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Cosmic Acceleration

TIARA winter school on cosmology
Taipei, February 2014

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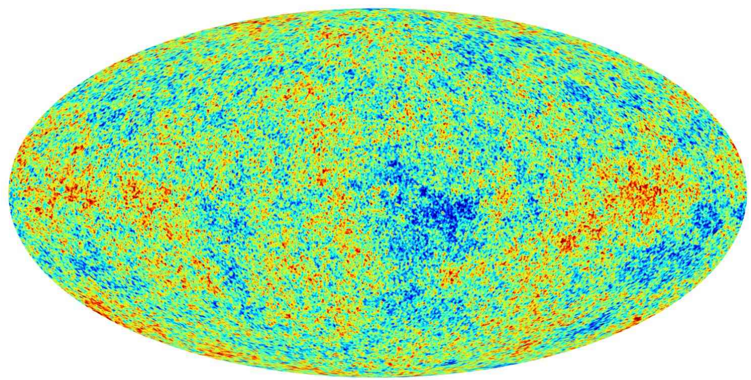

Equivalence Principle and Cosmology

Einstein's Equivalence Principle:

Gravity = Curvature = Acceleration

Gravity is equivalent to the curvature of spacetime geometry, and determines the motions of particles along geodesics.

Forces (acceleration) change the motions of particles can be viewed as affecting spacetime geometry. Locally, acceleration is equivalent to gravity.

Equivalence Principle

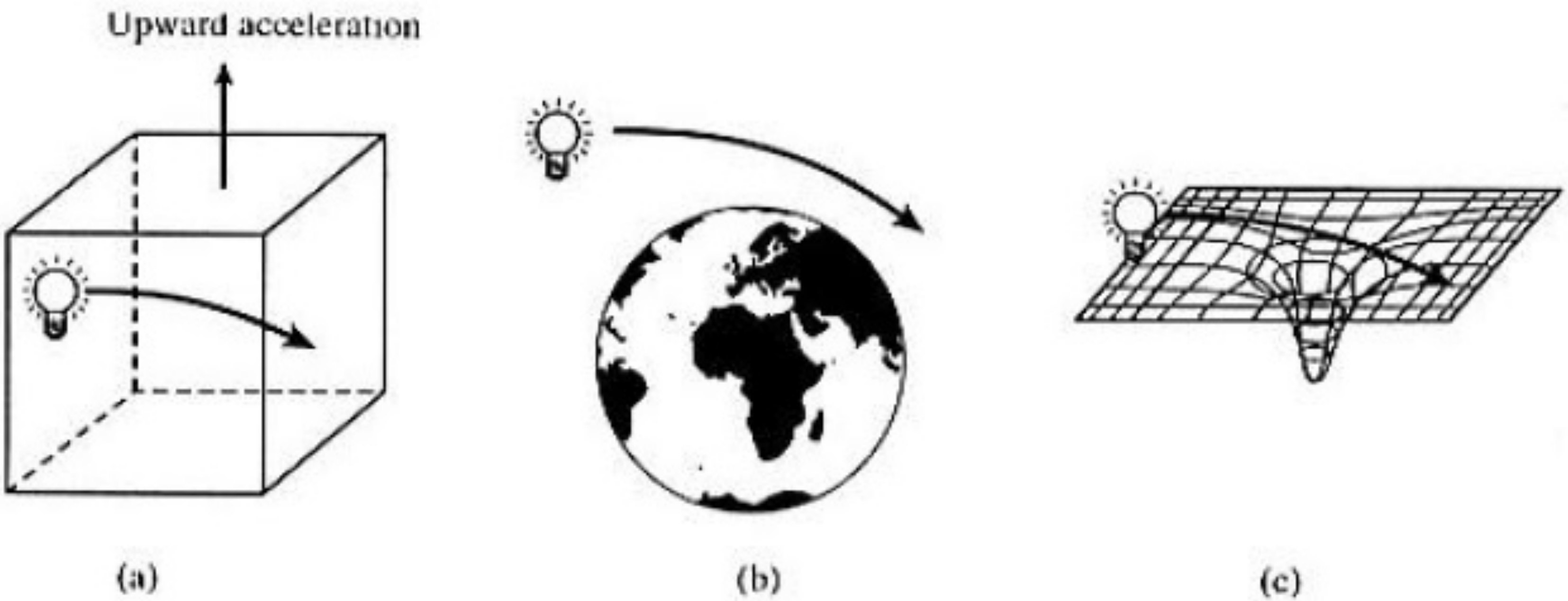

Figure 1.3 The motion of light equivalently interpreted as due to (a) acceleration in an elevator, (b) gravitational force of a mass, or (c) curvature of spacetime.

Acceleration = Gravity

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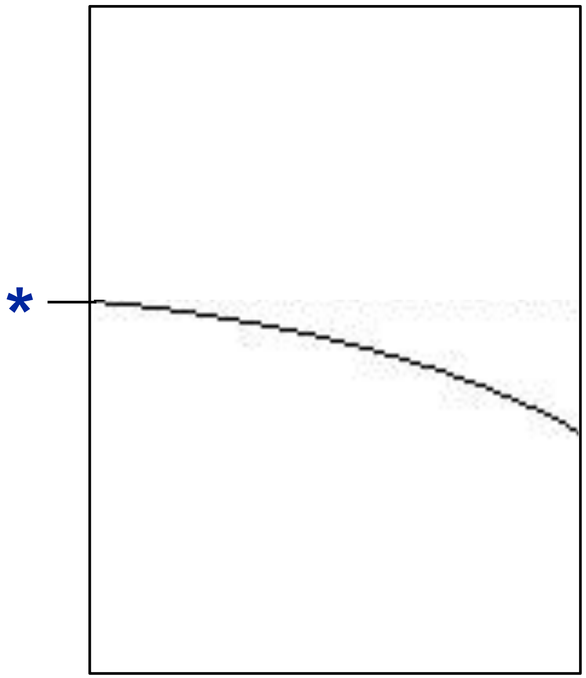

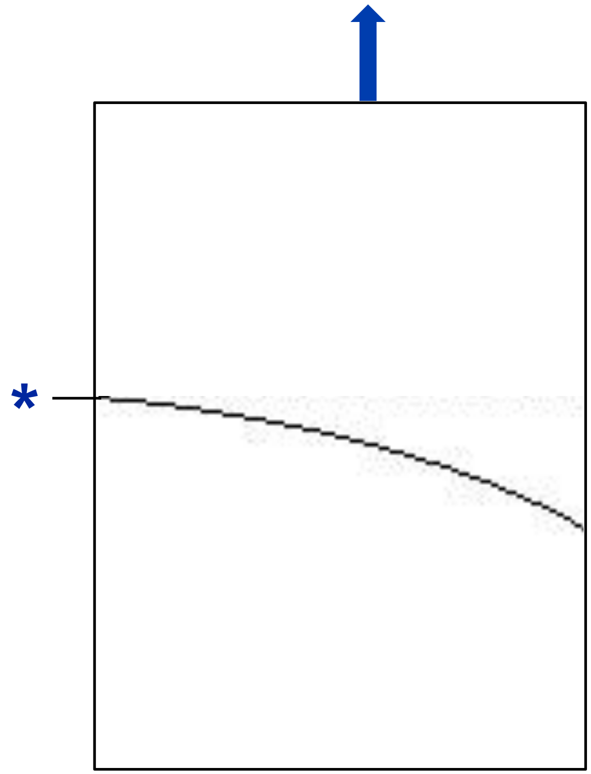

In the presence of gravity or of acceleration, light follows a curved path. Locally, they are equivalent.

Acceleration = Curvature

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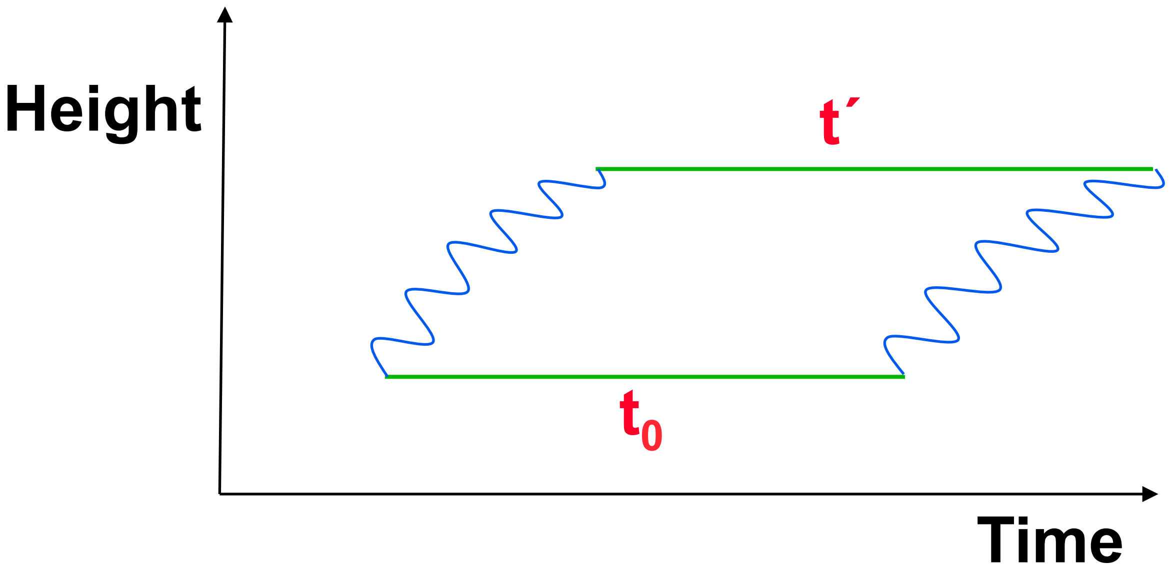
The Principle of Equivalence teaches that

Acceleration = Gravity = Curvature



Acceleration ⇒ over time will get $v=gh/c$,
so $z = v/c = gh/c^2$ (gravitational redshift).

But, $t' \neq t_0$ ⇒ parallel lines not parallel (curvature)!

Spacetime Geometry

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Therefore we describe cosmology in a spacetime with curvature. (This is separate from whether *space* has curvature.)

Homogeneity and isotropy determine the space to be maximally symmetric and the metric takes the Robertson-Walker form.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

The key ingredients are

constant parameter k – spatial curvature,

function of time a(t) – scale (expansion) factor.

Cosmic Expansion

Space flatness: k=0

Spacetime flatness: $\ddot{a} = 0$

The metric can be spatially flat (k=0) but the *spacetime* is curved if $\ddot{a} \neq 0$

This is exactly the Equivalence Principle:

Gravity = Curvature = Acceleration

All results coming directly from the metric (spacetime symmetries) are called kinematics.

We have not had to specify any laws of gravity!

(Results requiring force laws are called dynamics.)

Cosmic Acceleration

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Acceleration has:

- Direct (kinematic) effect on spacetime through a(t)

- Dynamic effects on objects within spacetime, e.g. growth, ISW

What appears in the metric is the cosmic scale factor a(t).

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

The metric can be spatially flat (k=0) but the *spacetime* is curved if $\ddot{a} \neq 0$

This is exactly the Equivalence Principle:

Gravity = Curvature = Acceleration

Cosmological Framework

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Equivalence Principle

→ **Metric description of spacetime**

Homogeneity and Isotropy

→ **Metric is Robertson-Walker (a,k)**

→ **Energy-momentum has perfect fluid form (ρ,p)**

Gravitational Field Eqs (General Relativity)
+ Homogeneity and Isotropy

→ **Friedmann equations for evolution of spacetime**

Equations of State + Friedmann equations

→ **Evolution of energy densities**

Gravitating Energy

Einstein says gravitating mass depends on energy-momentum tensor:

both energy density ρ and pressure p, as ρ+3p

Negative pressure can give negative "mass"

Newton's 2nd law: Acceleration = Force / mass

$$\ddot{R} = -GM/R^2 = -(4\pi/3)G\,\rho\,R$$

Einstein/Friedmann equation:

$$\ddot{a} = -(4\pi/3)G\,(\rho+3p)\,a$$

Negative pressure can accelerate the expansion

Expansion History

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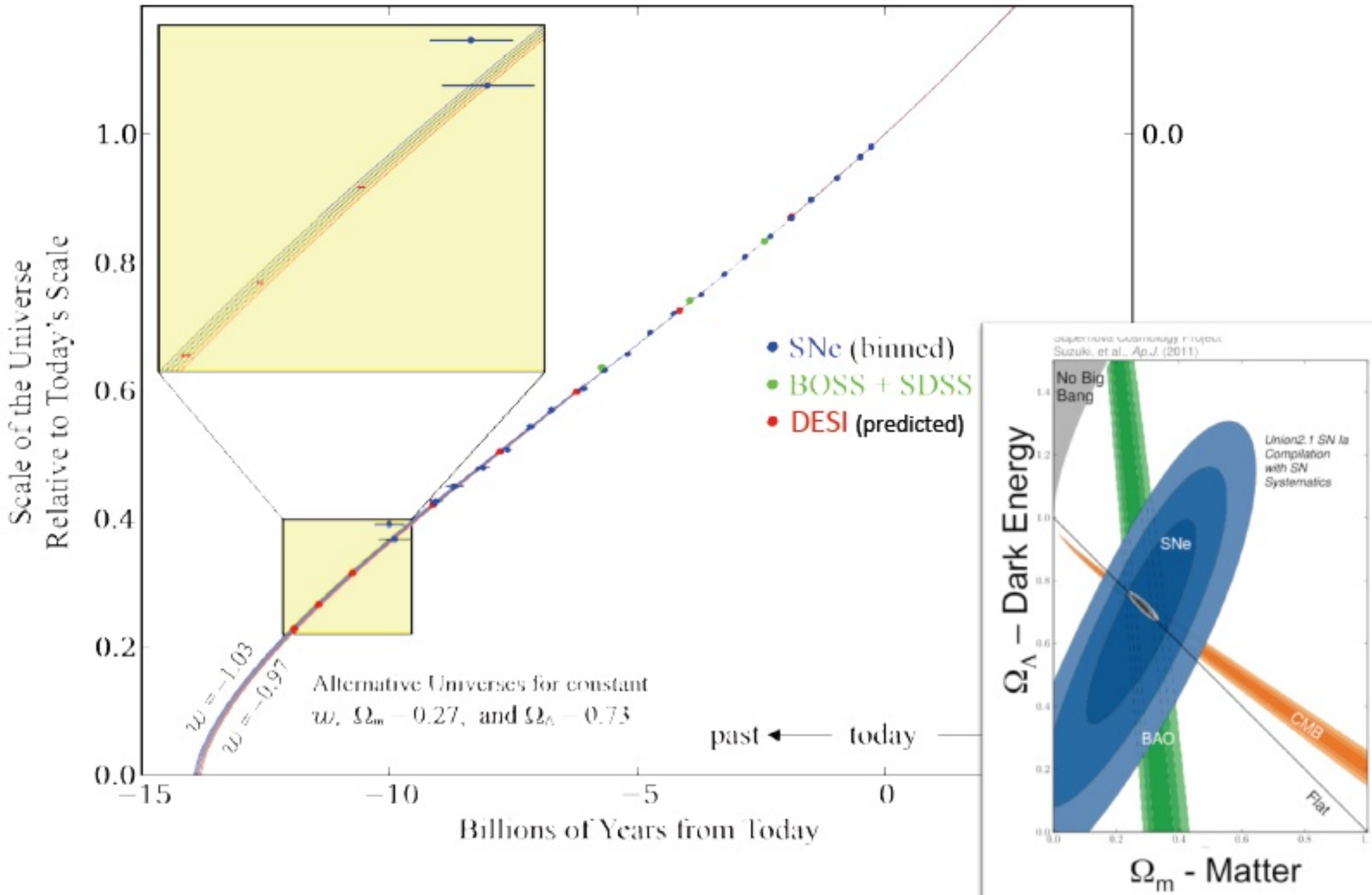
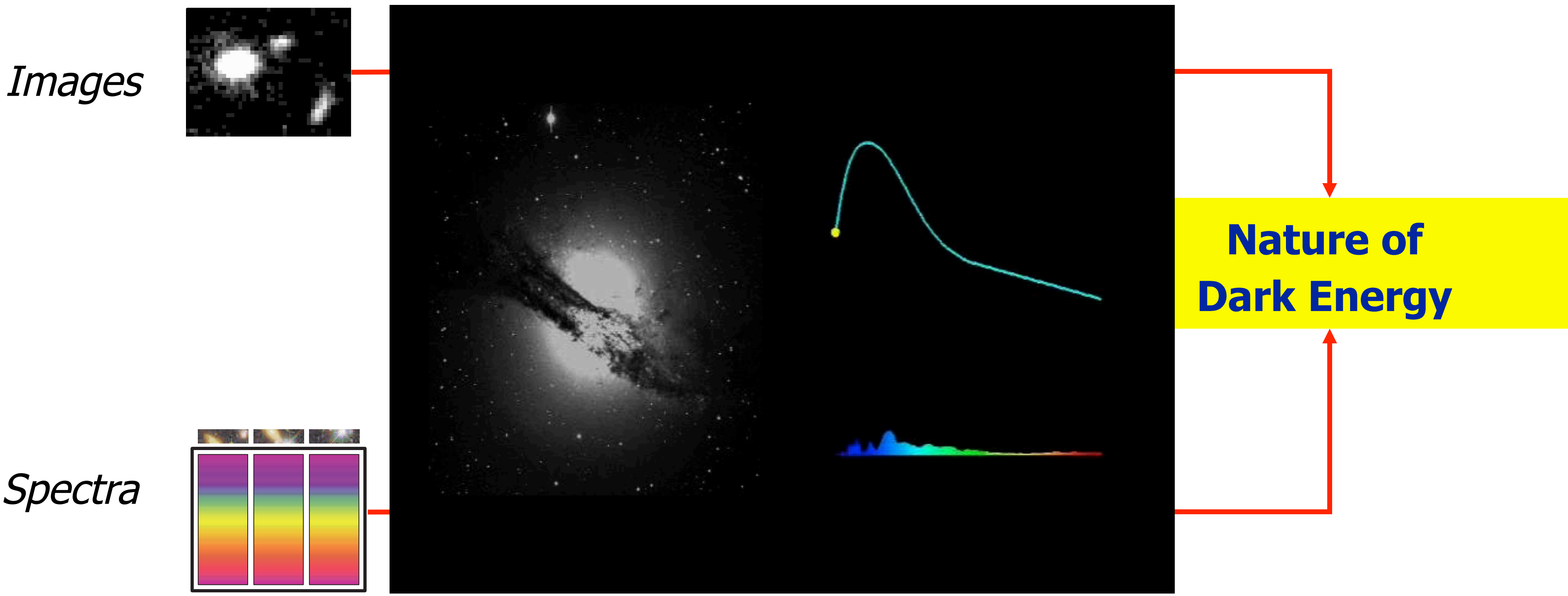
Distances come directly from the metric. Measuring them through, e.g., Type Ia supernovae, maps the expansion history a(t) and hence acceleration.



Type Ia Supernovae

Each supernova is "sending" us a rich stream of information about itself.



data → **analysis** → **physics**

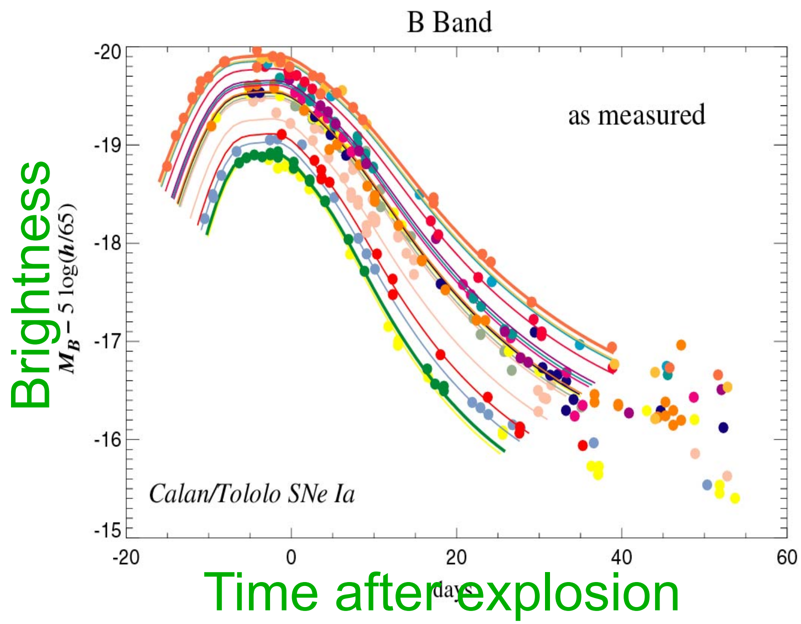
Supernova Distances

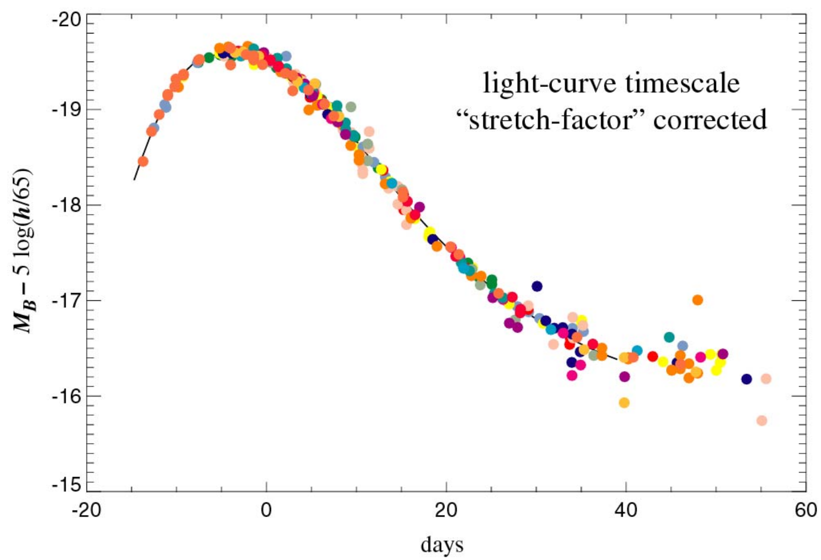
Luminosity distance using a "standardized candle".

Standardization:
empirical **(nuclear physics and degenerate electron gas).**

Advancement: 7% distance measure improves to 3% with spectroscopic data.

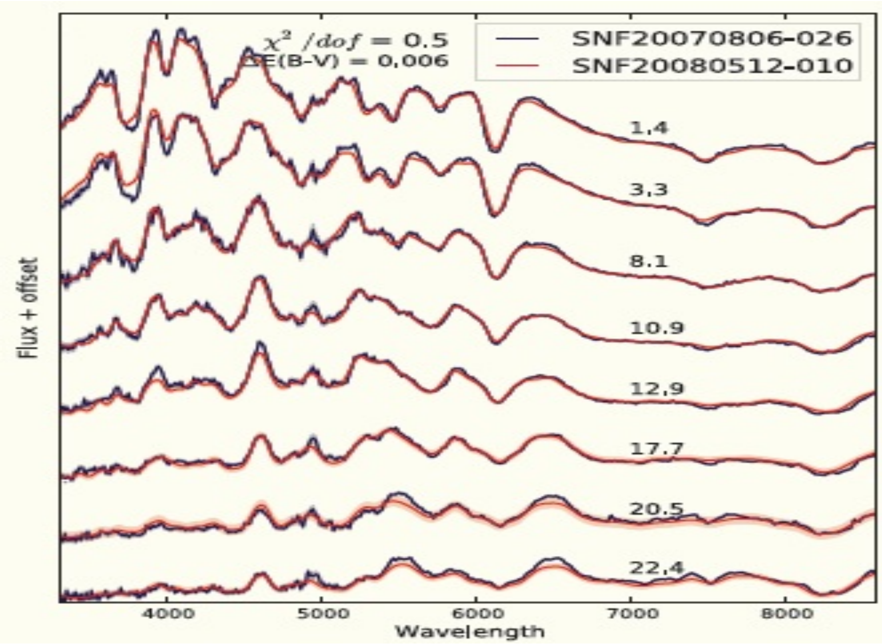
Main systematic: flux calibration.





Kim, *et al.* (1997)

Supernova Twinness



Some supernovae look nearly identical in spectra – twinness.

Comparing them at different redshifts gives tight Hubble diagram.

The "twinner", the more homogeneous. 3% distances per SN!

Fakhouri 2013
SNfactory

Supernova Data

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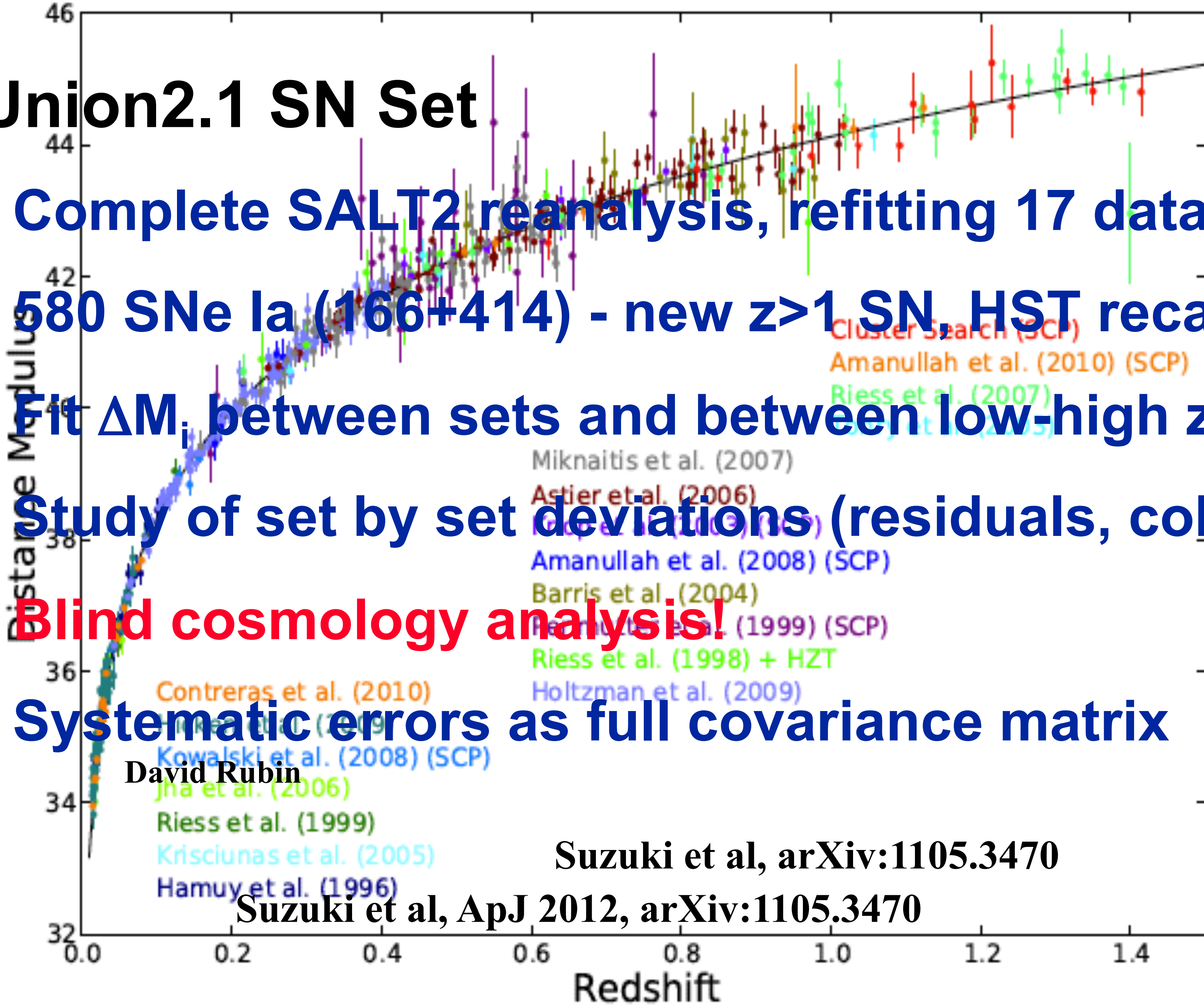
Union2.1 SN Set

- Complete SALT2 reanalysis, refitting 17 data sets
- 580 SNe Ia (166+414) - new z>1 SN, HST recalib
- Fit ΔM_i between sets and between low-high z
- Study of set by set deviations (residuals, color)
- **Blind cosmology analysis!**
- Systematic errors as full covariance matrix



David Rubin

Suzuki et al, arXiv:1105.3470

Suzuki et al, ApJ 2012, arXiv:1105.3470

Describing Our Universe

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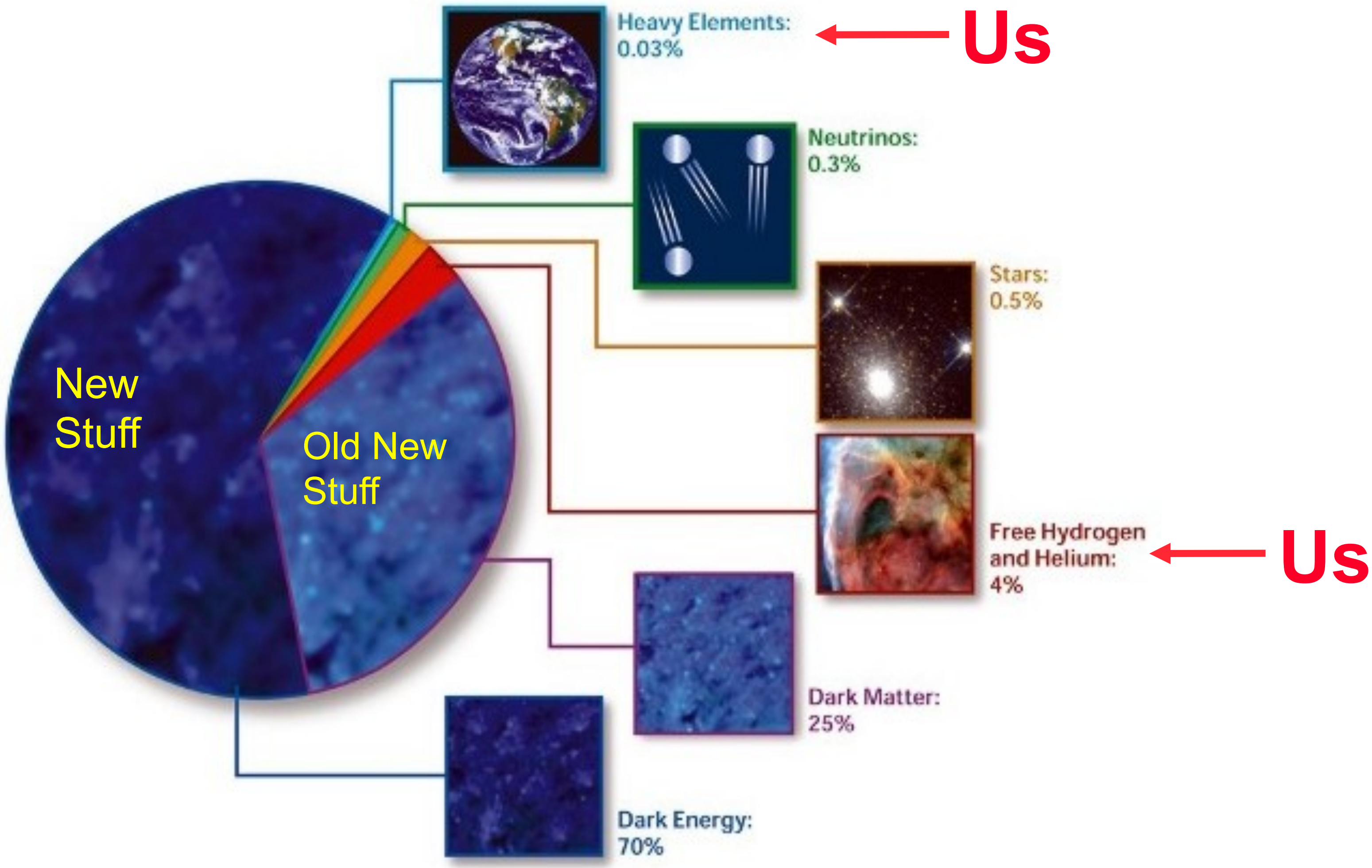

STScI

95% of the universe is unknown!

Role of Observations

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小红帽

But Λ, what big teeth you have!

Before we jump into bed with Λ, we should be sure it is not something more beastly.

Role of Theory

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Cosmic acceleration and dark energy are fundamental mysteries in understanding our universe.

Copernican Principle / Cosmic Modesty:

- **Our galaxy is not the center of the universe.**
- **Our particles are not the matter/energy of the universe.**
- **Is our vacuum the vacuum of the universe?**
- **Is our gravity the gravity of the universe?**

Finding Our Way in the Dark

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Dark energy is a completely unknown animal.

A new theory or a new component?

Track record:

Outer solar system motions → Neptune

Inner solar system motions → General Relativity

Galaxy rotation curves → Dark Matter

Nature of Acceleration

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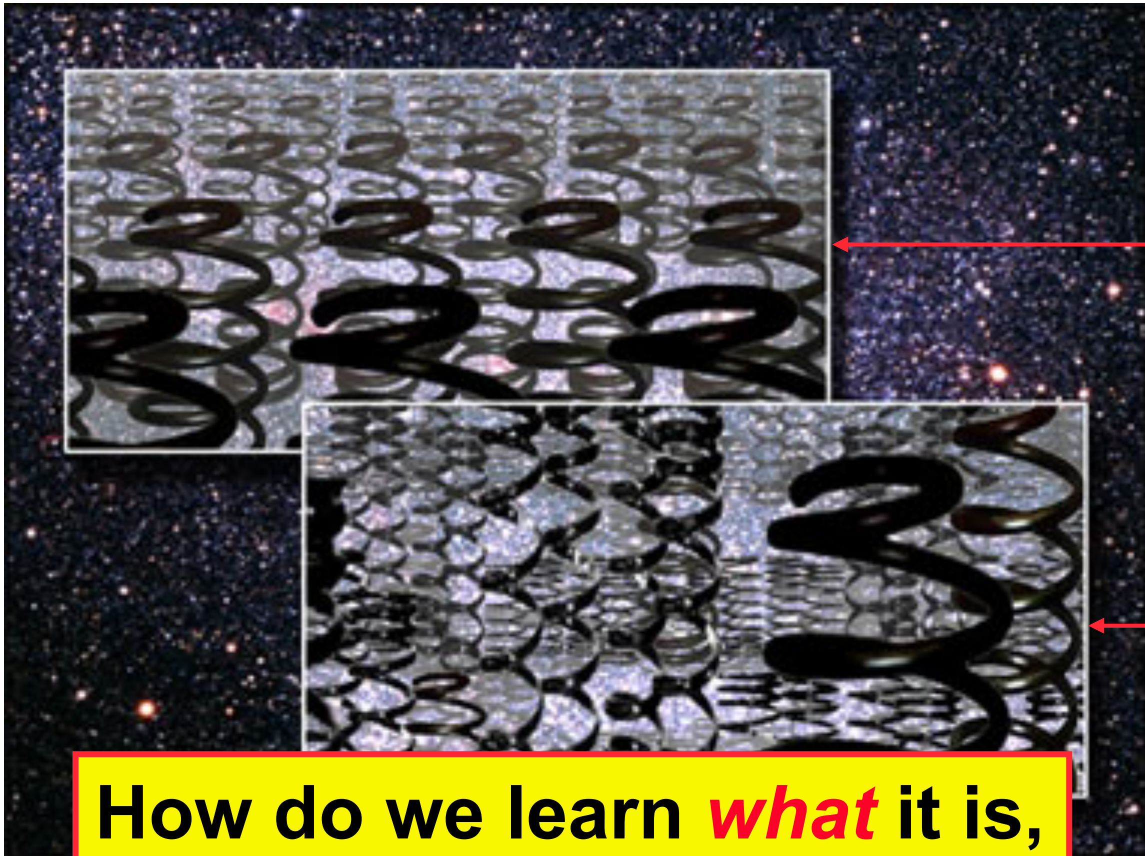

How do we learn ***what*** it is, not just ***that*** it is?

Is dark energy static? Einstein's cosmological constant Λ.

Is dark energy dynamic? A new, time- and space-varying field.

Is dark energy a change in gravity?

How much dark energy is there? Ω_{DE}

How springy/stretchy is it? $w=P/\rho$

A new law of gravity, or a new component? $G_N(k,z)$

Theory of Fields

Scalar field:

At every point in a field of grass, you can measure the height of the grass: a single number or scalar h(x).

Vector field:

At every point in a trampled field of grass, you can measure the length of the grass and the direction it is lying: a vector $\vec{g}(x)$.

Scalar Field Theory

Scalar field Lagrangian - canonical, minimally coupled

$$\mathcal{L}_\phi = (1/2)(\partial_\mu\phi)^2 - V(\phi)$$

Noether prescription → Energy-momentum tensor

$$T_{\mu\nu} = (2/\sqrt{-g})\,[\,\delta(\sqrt{-g}\,\mathcal{L})/\delta g_{\mu\nu}\,]$$

Perfect fluid form (from RW metric)

Energy density $\rho_\phi = (1/2)\,\dot{\phi}^2 + V(\phi) + (1/2)(\nabla\phi)^2$

Pressure $p_\phi = (1/2)\,\dot{\phi}^2 - V(\phi) - (1/6)(\nabla\phi)^2$

$+ (1/2)(\nabla\phi)^2$

Scalar Field Equation of State

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Equation of state ratio

$$w = p/\rho$$

Klein-Gordon equation (Lagrange equation of motion)

$$\ddot{\phi} + 3H\dot{\phi} = -dV(\phi)/d\phi$$

Continuity equation follows KG equation

$$[(1/2)\dot{\phi}^2]^{\cdot} + 6H\,[(1/2)\dot{\phi}^2] = -\dot{V}$$

$$\dot{\rho} - \dot{V} + 3H\,(\rho+p) = -\dot{V}$$

$$d\rho/d\,ln\,a = -3(\rho+p) = -3\rho\,(1+w)$$

$$\rho_i(a) = \rho_i\, e^{-3\int_0^{\ln a} d\ln a'\,[1+w_i(a')]} \sim a^{-3(1+w_i)}$$

Equation of State

Limits of (canonical) Equations of State:

$$w = (K-V) / (K+V)$$

Potential energy dominates (slow roll)

$$V >> K \Rightarrow w = -1$$

Kinetic energy dominates (fast roll)

$$K >> V \Rightarrow w = +1$$

Oscillation about potential minimum (or coherent field, e.g. axion)

$$\langle V \rangle = \langle K \rangle \Rightarrow w = 0$$

Equation of State

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Reconstruction from EOS:

$\rho(a) = \Omega_\phi \rho_c \exp\{ 3 \int d\ln a\, [1+w(z)] \}$

$\phi(a) = \int d\ln a\, H^{-1} \mathrm{sqrt}\{ \rho(a) [1+w(z)] \}$

$V(a) = (1/2)\, \rho(a)\, [1-w(z)]$

$K(a) = (1/2)\dot{\phi}^2 = (1/2)\, \rho(a)\, [1+w(z)]$

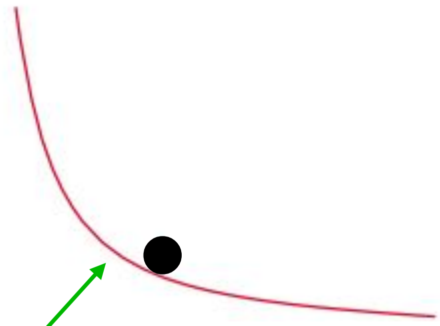
But, $\dot{\phi} \sim \sqrt{[(1+w)\rho]} \sim \sqrt{(1+w)}\, HM_p$

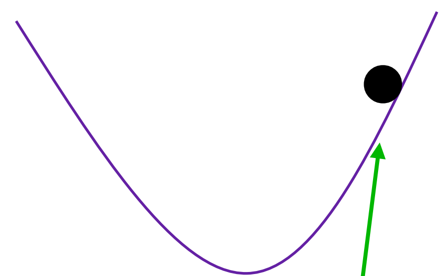
So if 1+w << 1, then $\Delta\phi \sim \dot{\phi}/H << M_p$.

It is very hard to directly reconstruct the potential.

Goldilocks problem: Dark energy is unlike Inflation!

Dynamics of Quintessence





Equation of motion of scalar field

$$\ddot{\phi} + 3H\dot{\phi} = -dV(\phi)/d\phi$$

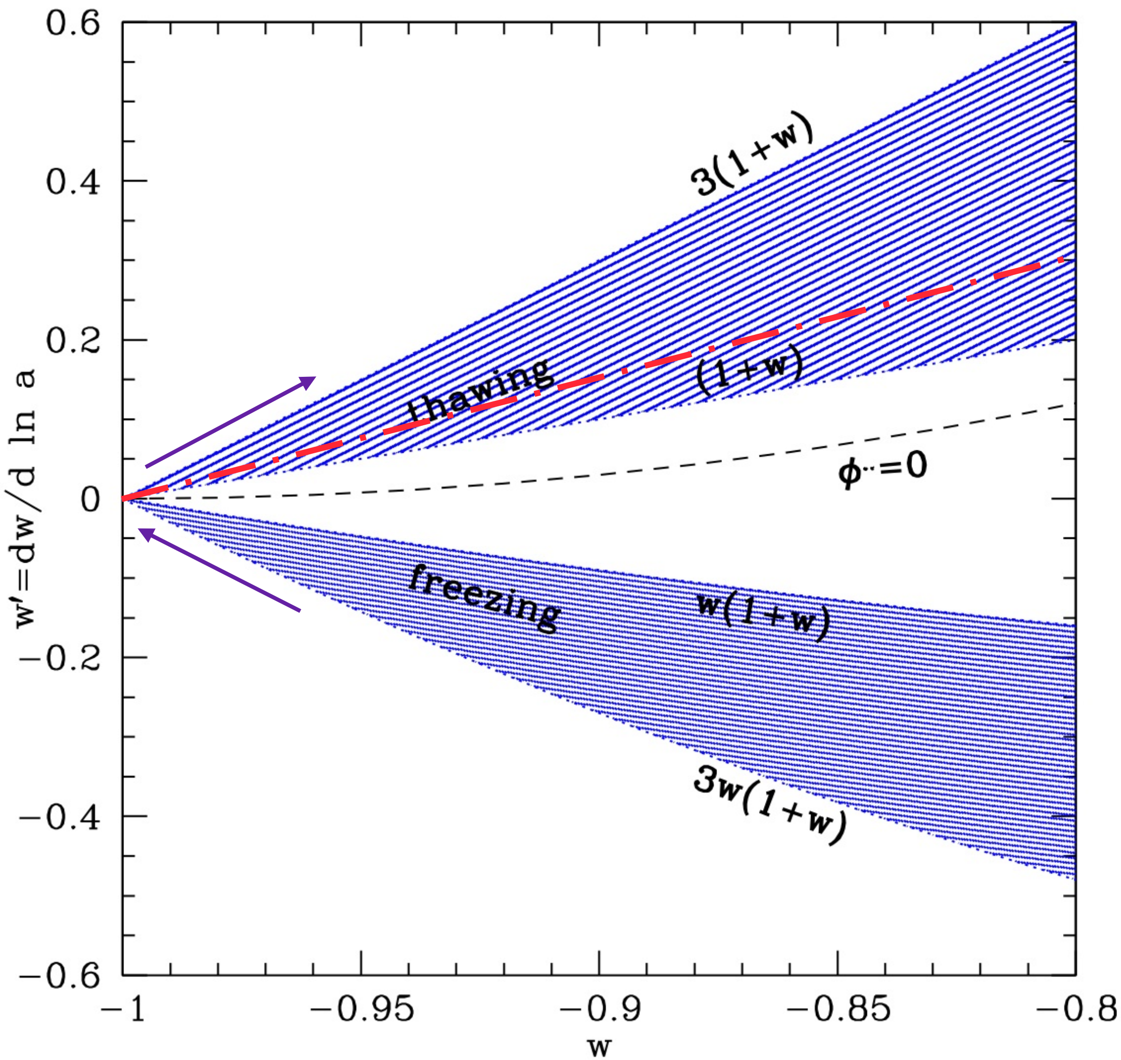
- driven by steepness of potential
- slowed by Hubble friction

Broad categorization – which term dominates:

- field rolls but decelerates as dominates energy
- field starts frozen by Hubble drag and then rolls

Freezers vs. Thawers

Limits of Quintessence



$$w = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$$

Distinct, narrow regions of w-w′

Caldwell & Linder 2005
PRL 95, 141301

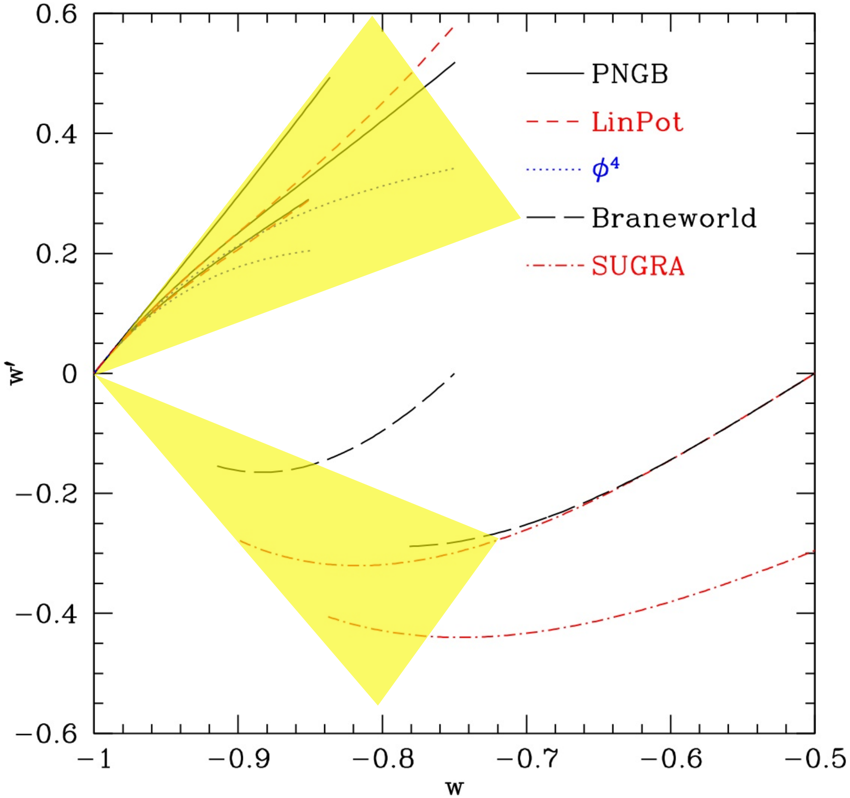
Entire "thawing" region looks like <w> = -1 ± 0.05.

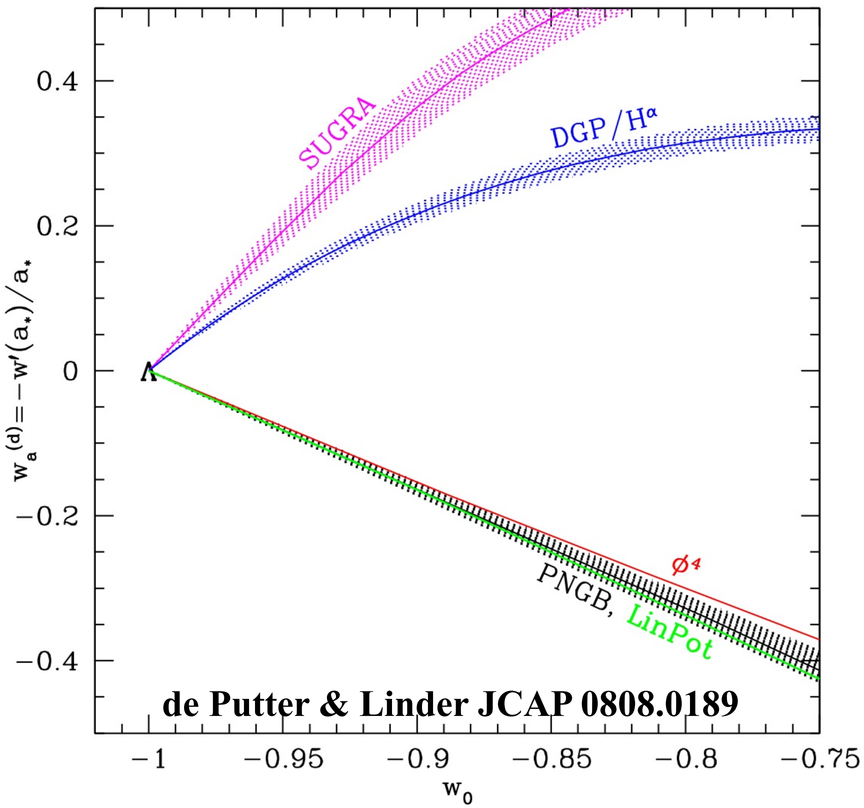
Need w′ experiments with $\sigma(w') \approx 2(1+w)$.

Calibrating Dark Energy

Models have a diversity of behavior, within thawing and freezing.

But we can calibrate w' by "stretching" it: $w' \rightarrow w'(a_*)/ a_*$. Calibrated parameters w_0, w_a.





The two parameters w_0, w_a achieve 10^{-3} level accuracy on observables d(z), H(z).

$w(a)=w_0+w_a(1-a)$

This is from physics (Linder 2003). It has *nothing* to do with a Taylor expansion.

Solving the Equation of Motion

Klein-Gordon equation

$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi}$$

Transform to new variables

$$x \equiv \frac{\kappa\dot{\phi}}{\sqrt{6}\,H} \quad ; \quad y \equiv \frac{\kappa\sqrt{V}}{\sqrt{3}\,H}$$

$$' = \frac{d}{d\ln a}$$

$$H^2 = (\kappa^2/3)[\rho_m + (1/2)(\dot{\phi})^2 + V]$$

Autonomous system

$$x' = -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right]$$

$$y' = -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right] \ ,$$

Copeland, Liddle, Wands 1998
Phys. Rev. D 57, 4686

where

$$\kappa^2 = 8\pi G \quad ; \quad \gamma = 1 + w_b \quad ; \quad \lambda = \frac{-V_{,\phi}}{\kappa V}$$

Transform solution to

$$\Omega_\phi = x^2 + y^2 \quad ; \quad w = \frac{x^2 - y^2}{x^2 + y^2}$$

Can add equation for EOS dynamics

$$w' = -3(1 - w^2) + \lambda(1 - w)\sqrt{3(1 + w)\Omega_\phi}$$

Caldwell & Linder 2005
Phys. Rev. Lett 95, 141301

Physics of Growth

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Growth g(a)=(δρ/ρ)/a depends purely on the expansion history H(z) – *and* gravity theory.

$$g'' + [5 + \tfrac{1}{2}\frac{d\ln H^2}{d\ln a}]g'a^{-1} + [3 + \tfrac{1}{2}\frac{d\ln H^2}{d\ln a} - \tfrac{3}{2}G\,\Omega_m(a)]\,ga^{-2} = 0$$

Within general relativity ($G=G_N=1$), expansion determines growth and vice versa.

Acceleration suppresses growth in two ways:

1) the friction term ~ (3-q) so q<0 slows growth,

2) the source term $\Omega_m(a)$ is diminished.

ISW Potentials and Acceleration

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The Integrated Sachs Wolfe (ISW) is the redshift of light passing thru growing gravitational potentials. It has been claimed to be a direct probe of acceleration. **Is it?**

Gravitational potential ϕ stays constant during matter domination.

$$\nabla^2\phi \to (k/a)^2\phi = 4\pi G\,\delta\rho_{\rm tot} \approx 4\pi G\rho_{\rm m}(\delta\rho/\rho)_{\rm m}$$

For matter domination, $\delta \sim a$, so $\phi \sim$ const.
ISW arises from $\dot{\phi}=H\phi(f-1)$ so no effect in matter domination.

ISW only shows breakdown of matter domination, *not* acceleration.

The Direction of Gravity

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Cosmic acceleration: Gravity is pulling *out* not down!

Is gravity (G_{Newton}) constant, or strengthening, or weakening with time?

Does gravity govern the growth of large scale structure exactly as it does for cosmic expansion, or are there more degrees of freedom?

Effect of gravity on light (strong/weak lensing).

Does gravity behave the same on all scales?

Dark energy motivates us to ask "what happens when gravity no longer points down?".

Expansion History

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Observations that map out expansion history a(t), or w(a), tell us about the fundamental physics of dark energy.

Alterations to Friedmann framework → *w(a)*

Suppose we admit our ignorance:

$$H^2 = (8\pi/3)\,\rho_m + \delta H^2(a)$$

gravitational extensions or high energy physics

Effective equation of state:

$$w(a) = -1 - (1/3)\, d\ln(\delta H^2) / d\ln a$$

Modifications of the expansion history are equivalent to time variation w(a). ***Period.***

Expansion History

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For modifications δH^2, define an effective scalar field with

$$V = (3M_P^2/8\pi)\ \delta H^2 + (M_P^2 H_0^2/16\pi)\ [\ d\ \delta H^2/d\ ln\ a]$$

$$K = -\ (M_P^2 H_0^2/16\pi)\ [\ d\ \delta H^2/d\ ln\ a]$$

Example: $\delta H^2 = A(\rho_m)^n$

$$w = -1+n$$

Example: $\delta H^2 = (8\pi/3)\ [g(\rho_m) - \rho_m]$

$$w = -1 + (g'-1)/[\ g/\rho_m - 1\ ]$$

Cosmological Framework

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Comparing cosmic expansion history vs. cosmic growth history is one of the major tests of the cosmological framework.

If do *not* simultaneously fit then deviation in one biases the other, e.g. looks like non-GR or non-Λ.

Approach 1: Separate out the expansion influence on the growth – gravitational growth index γ.

Approach 2: Parametrize equations of motion, i.e. Poisson equation and lensing equation – gravity functions $G_{matter}(k,a)$, $G_{light}(k,a)$.

Physics of Growth

Growth $g(a)=(\delta\rho/\rho)/a$ depends purely on the expansion history $H(z)$ -- *and* gravity theory.

$$g'' + [5 + \tfrac{1}{2}\frac{d\ln H^2}{d\ln a}]g' a^{-1} + [3 + \tfrac{1}{2}\frac{d\ln H^2}{d\ln a} - \tfrac{3}{2}G\,\Omega_m(a)]\,g a^{-2} = 0$$

Expansion effects via $w(z)$, but *separate* effects of gravity on growth.

$$g(a) = \exp\{\int_0^a d\ln a\,[\Omega_m(a)^\gamma - 1]\}$$

Linder 2005

Growth index γ is valid parameter to describe modified gravity. Accurate to 0.1% in numerics.

Similar to Peebles 1980 ($\gamma=0.6$) and Wang & Steinhardt 1998 (constant w).

Testing Gravity

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Test gravity in model independent way.

Gravity and growth: $\nabla^2\phi = 4\pi G\, a^2\, \delta\rho$

Gravity and acceleration: $-\vec{\nabla}\psi = \ddot{x}$

Are ϕ and ψ the same? (yes, in GR)

Tie to observations via modified Poisson equations:

$$\nabla^2(\phi+\psi) = 8\pi G_N\, a^2\, \delta\rho \times G_{\rm light}$$

$$\nabla^2\psi = 4\pi G_N\, a^2\, \delta\rho \times G_{\rm matter}$$

$G_{\rm light}$ tests how light responds to gravity: central to lensing and integrated Sachs-Wolfe.

cf Bertschinger & Zukin 2008

$G_{\rm matter}$ tests how matter responds to gravity: central to growth and velocities (γ is closely related).

What is Dark Energy?

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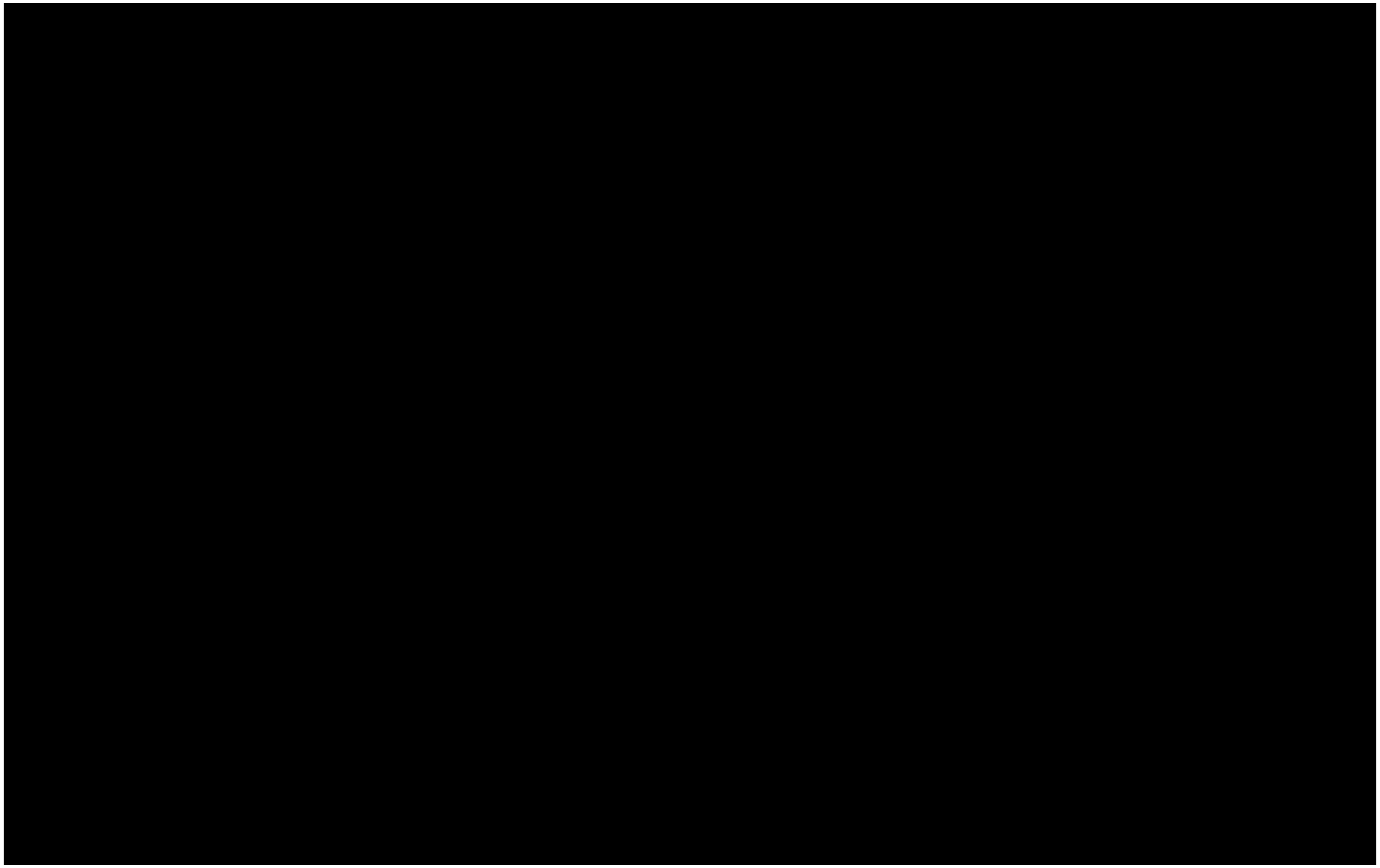

How many dark rectangles do you see?

Dark Energy Properties

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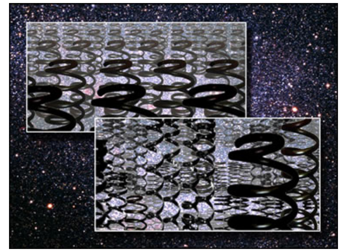
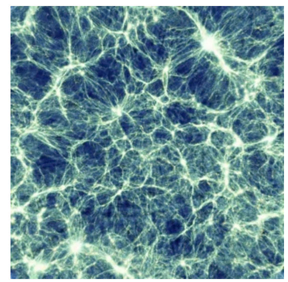
Dark energy is very much *not* the search for one number, "w".

Dynamics: Theories other than Λ give time variation $w(z)$. Form $w(z)=w_0+w_a z/(1+z)$ accurate to 0.1% in observable.



Degrees of freedom: Quintessence determines sound speed $c_s^2=1$. Barotropic DE has $c_s^2(w)$. But generally have $w(z)$, $c_s^2(z)$. Is DE cold ($c_s^2<<1$)? Cold DE enhances perturbations.



Persistence: Is there early DE (at $z>>1$)? $\Omega_\Lambda(z_{CMB})\sim10^{-9}$ but observations allow 10^{-2}.

Observational Leverage

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Dynamics: High+low redshift, complementarity (e.g. SN+SL, SN+CMB/BAO)

Degrees of freedom: Sensitivity to perturbations (CMB lensing, Galaxy clustering)

Persistence: High z probes (CMB lensing, Crosscorrelate CMB x Galaxies)

Test Gravity: Expansion vs growth (SN/BAO + CMBlens/Gal/WL)

Very much a *program*:
Multiple, complementary, diverse observations.
Coordinated Theory/Simulation/Observation essential.

Think About

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Exercise 1: Since the scale factor changes with time, a(t), the expansion rate H also changes with time. Show that the sign of redshift drift

$$dz/dt_0=d[a(t_0)/a(t_e)]/dt_0$$

gives the sign of acceleration (and write in terms of H).

Exercise 2: Show that $\ddot{a}$ = 0 is equivalent to a flat (Minkowski) spacetime. [Not as easy as it may look!]

Exercise 3: Solve the dynamics for a DBI scalar field

$$\mathcal{L}_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2}$$

$$H^2 = \frac{\kappa^2}{3}\left[\rho_m + \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} - V(\phi)\sqrt{1-\dot{\phi}^2}\right]$$

Exercise 4: Put a fundamental scale in the Friedmann equation for H(a), say as a power law $\delta H^2=(H/r_c)^n$. What is w(a) and the early/late time behavior?

Further Reading

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For more cosmic acceleration resources, see

http://supernova.lbl.gov/~evlinder/scires.html

Resource Letter on Dark Energy http://arxiv.org/abs/0705.4102

Mapping the Cosmological Expansion http://arxiv.org/abs/0801.2968

Frieman, Turner, Huterer 2008, *Dark Energy and the Accelerating Universe* http://arxiv.org/abs/0803.0982

and the references cited therein.

For resources on dark energy as a field, see

Copeland, Sami, Tsujikawa 2006, *Dynamics of Dark Energy* http://arxiv.org/abs/hep-th/0603057 and the references cited therein.

For resources on dark energy as gravity, see

Jain & Khoury 2010, *Cosmological Tests of Gravity* http://arxiv.org/abs/1004.3294 and the references cited therein.