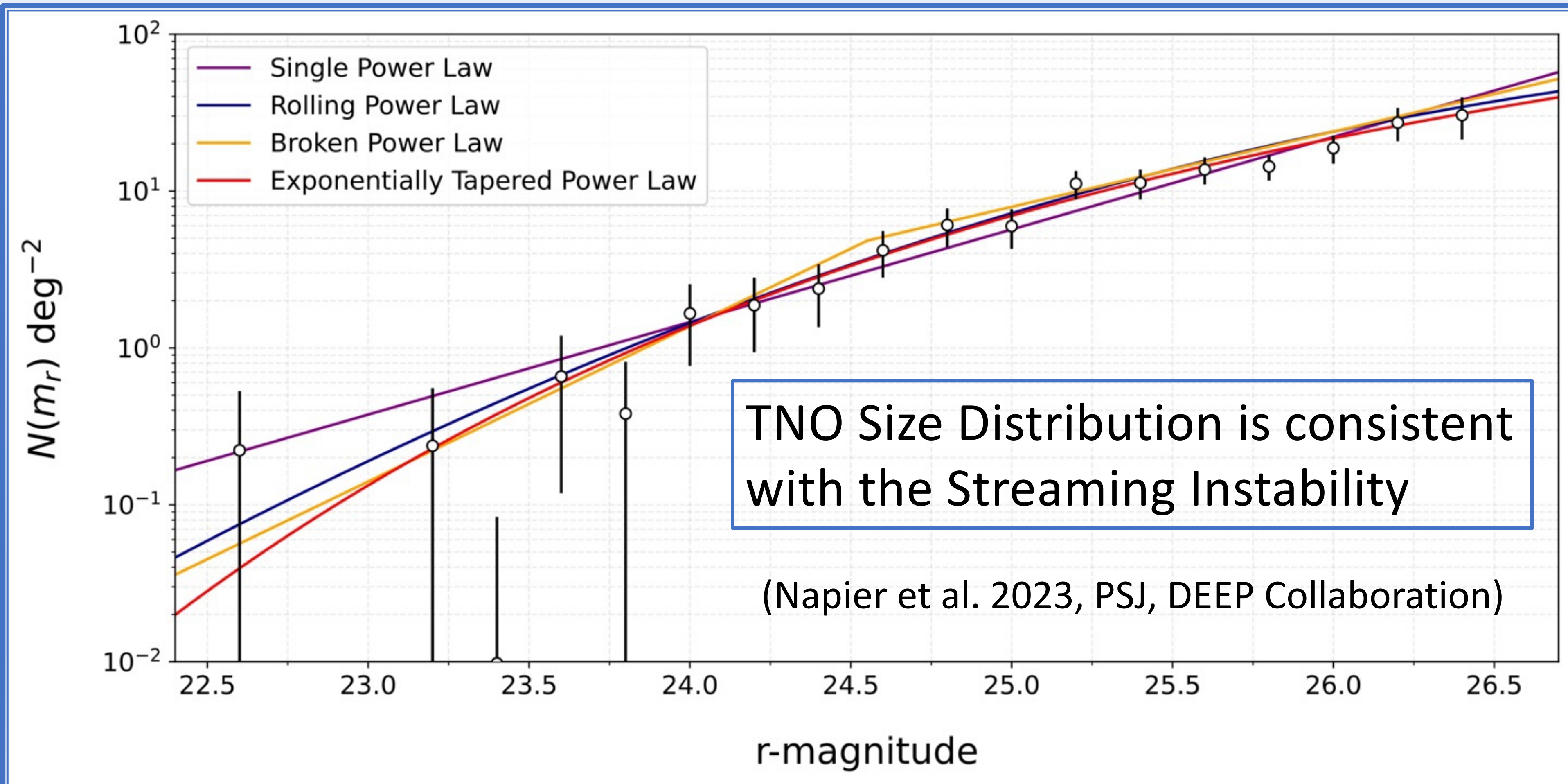
(Weiss et al. 2023, Protostars & Planets VII)

Rocks in Our Solar System

Mosaic Image of 2300 new TNO detections from the *DEEP collaboration* (Napier et al.) arXiv:2309.09478



Rocks in Our Solar System



Fundamental Mass Scale → Super Earth

[1] Pebble Isolation mass: $M_P = \frac{\alpha^{3/2}}{\sqrt{24}} \left(\frac{h}{r}\right)^3 M_* \approx 8M_\oplus$

[2] Mass Scale for Gas Retention: $M_P \approx 10M_\oplus$

[3] Peak/turnover of obs mass distribution (tentative):

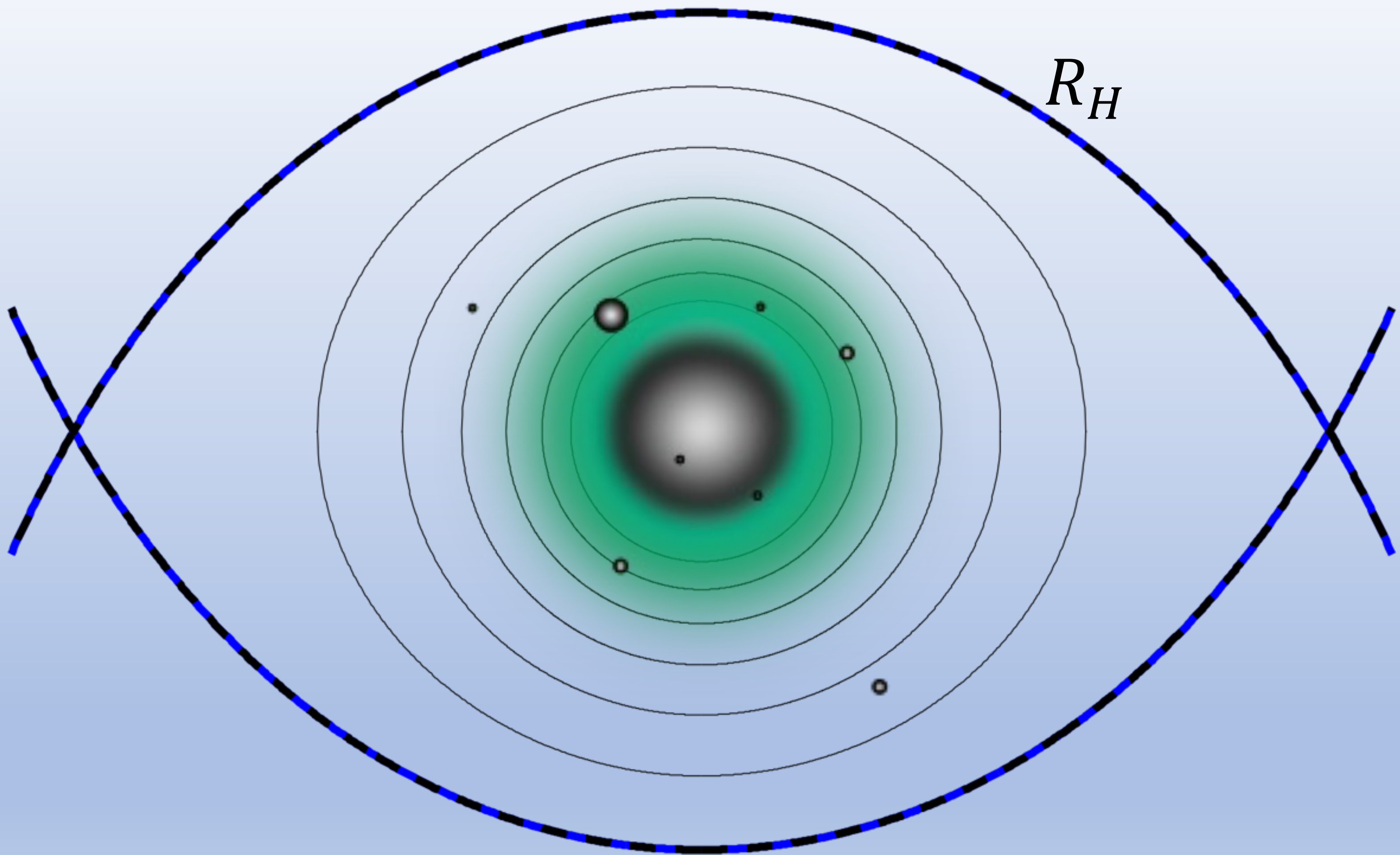
$$M_P \approx 5 - 10\ M_\oplus$$

Most rocky material tends to end up in bodies with masses $\sim$ SuperEarths

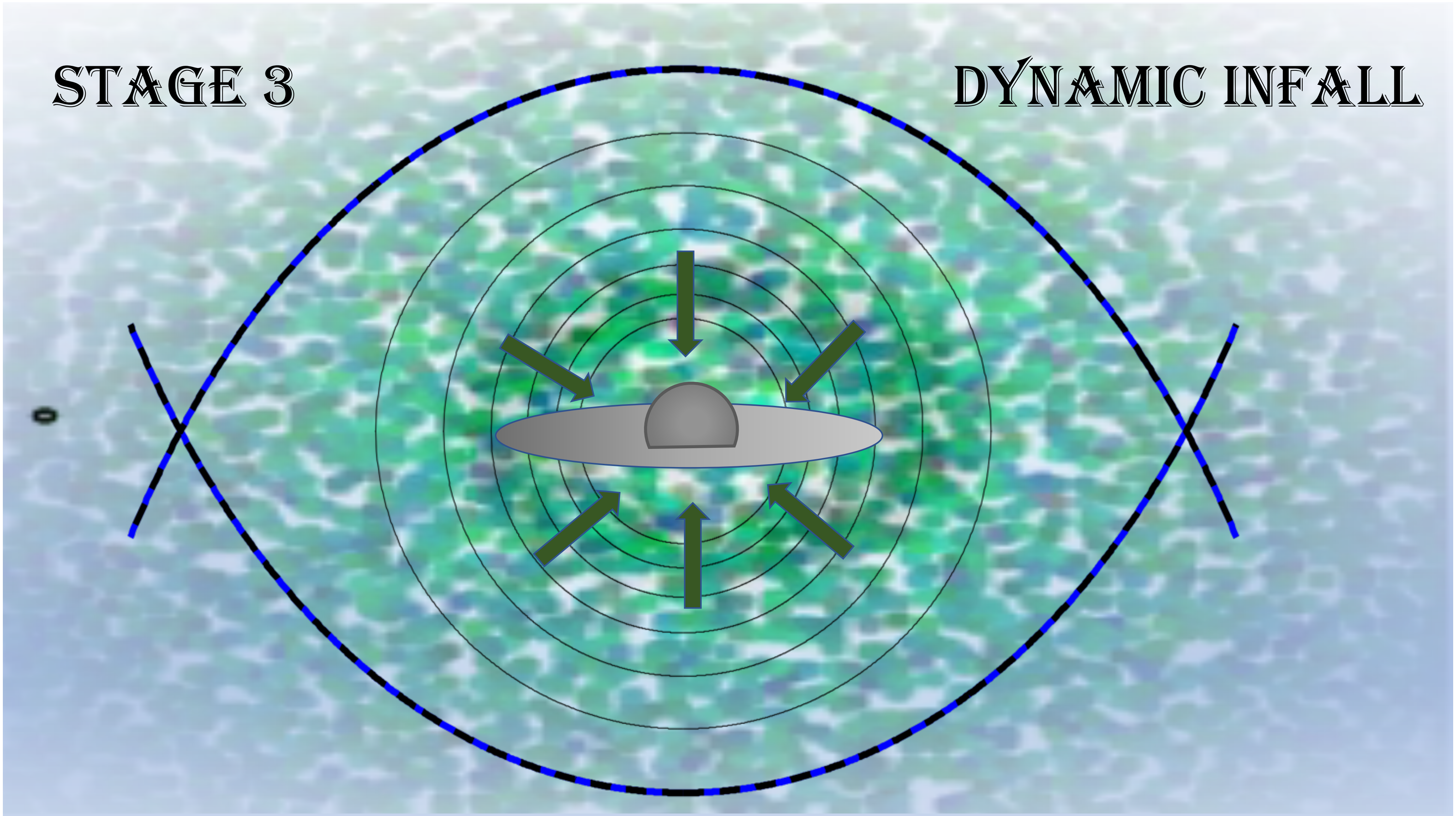
[a] Vast majority of rocky mass in our Solar System resides in giant planet cores (which are SuperEarths)

[b] For mass distribution $\mathrm{dN}/\mathrm{d}m \sim m^{-p}$, most of the mass is in largest bodies (SuperEarths) unless $p \geq 2$

[c] If body grows larger than SuperEarth, it accretes gas instead of rock (mostly)



STAGE 2: HYDROSTATIC EQUILIBRIUM


STAGE 3
DYNAMIC INFALL

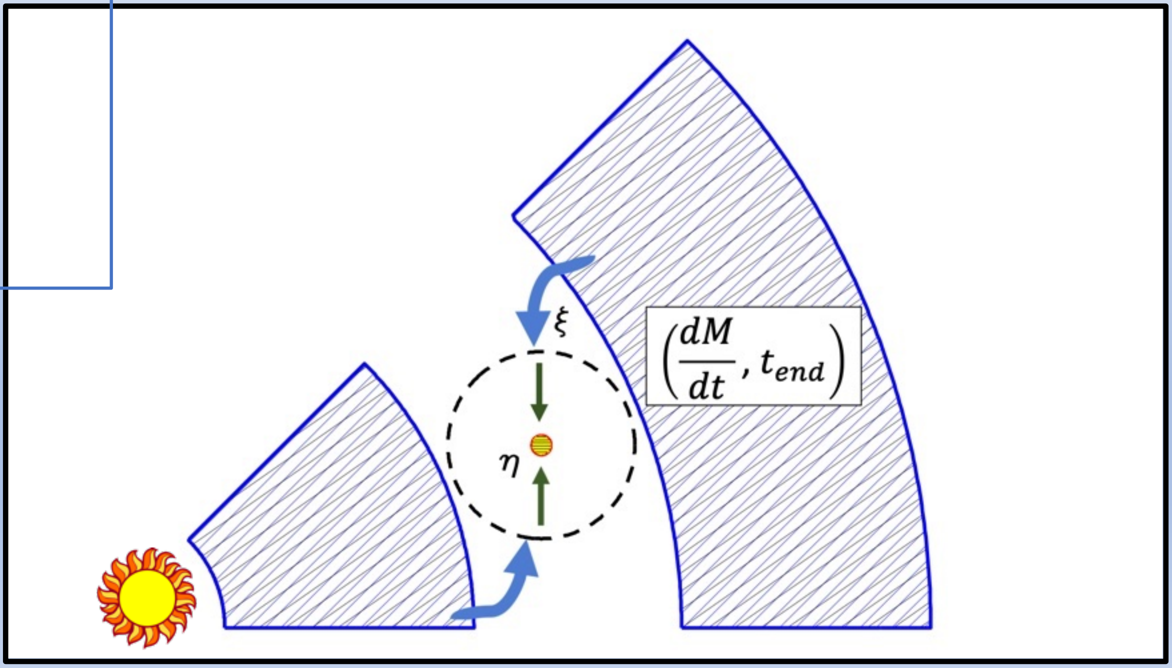
Late Stages of Giant Planet Formation

- In core accretion paradigm, most of the final planetary mass is acquired during the third and final infall phase
- Most material falls initially onto circumplanetary disk
- Find solutions for the density and velocity of rotating infall region
- Find solutions for the structure and evolution of the disk
- Inner disk is truncated by strong planetary magnetic fields, which also determine capture radius for direct infall
- Luminosity has several contributions (determined herein)
- Find spectral energy distributions of these forming planets

Adams & Batygin 2022, ApJ, 934, 111

Outer Boundary(ies)

- Hill Radius $R_H = a\left(\frac{m}{3M}\right)^{1/3}$
- Bondi Radius $R_B = \left(\frac{Gm}{v^2}\right)$
- Disk Scale Height $H = \frac{v}{\Omega}$



Infall Solution

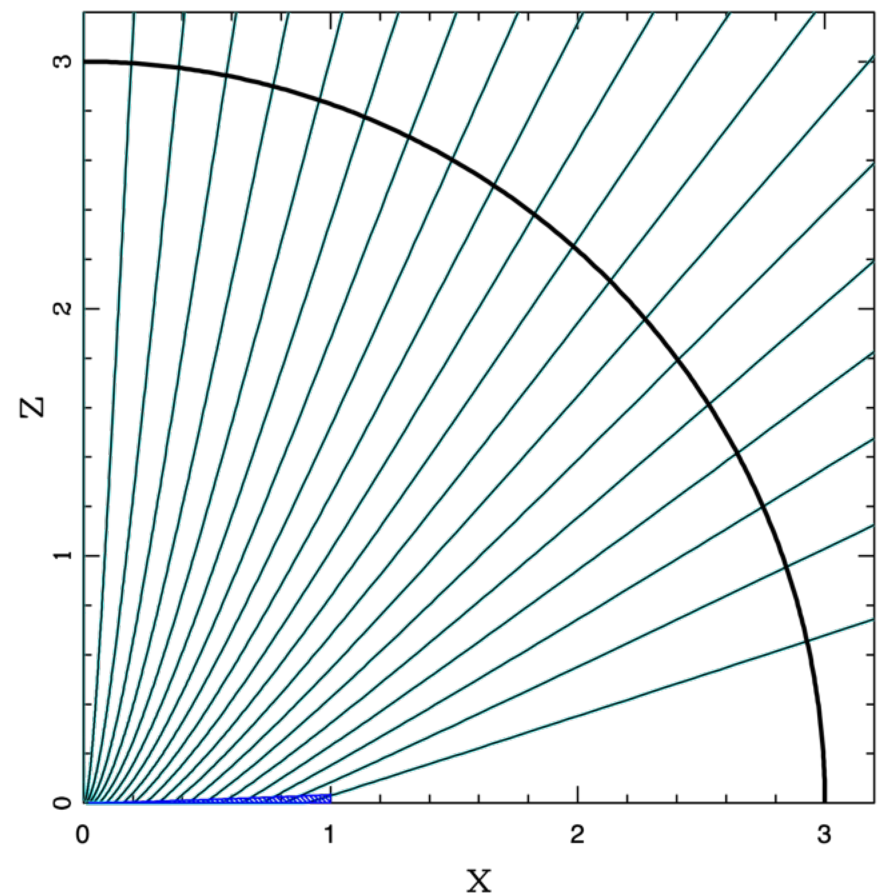
$$R_C \equiv \frac{j_\infty^2}{GM_\mathrm{p}}\,, \qquad \zeta \equiv \frac{j_\infty^2}{GM_\mathrm{p} r}\,, \qquad v_0 \equiv (GM_\mathrm{p}/r)^{1/2}$$

$$v_r = -v_0\left[2 - \zeta(1-\mu_0^2)\right]^{1/2}$$

$$v_\theta = v_0\left[\frac{1-\mu_0^2}{1-\mu^2}(\mu_0^2-\mu^2)\zeta\right]^{1/2}$$

$$v_\phi = v_0(1-\mu_0^2)(1-\mu^2)^{-1/2}\zeta^{1/2}$$

$$\rho(r,\mu) = \frac{\dot{M}_\mathrm{p}}{4\pi r^2|v_r|}\frac{d\mu_0}{d\mu} = \frac{\dot{M}_\mathrm{p}}{4\pi r^2|v_r|}\left[1+2\zeta P_2(\mu_0)\right]^{-1}$$

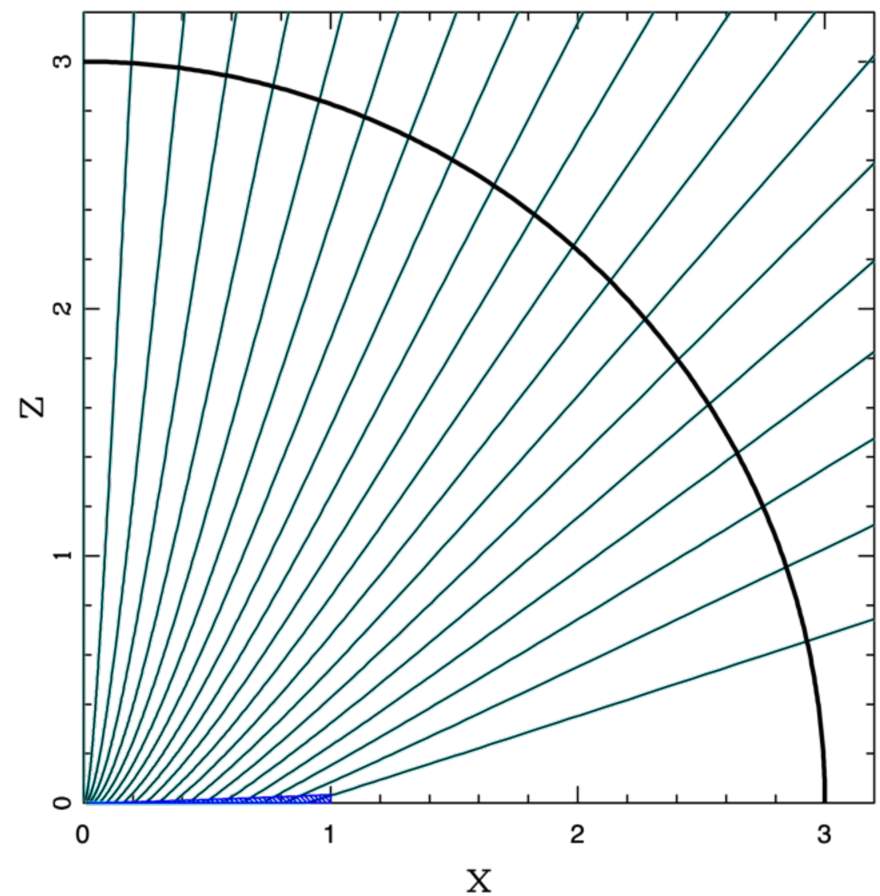


Infall Solution

$$\rho(r,\theta)$$

$$\rho(r) \sim r^{-\frac{3}{2}}, r \gg R_d$$

$$\rho(r) \sim r^{-\frac{1}{2}}, r \ll R_d$$



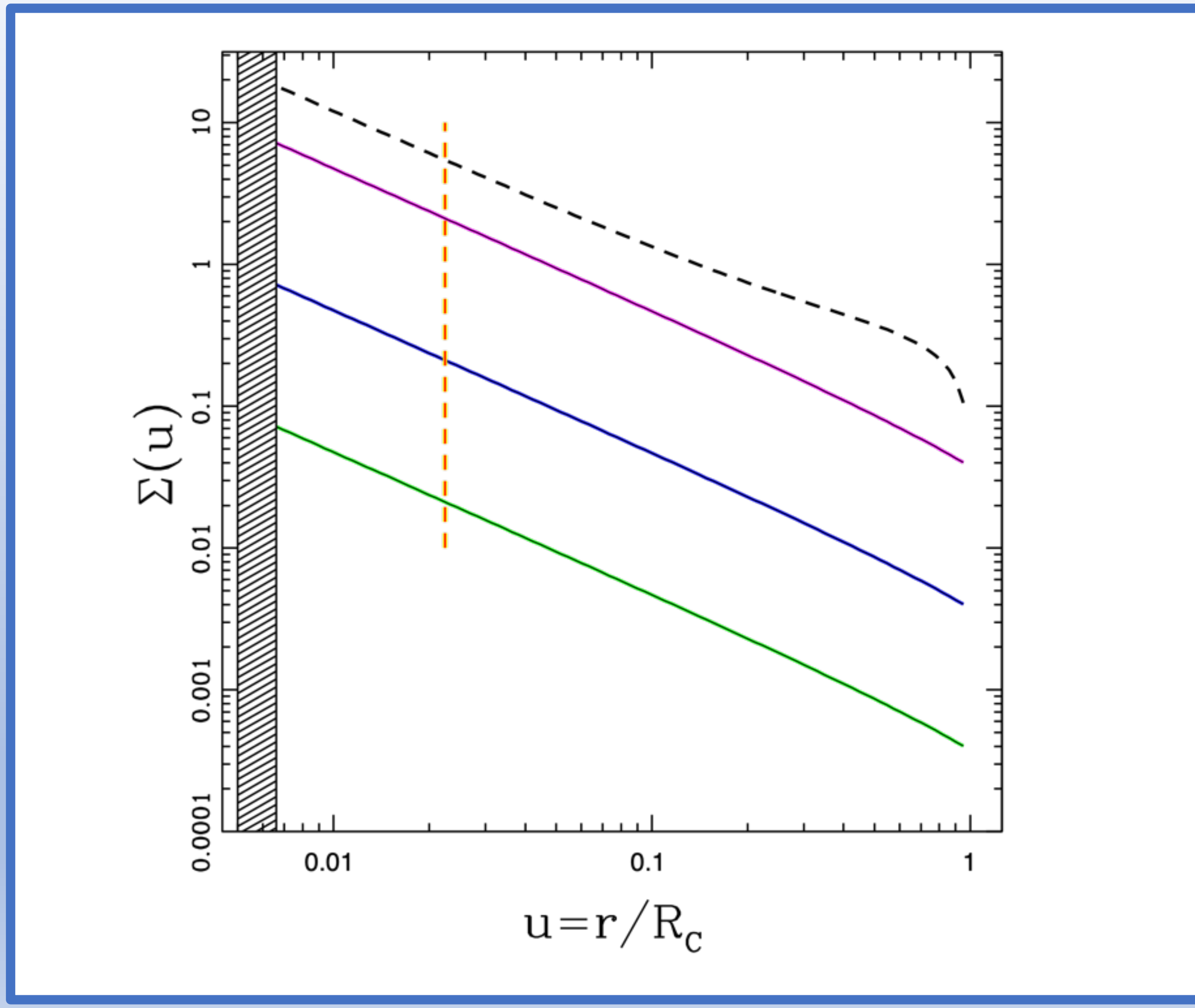
Disk Solutions

$$\frac{\partial\Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(r^{1/2}\nu\Sigma\right)\right] + \frac{\dot{M}_{\rm p}}{4\pi r^2}\frac{1}{(\zeta^2-\zeta)^{1/2}}$$

$$\Sigma_0(u) = \frac{3M_{\rm p}}{16\pi R_C^2}\left\{\frac{\sqrt{1-u}(2+3u)}{u} + 3u\tanh^{-1}(\sqrt{1-u})\right\}$$

$$\Sigma_\nu(u) = \frac{\dot{M}_{\rm p}}{6\pi\nu}\frac{1}{\sqrt{u}}\left[\sin^{-1}\sqrt{u} + \sqrt{u(1-u)}\right]$$

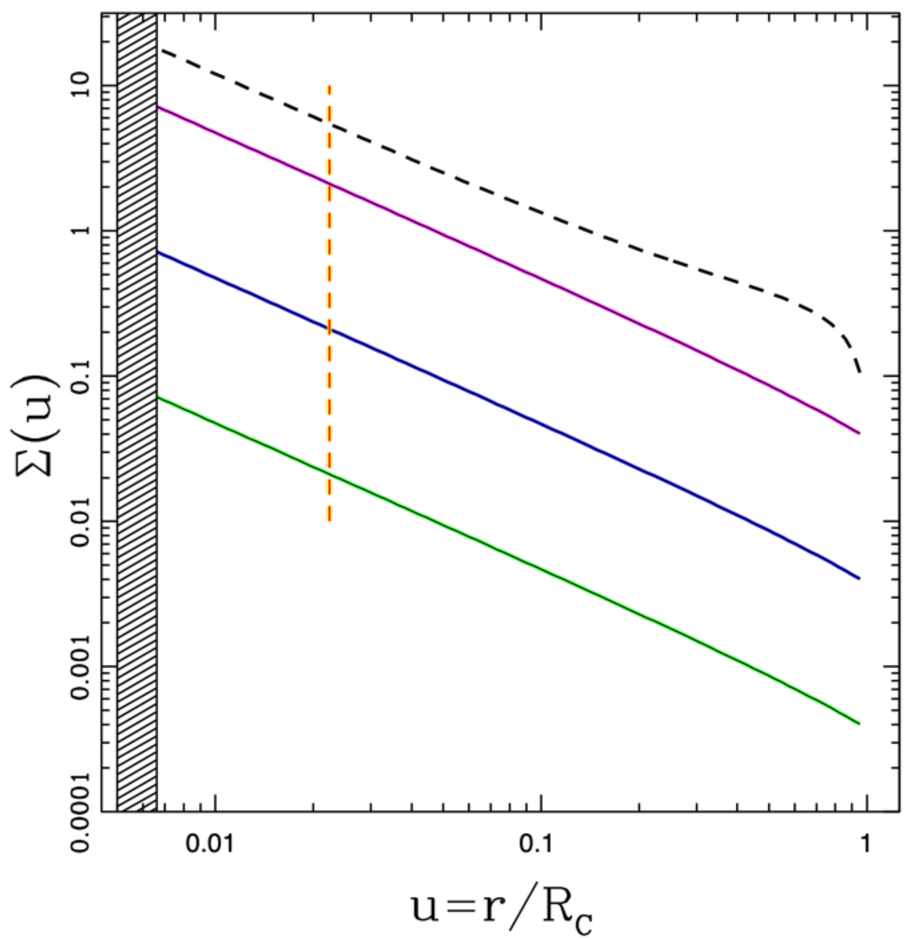
$$\alpha = 10^{-4}, 10^{-3}, 10^{-2}$$



Disk Solutions

$$\Sigma \sim \frac{1}{r}$$

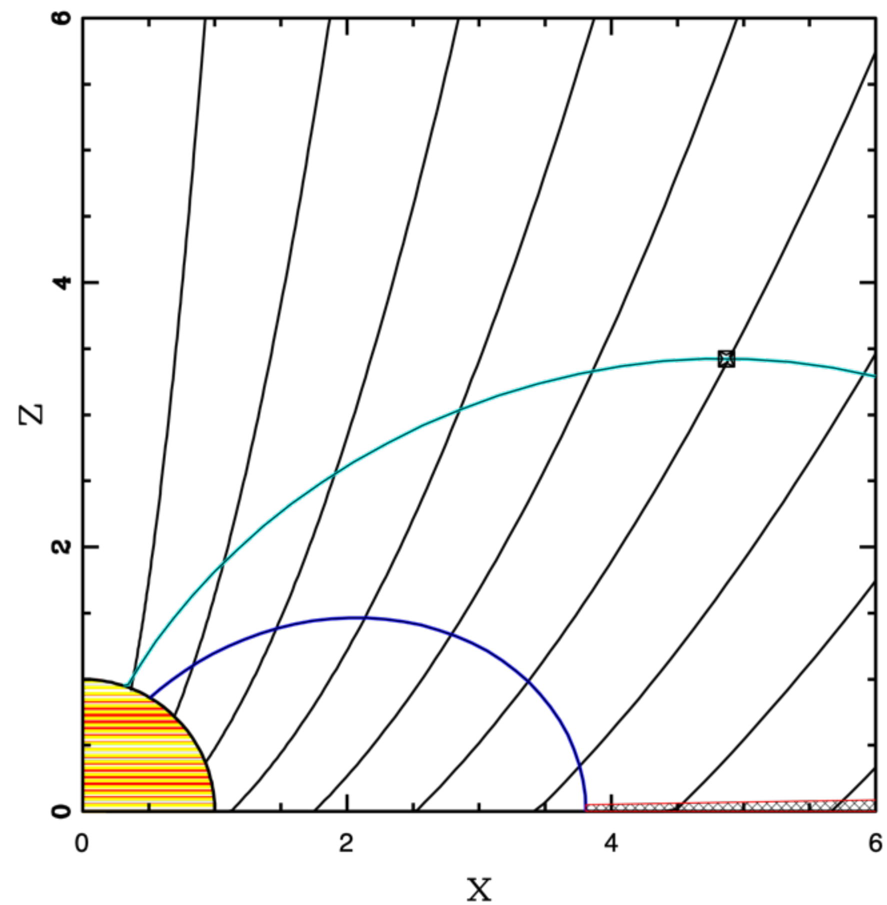
$$\alpha = 10^{-4}, 10^{-3}, 10^{-2}$$



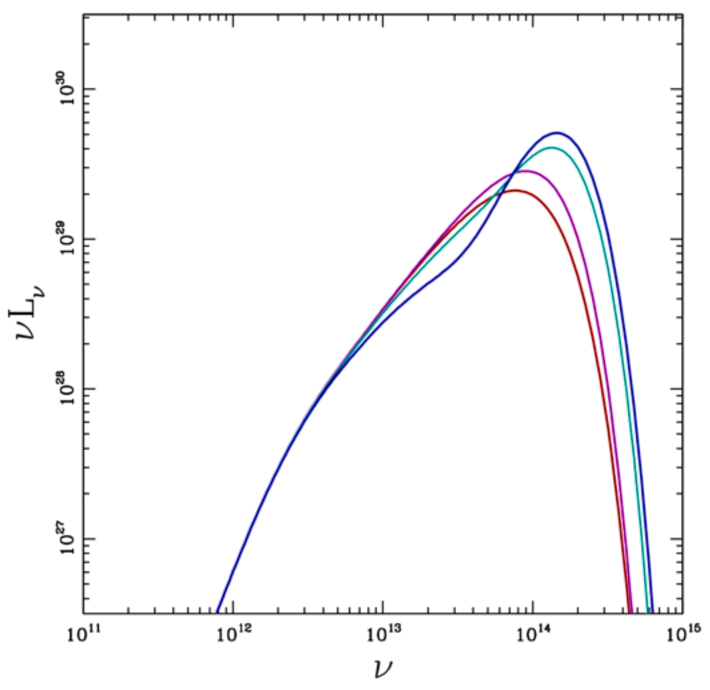
Magnetic Radii

$$R_X = \omega \left(\frac{B_\mathrm{p}^4 R_\mathrm{p}^{12}}{G M_\mathrm{p} \dot{M}_\mathrm{p}^2} \right)^{1/7}$$

$$\frac{R_\sigma}{R_\mathrm{p}} = \tilde{\omega} \left(\frac{B_\mathrm{p}^4 R_\mathrm{p}^3 R_C^2}{G M_\mathrm{p} \dot{M}_\mathrm{p}^2} \right)^{1/9}$$



Luminosity Contributions



$$L_0 \equiv \frac{G M_\mathrm{p} \dot{M}_\mathrm{p}}{R_\mathrm{p}}$$

$$L_\mathrm{p}^\mathrm{dir} = \left(1 - \sqrt{1-u_\mathrm{x}}\right) \left(1 - \frac{1}{2} \langle \sin^2\theta \rangle \frac{R_\mathrm{p}^3}{R_X^3}\right) L_0$$

$$L_\mathrm{p}^\mathrm{acc} = \sqrt{1-u_\mathrm{x}} \left(1 - \frac{R_\mathrm{p}}{R_X}\right) \left(1 - \frac{R_\mathrm{p}^3}{3R_X^3}\right) L_0$$

$$L_\mathrm{p} = \left(1 - \sqrt{1-u_\mathrm{x}} \frac{R_\mathrm{p}}{R_X}\right) \left(1 - \frac{R_\mathrm{p}^3}{3R_X^3}\right) L_0$$

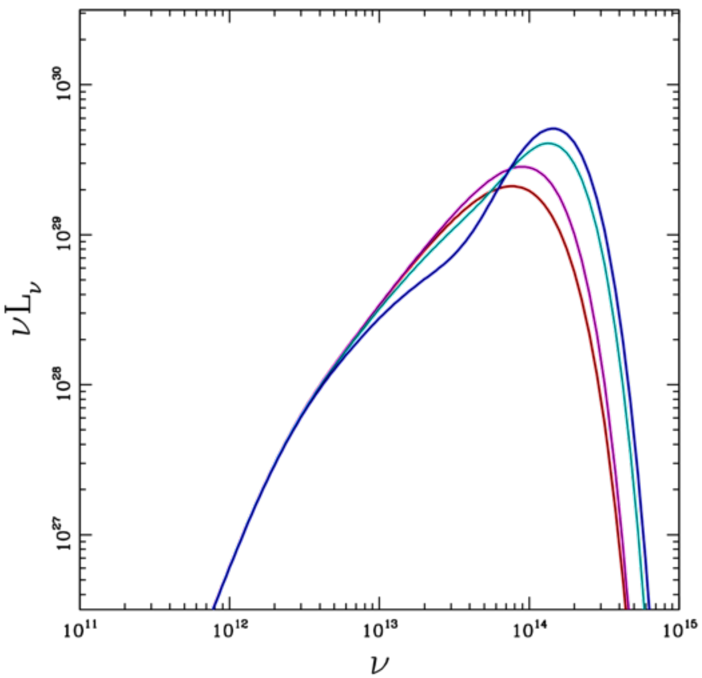
$$L_\mathrm{d}^\mathrm{surf} = \int_A \left(\frac{1}{2} v_\theta^2\right) \rho |v_\theta| dA = \frac{u_\mathrm{x}}{4} \left\{ \ln \left[\frac{1+\sqrt{1-u_\mathrm{x}}}{1-\sqrt{1-u_\mathrm{x}}}\right] - 2\sqrt{1-u_\mathrm{x}} \right\} L_0$$

$$L_\mathrm{d}^\mathrm{mix} = \frac{u_\mathrm{x}}{2} \left\{ \ln \left[\frac{1+\sqrt{1-u_\mathrm{x}}}{1-\sqrt{1-u_\mathrm{x}}}\right] + \sqrt{1-u_\mathrm{x}} + \pi - 2\tan^{-1} \left[\frac{u_\mathrm{x}}{1-u_\mathrm{x}}\right]^{1/2} \right\} L_0$$

$$L_\mathrm{d}^\mathrm{dir} = \frac{u_\mathrm{x}}{4} \left\{ 3\ln \left[\frac{1+\sqrt{1-u_\mathrm{x}}}{1-\sqrt{1-u_\mathrm{x}}}\right] + 2\pi - 4\sin^{-1}\sqrt{u_\mathrm{x}} \right\} L_0$$

$$L_\mathrm{d}^\mathrm{visc} = L_0 \frac{R_\mathrm{p}}{2R_X} \sqrt{1-u_\mathrm{x}} - L_\mathrm{d}^\mathrm{dir}$$

Luminosity Contributions



Direct accretion onto planet
Direct accretion onto disk
Mixing Luminosity in disk
Mixing Luminosity in planet
Viscous dissipation in disk
Intrinsic contraction luminosity

Envelope Contribution

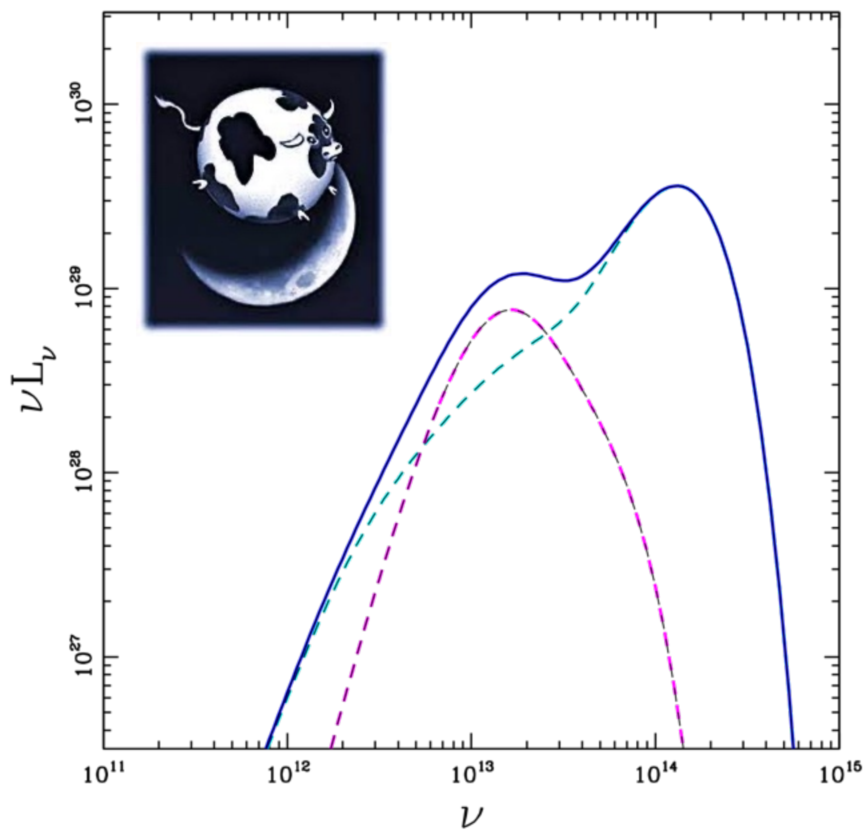
$$\mathcal{L}_\nu = f_e \int_{R_X}^{R_H} \rho \kappa_\nu 4\pi B_\nu(T)\, 4\pi r^2 dr$$

$$\mathcal{L} = 16\pi \int_{R_X}^{R_H} \rho \kappa_P \sigma T^4 r^2 dr$$

$$\mathcal{L} = 16\pi\, \sigma\, C b_\kappa T_C^5 R_C^{3/2} \int_{u_\times}^{3} u^{-3/2} \mathcal{A}(u) du = 16\pi R_C^2\, \sigma\, b_\kappa T_C^5 N_{\rm col}$$

$$T_C = \left(\frac{\mathcal{L}}{16\pi R_C^2 \sigma N_{\rm col} b_\kappa} \right)^{1/5}$$

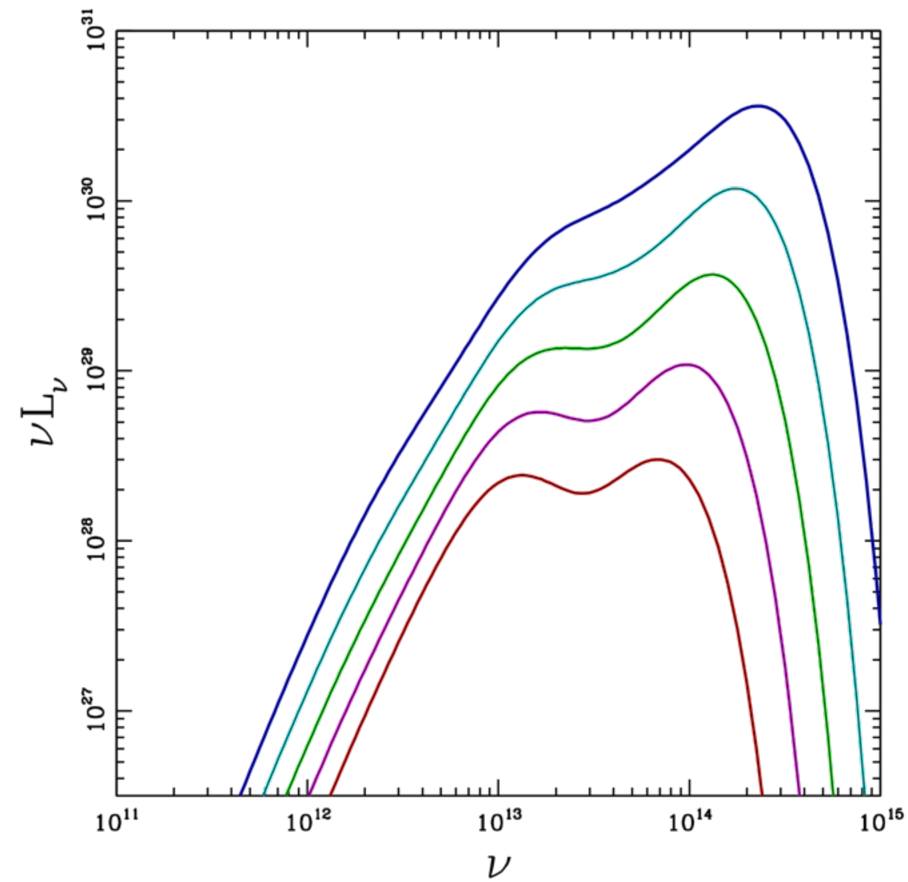
$$\mathcal{L} = \left[L_{\rm p} + L_{\rm d} - \int_0^\infty d\nu (L_{{\rm p}\nu} + L_{{\rm d}\nu}) \exp(-\tau_\nu) \right]$$



Spectral Energy Distributions vs Planet Mass

$$M = 0.1 - 10\ M_J$$
$$\dot{M} = 1 M_J/\text{Myr}$$
$$a = 5\ \text{AU}$$
$$B_P = 500\ gauss$$

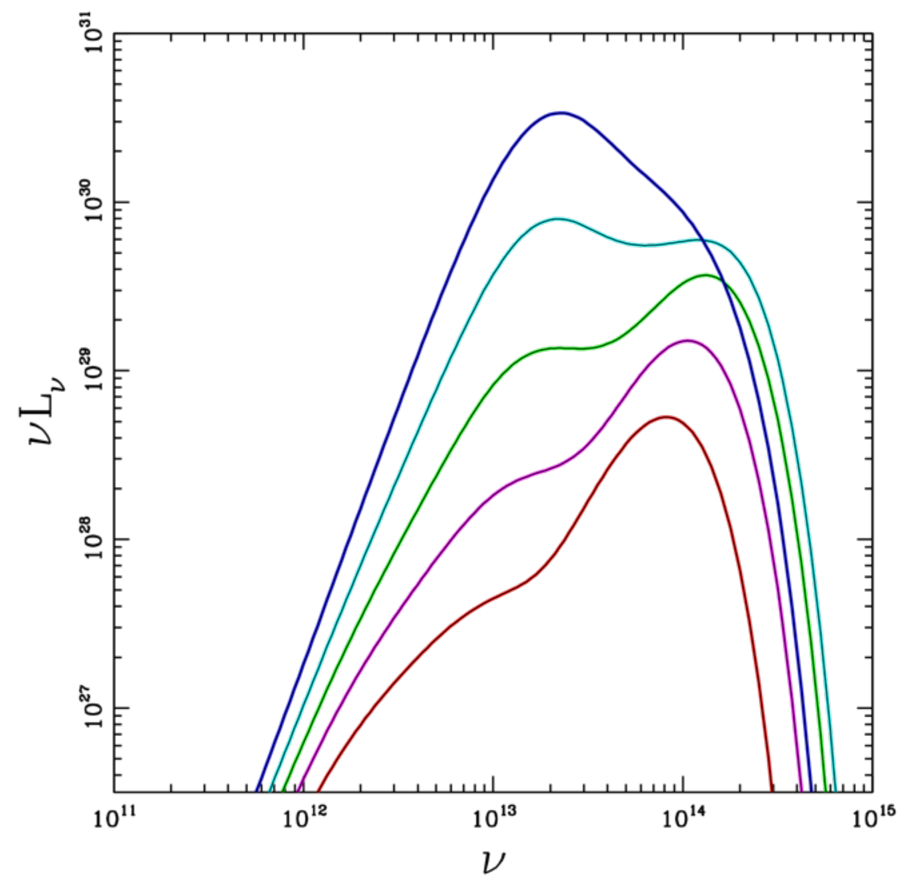
larger mass → brighter and bluer



Spectral Energy Distributions vs Mass Accretion

$$\dot{M} = 0.1 - 10 M_J/\text{Myr}$$
$$M = 1\ M_J$$
$$a = 5\ \text{AU}$$
$$B_P = 500\ gauss$$

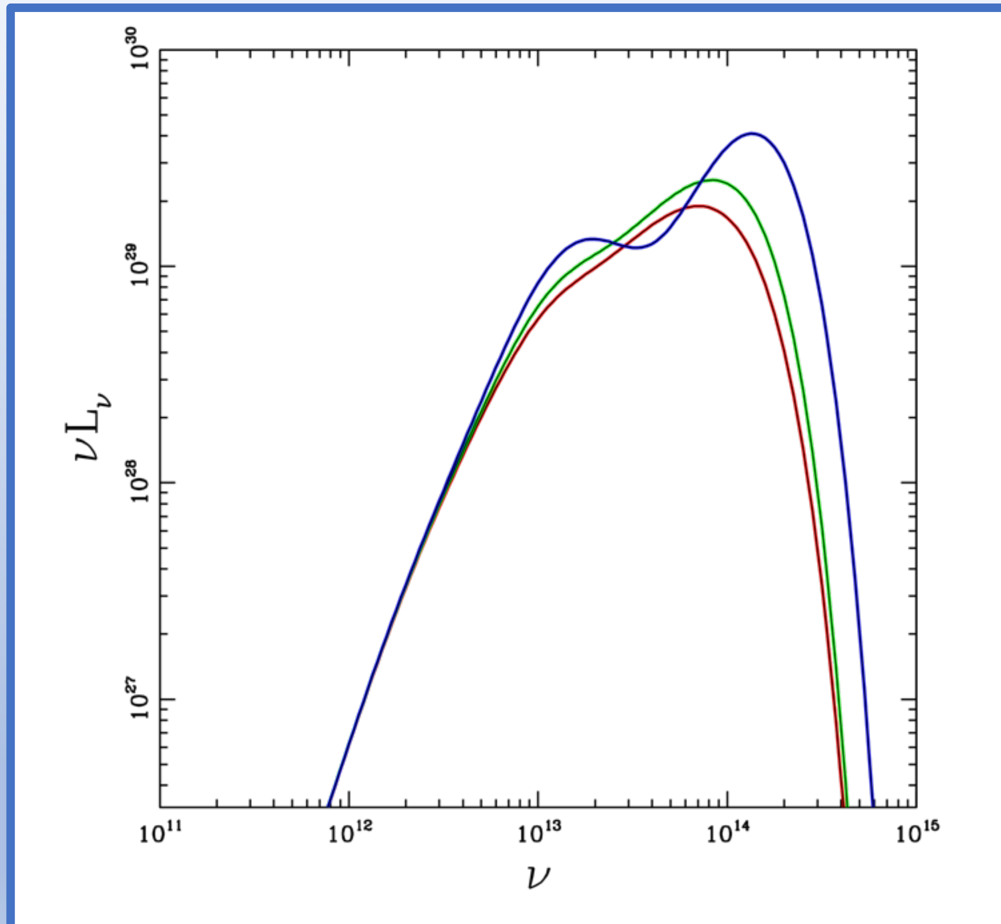
larger accretion rate → brighter and redder



Spectral Energy Distributions vs Planet Magnetic Field Strength

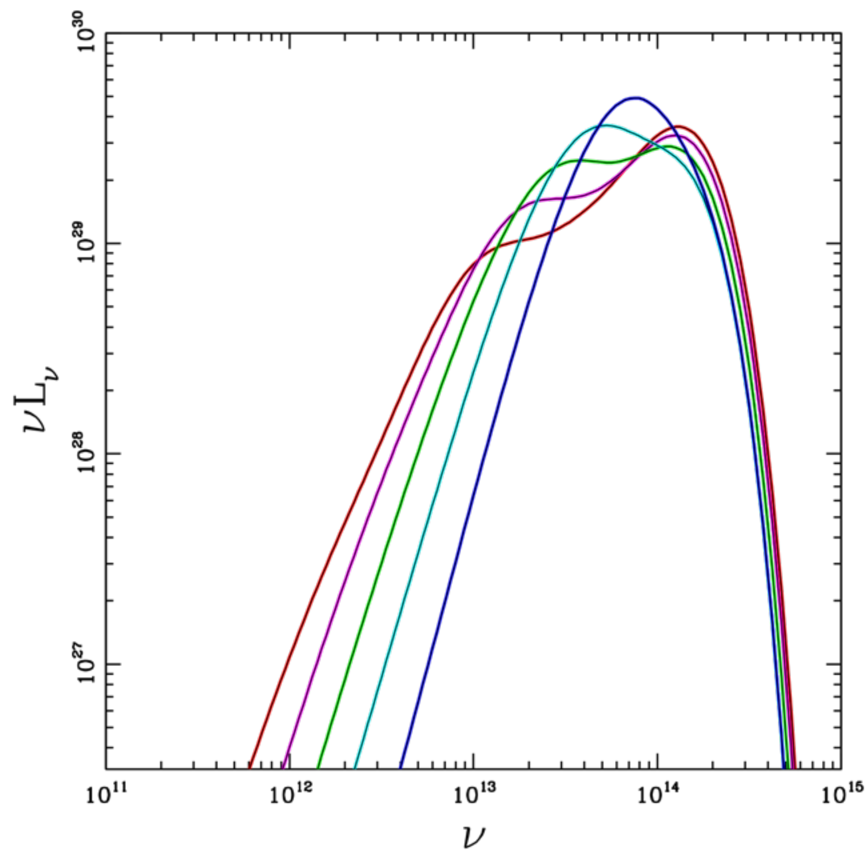
$$B_P = 10 - 1000\ gauss$$
$$M = 1\ M_J$$
$$\dot{M} = 1 M_J/\text{Myr}$$
$$a = 5\ \text{AU}$$

more B field → brighter and bluer

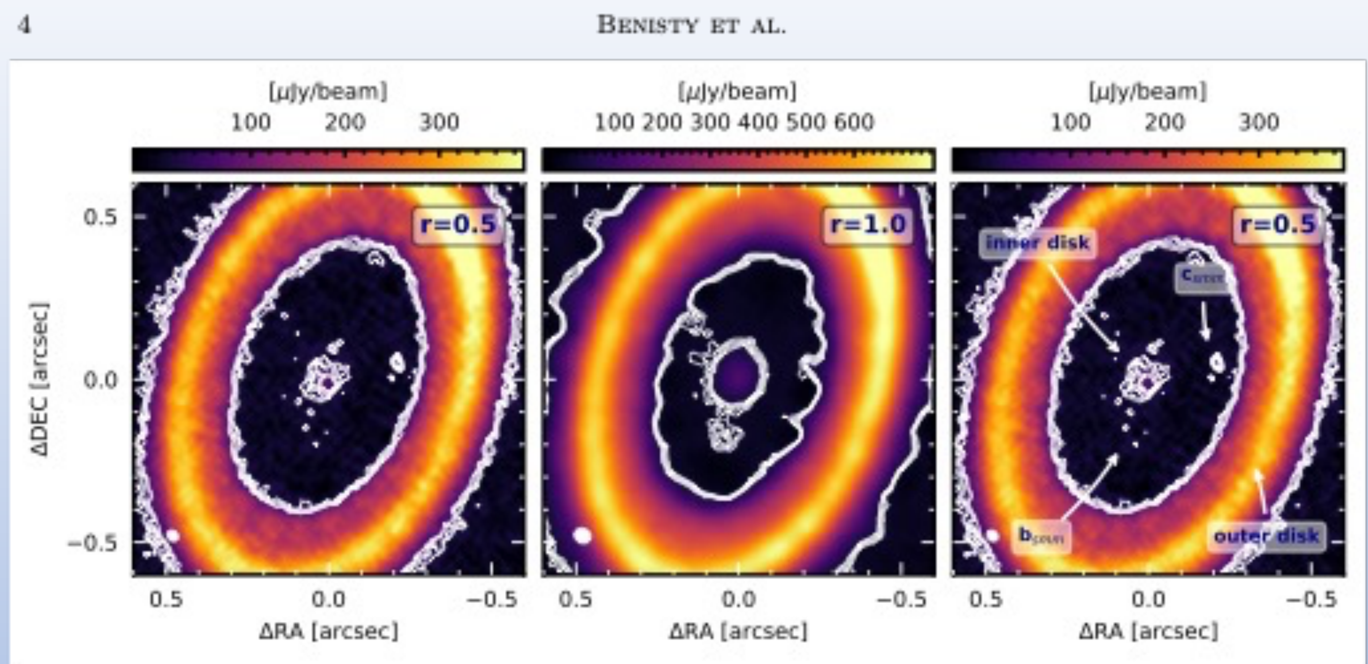


Spectral Energy Distributions vs Semimajor Axis

$$a = 0.1 - 10\ \text{AU}$$
$$M = 1\ M_J$$
$$\dot{M} = 1 M_J/\text{Myr}$$
$$B_P = 500\ gauss$$



larger semimajor axis → wider SED



We can now detect planets forming in circumstellar disks
Next step is to measure spectral energy distributions

Milestones

- Analytic infall solution (density and velocity fields)
- Analytic disk surface density – for infall only and steady state viscous accretion
- Magnetic truncation of inner disk edge
- Magnetic field sets capture radius for planet
- Determination of luminosity contributions
- Spectral energy distributions for forming planets

Solutions for instantaneous values of $(M, \dot{M}, a, B)$
One can construct evolutionary models with $(M, \dot{M}, a, B)[t]$

Discussion

Compare the star formation problem with the giant planet formation problem:

- In both cases, most incoming mass falls onto the disk due to conservation of angular momentum
- SF: dynamic range disk/star=10,000, time = 0.1 Myr
- PF: dynamic range disk/planet=180, time = 3-10 Myr
- Viscous accretion disks provide viable solution to solve angular momentum issue in planet formation

The Paradigm is Incomplete

- Migration – planets can move as they form
- Competition – planets form as part of solar systems
- Flow through the outer boundary is affected by magnetic fields, turbulence, larger structures
- These same agents can exert torques and change the angular momentum budget of forming body
- What ultimately stops mass accretion onto the planet?

The Paradigm is Incomplete

- Migration – planets can move as they form
- Competition – planets form as part of solar systems
- Flow through the outer boundary is affected by magnetic fields, turbulence, larger structures
- These same agents can exert torques and change the angular momentum budget of forming body
- **What ultimately stops mass accretion onto the planet?**