

Large Scale Structure

Lecture One

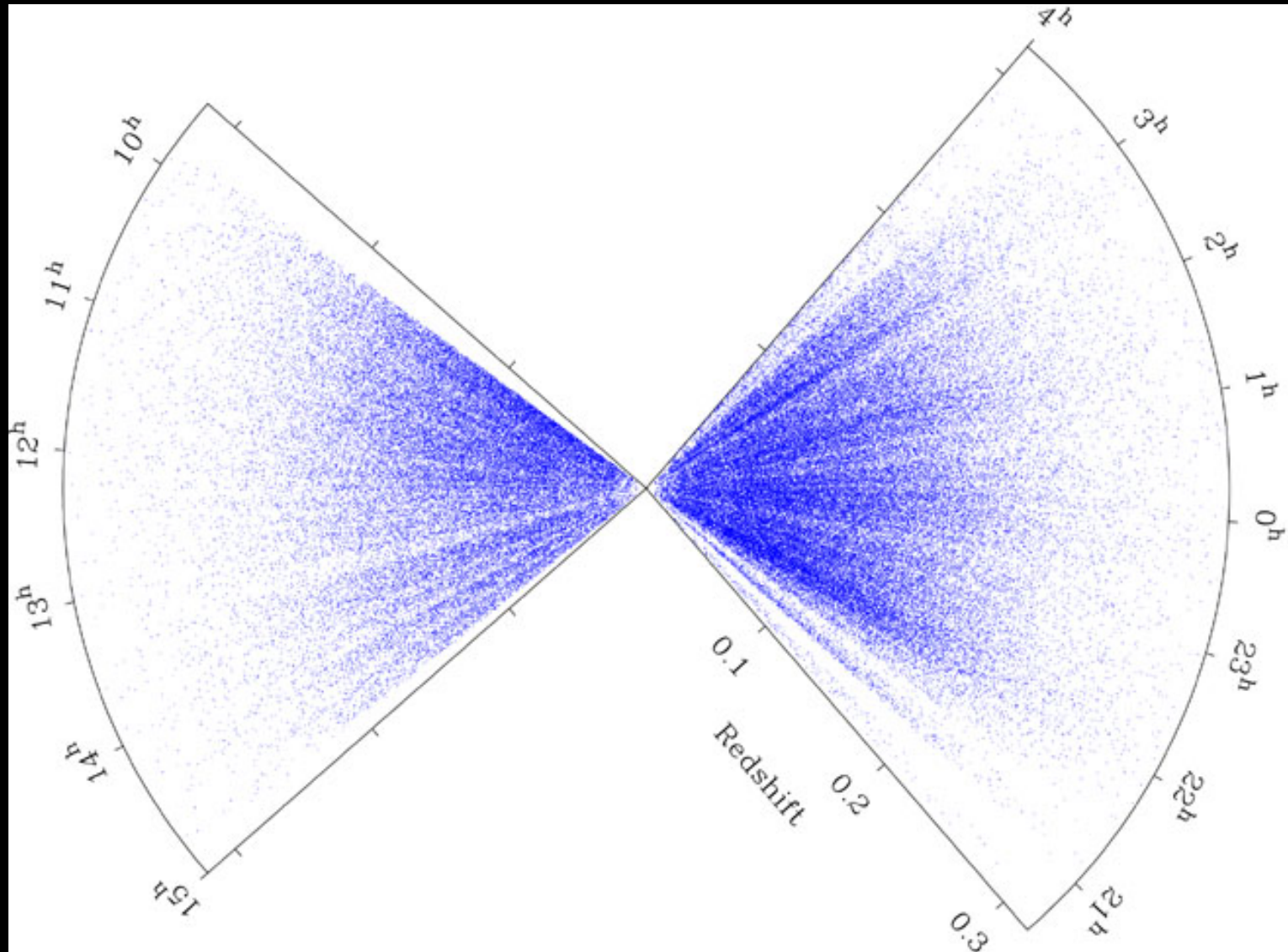
Theory, BAO, and Growth

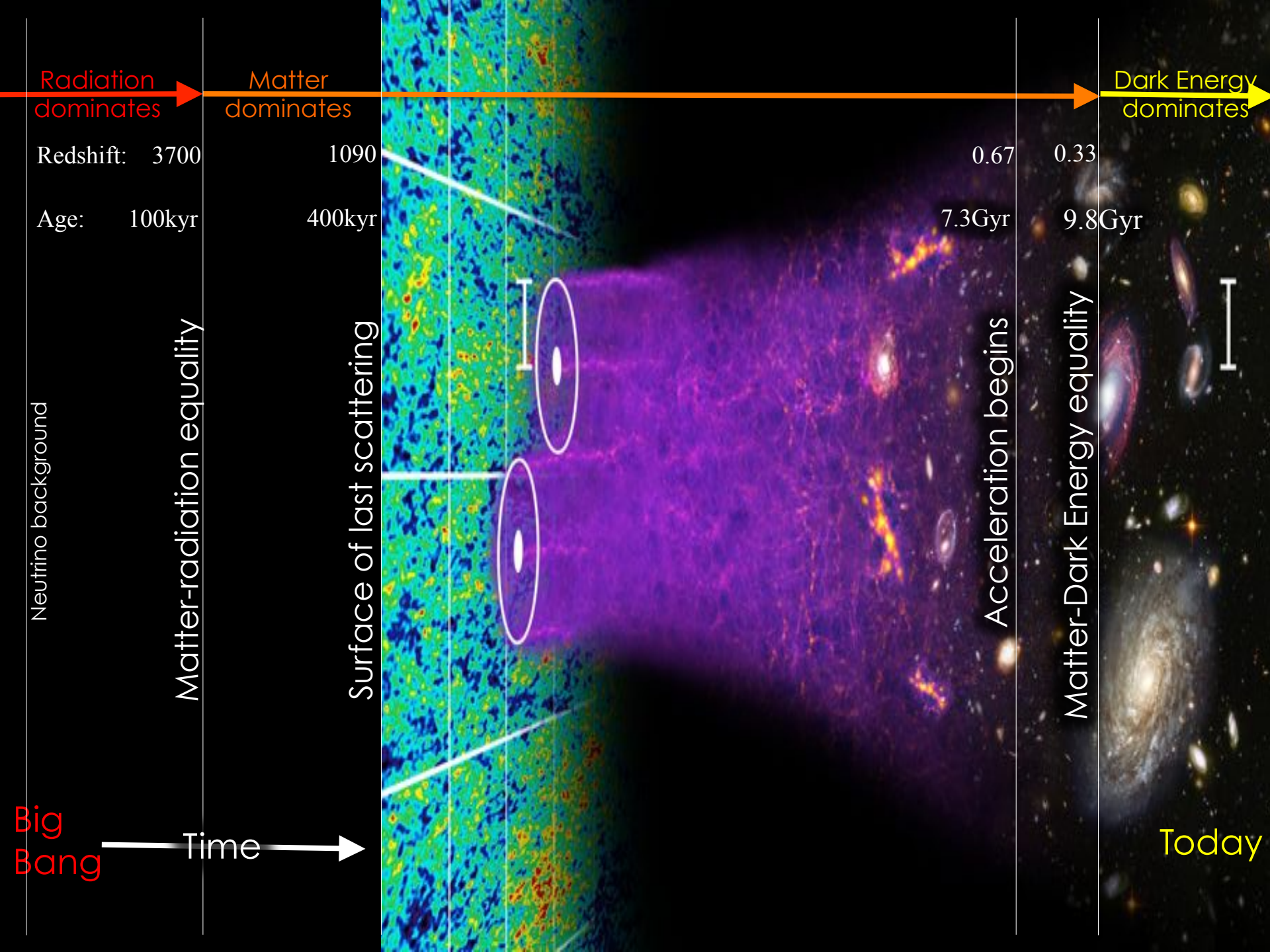
TIARA Winter School, Taipei, February 2014

A/Prof. Tamara Davis

University of Queensland

Where does structure come from?





Initial density fluctuations:

Adiabatic &

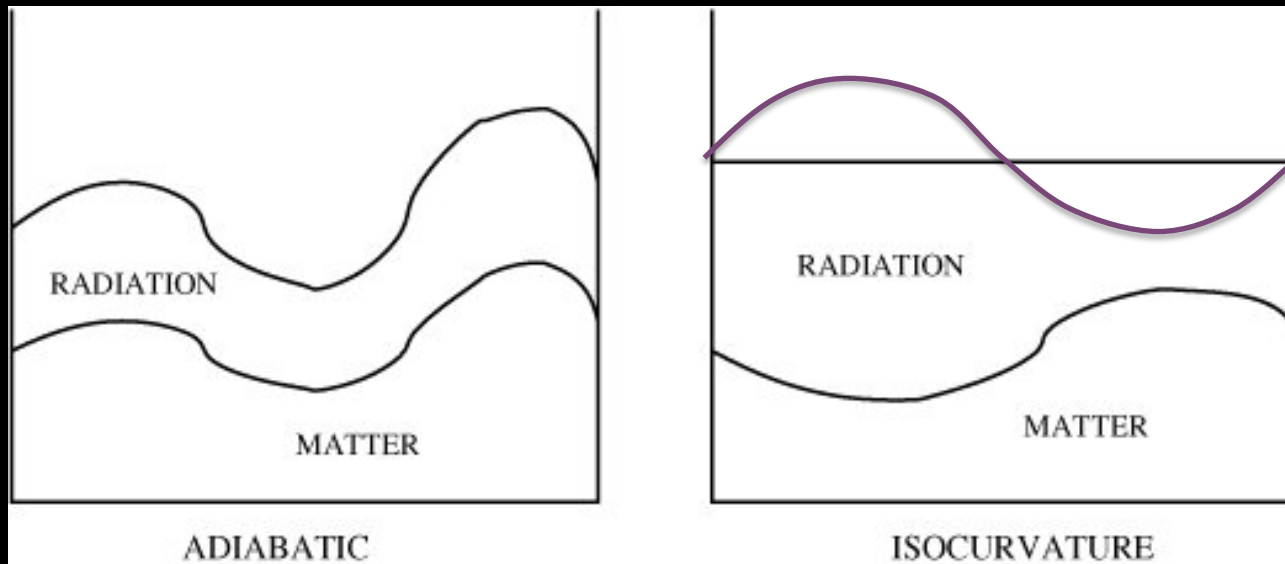
- Fluctuations of all species trace one another
- No energy exchange
- Complicated growth history before decoupling

Observations strongly suggest fluctuations in our universe are adiabatic

Isocurvature

- Different species distributed differently
- Sum of 4-density fluctuations is zero
- Potential for density increase (e.g. pressure) inhibited by interaction with the radiation field
- Could only grow after decoupling

All types of fluctuations can be described by a superposition of these two



Definition of a density fluctuation

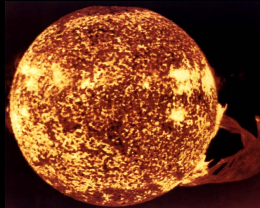
$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$



$\delta \sim 200$



$\delta \sim 10^5$



$\delta \sim 10^{30}$



$\delta \sim 10^{30}$

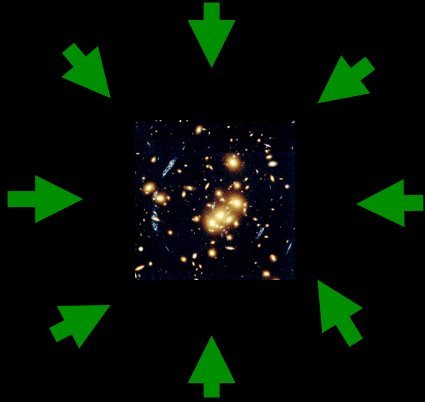


$\delta \sim 10^{30}$

} Rough

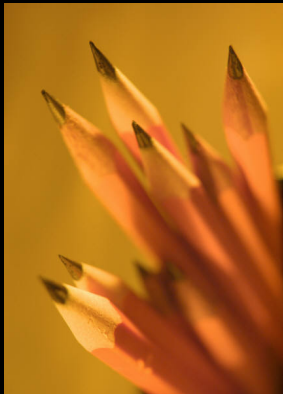
Linear vs non-linear

Linear: $\delta \ll 1$

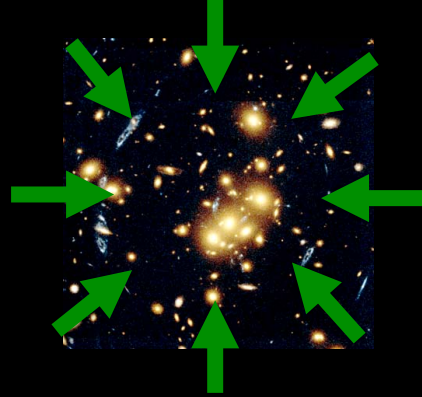


Perturbation theory
applies

To solve:



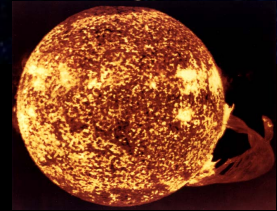
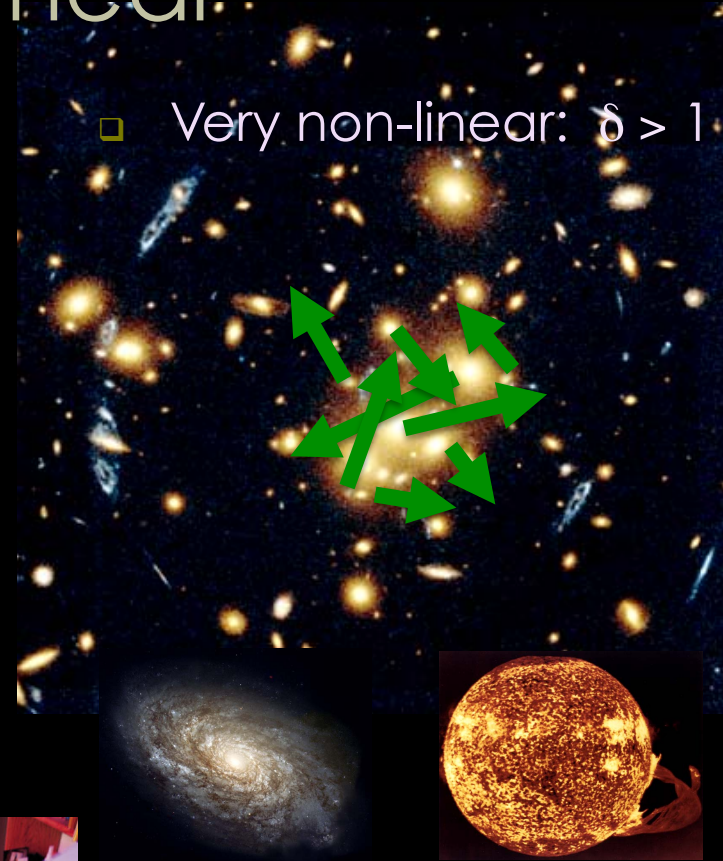
Non-linear: $\delta \sim 1$



To solve:



Very non-linear: $\delta > 1$



How do density fluctuations grow?

Static Solution ($\bar{\rho} = \text{const}$)

- Grav accel at surface

(due to excess mass)

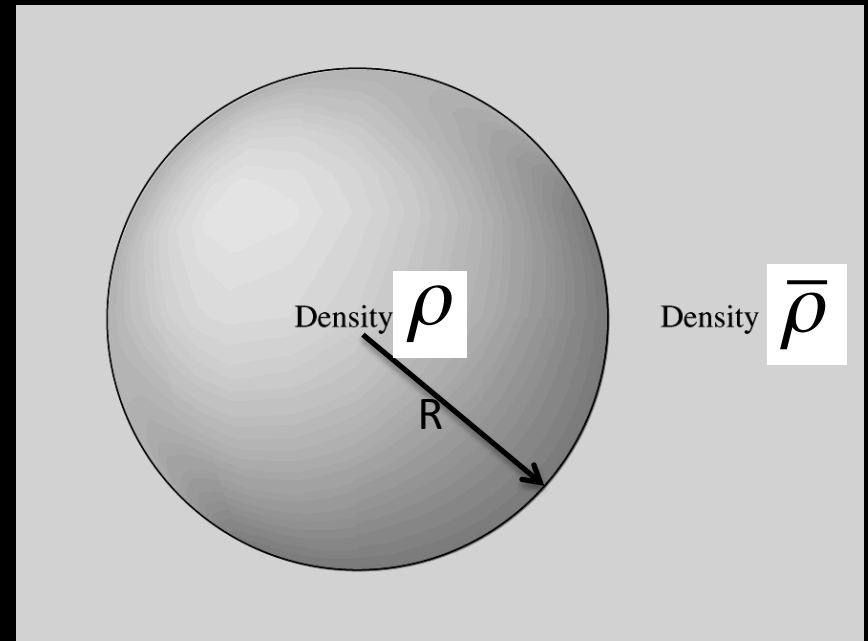
$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \bar{\rho} \delta$$

- Conservation of mass

$$\frac{\ddot{R}}{R} \approx -\frac{\ddot{\delta}}{\delta}$$

- Leaves differential equation

$$\ddot{\delta} = 4\pi G \bar{\rho} \delta$$



- Which has solution:

Growing mode

Decaying mode

$$\delta(t) = A_1 e^{t/t_{dyn}} + A_2 e^{-t/t_{dyn}}$$

$$t_{dyn} = (4\pi G \bar{\rho})^{-1/2}$$

Coefficients determined by initial conditions

Dynamical timescale

$$t_{dyn} = (4\pi G \bar{\rho})^{-1/2}$$

- The density in this room is $\sim 1 \text{ kg/m}^3$
- On what timescale will it collapse gravitationally?
- $G = 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$

About 9 hours!

Competing effects



DON'T LET GO

FROM DIRECTOR ALFONSO CUARÓN

G R A V I T Y



P R E S S U R E

Pressure to the rescue

Hydrostatic equilibrium = when pressure balances gravity

- Pressure builds up at the speed of sound, so

Where

$$c_s \approx \sqrt{\frac{dP}{d\rho}} = c \sqrt{w}$$

Equation
of state

$$t_{pre} \approx \frac{R}{c_s}$$

- For pressure to save the day, need $t_{pre} < t_{dyn}$

$$R / c_s < (4\pi G \bar{\rho})^{-1/2}$$

i.e. radius less than Jeans length

$$R < \frac{c_s}{\sqrt{4\pi G \bar{\rho}}} = \lambda_J$$

Jeans
Length

Atmosphere is stable

- The density in this room is $\sim 1 \text{ kg/m}^3$
- What is the Jeans length of the air in this room?
- $c_s \sim 330 \text{ m/s}$
- $G = 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$

$$\lambda_J = \frac{c_s}{\sqrt{4\pi G \bar{\rho}}}$$

About 20,000km!

(Stratosphere goes to altitude of about 50km.)

- Note that when sound speed goes to zero, so does the Jeans length, so all overdensities collapse.

Sound speed

- In general, sound speed given by:

$$c_s \approx \sqrt{\frac{dP}{d\rho}} = c\sqrt{w}$$

Equation
of state

- Relativistic fluid has $w=1/3$

$$c_s = \frac{c}{\sqrt{3}} \approx 0.58c$$

$$\lambda_J = \frac{c_s}{\sqrt{4\pi G\bar{\rho}}}$$

Jeans Length is greater
than horizon size

No Collapse!

- Including Baryons

$$c_s = \frac{c}{\sqrt{3(1 + 3\Omega_b / 4\Omega_\gamma)}}$$

Competing effects



DON'T LET GO

FROM DIRECTOR ALFONSO CUARÓN

G R A V I T Y

P R E S S U R E

EXPANSION

HORIZON



Growth including expansion

c/H

Expanding Solution

- Grav accel at surface

(due to
total mass)

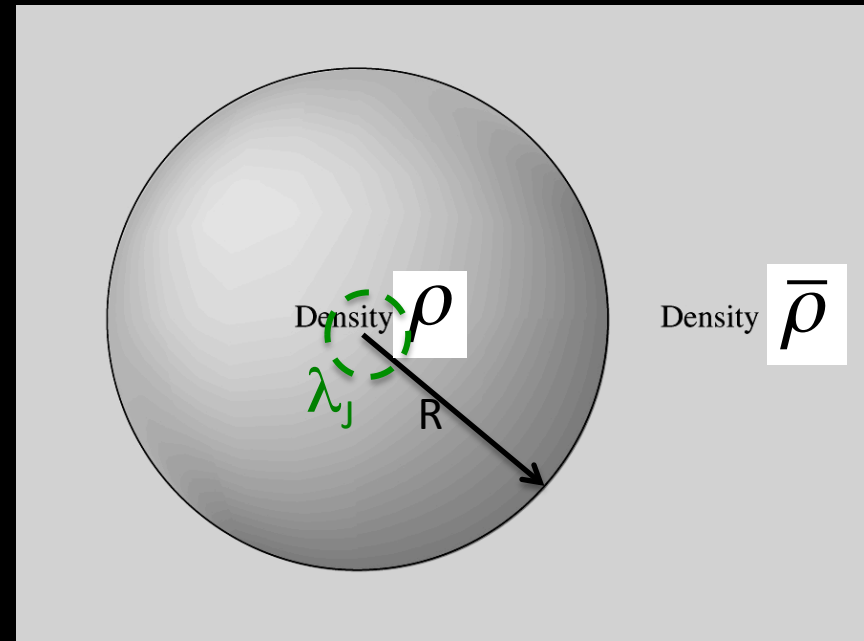
$$\frac{\ddot{R}}{R} = \frac{4\pi G}{3} R \bar{\rho} (1 - \delta)$$

- Conservation of mass

$$\frac{\ddot{R}}{R} \approx \frac{\ddot{a}}{a} - \frac{\ddot{\delta}}{3} - \frac{2}{3} \left(\frac{\dot{a}}{a} \right) \dot{\delta}$$

- Leaves differential equation

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G \bar{\rho} \delta$$



Same as static case, but with an extra term,

$$2H\dot{\delta}$$

known as the “Hubble friction” term

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\Omega_M H^2 \delta = 0$$

Radiation dominated

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\Omega_M H^2 \delta = 0$$

- Radiation dominated: $\Omega_M \ll 1, H=1/2t$

$$\ddot{\delta} + \frac{1}{t}\dot{\delta} = 0$$

- Has solution:

Constant Growing mode

$$\delta(t) = B_1 + B_2 \ln(t)$$

Grows logarithmically (i.e. slowly)

Matter dominated

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\Omega_M H^2 \delta = 0$$

- Matter dominated: $\Omega_M=1, H=2/(3t)$

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0$$

- Has solution:

$$\delta(t) = \overset{\text{Growing mode}}{D_1 t^{2/3}} + \overset{\text{Decaying mode}}{D_2 t^{-1}}$$

Grows as $t^{2/3}$, linearly proportional to scalefactor

Only during matter domination can density fluctuations grow significantly

Lambda dominated

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\Omega_M H^2 \delta = 0$$

- Lambda dominated: $\Omega_M \ll 1, H=H_\Lambda$

$$\ddot{\delta} + 2H_\Lambda \dot{\delta} = 0$$

- Has solution:

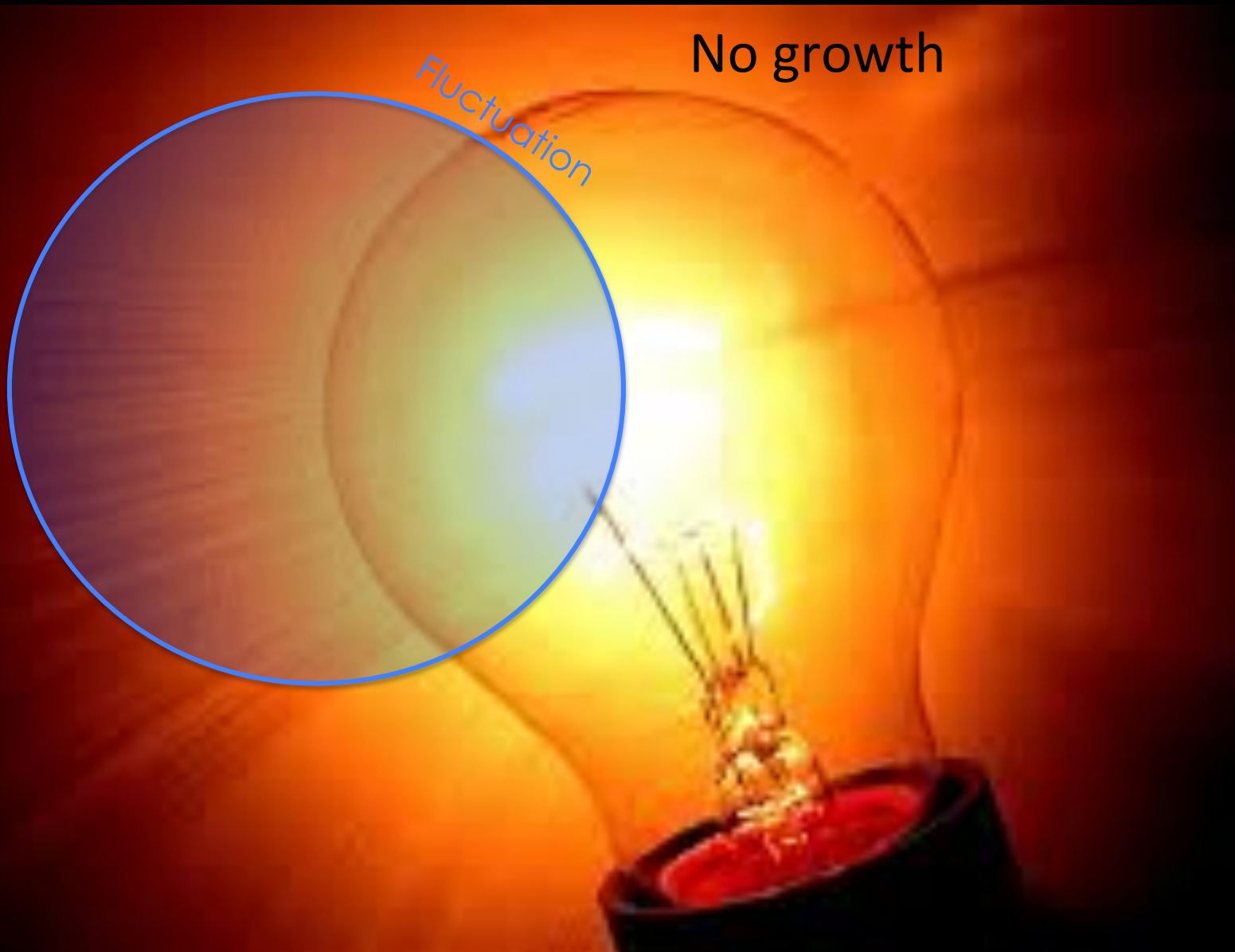
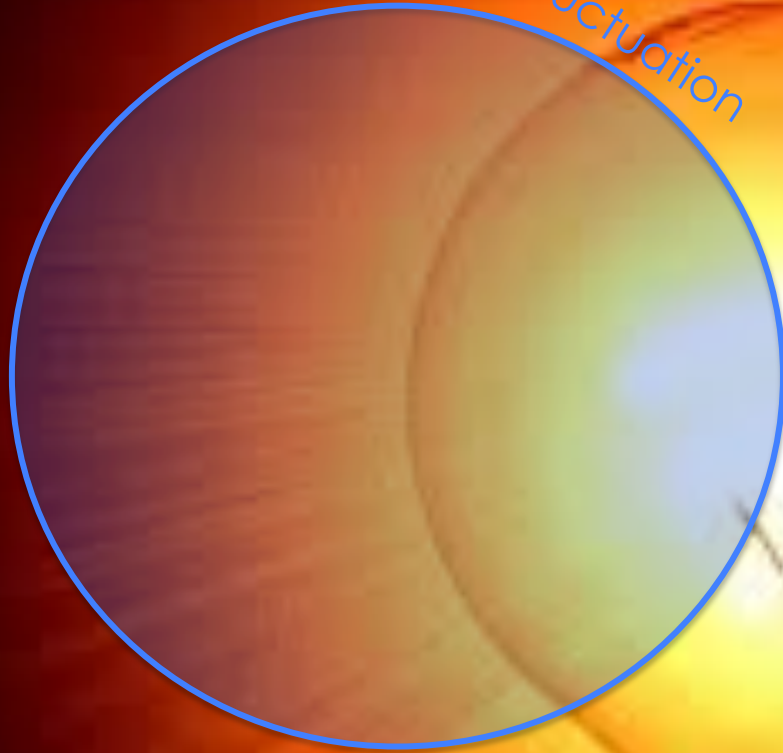
$$\delta(t) = \overset{\text{Constant}}{C_1} + \overset{\text{Decaying mode}}{C_2 e^{-2H_\Lambda t}}$$

Density contrast is constant

Radiation domination

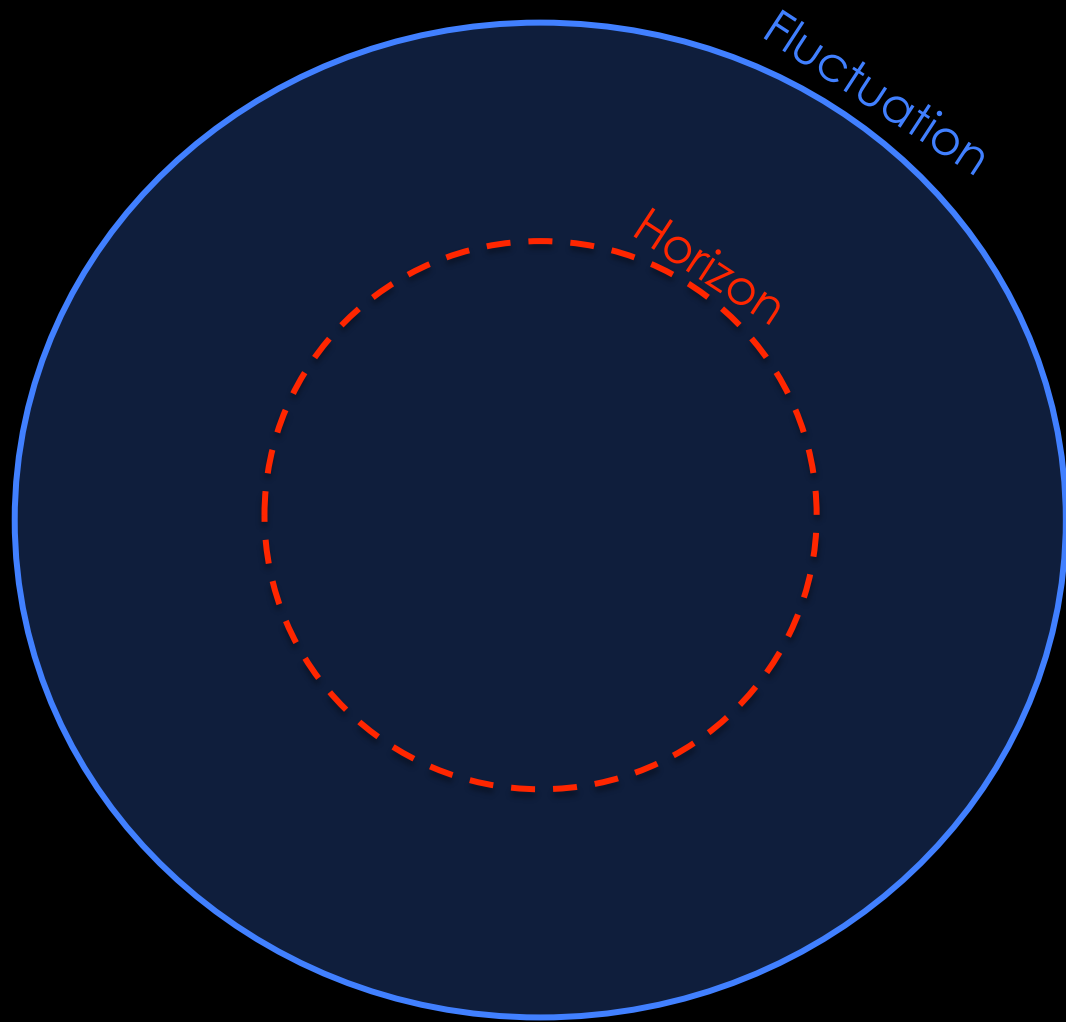
No growth

Fluctuation

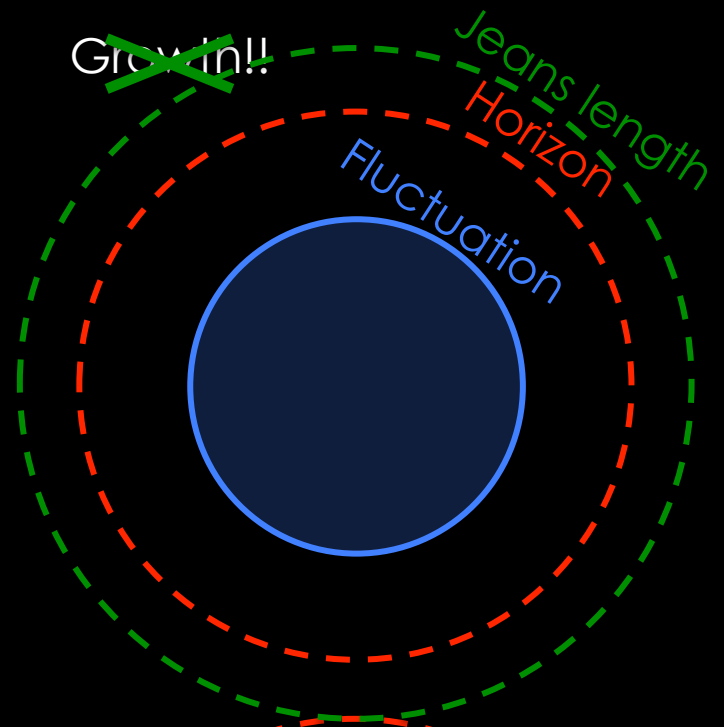


Matter domination

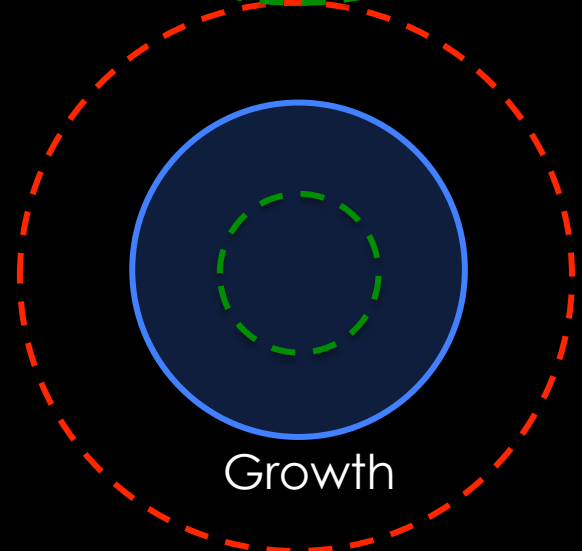
No Growth

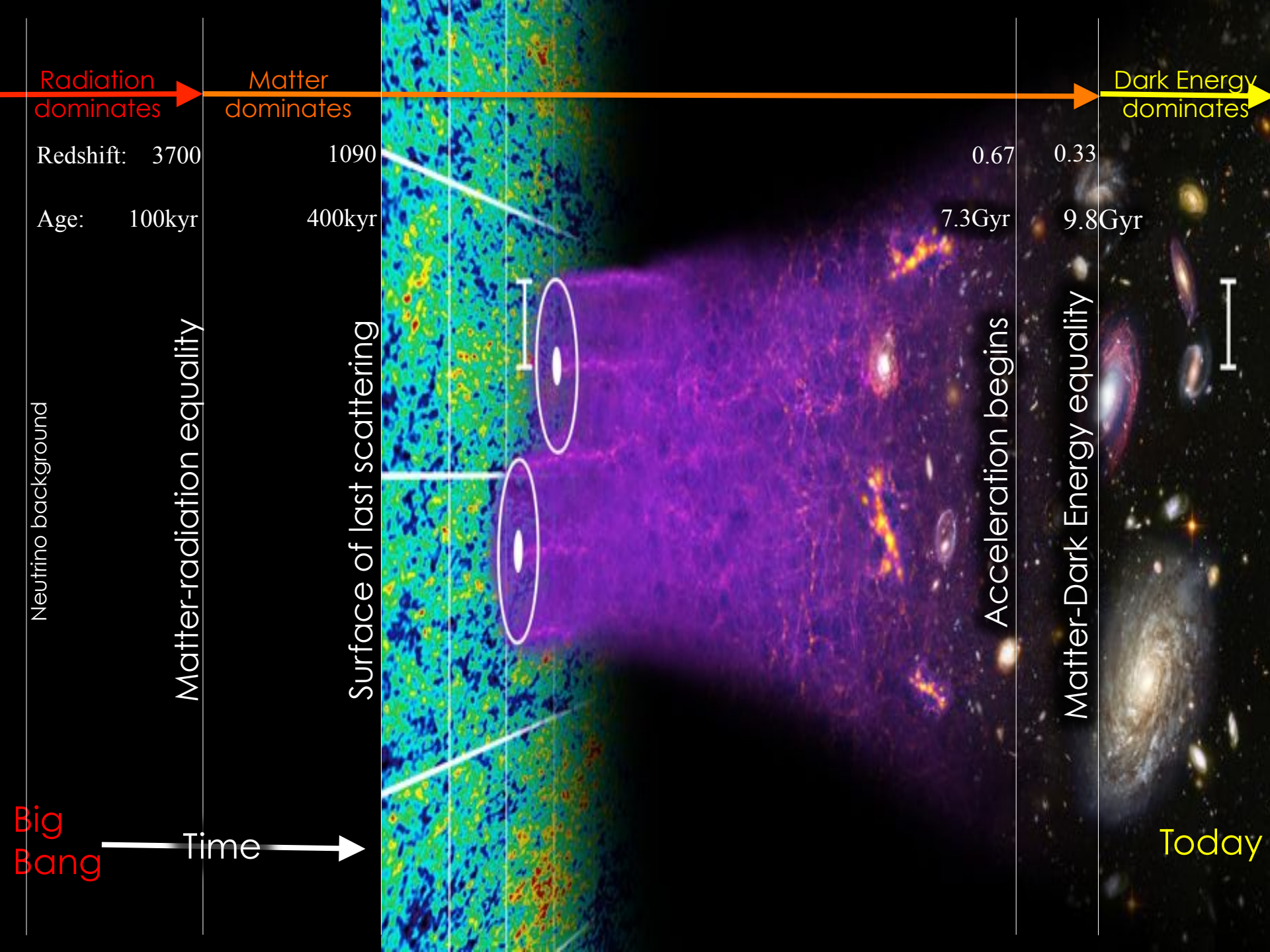


~~Growth!!~~



Growth





Timing of Structure Formation

(see Carroll and Ostlie Example 30.2.1)

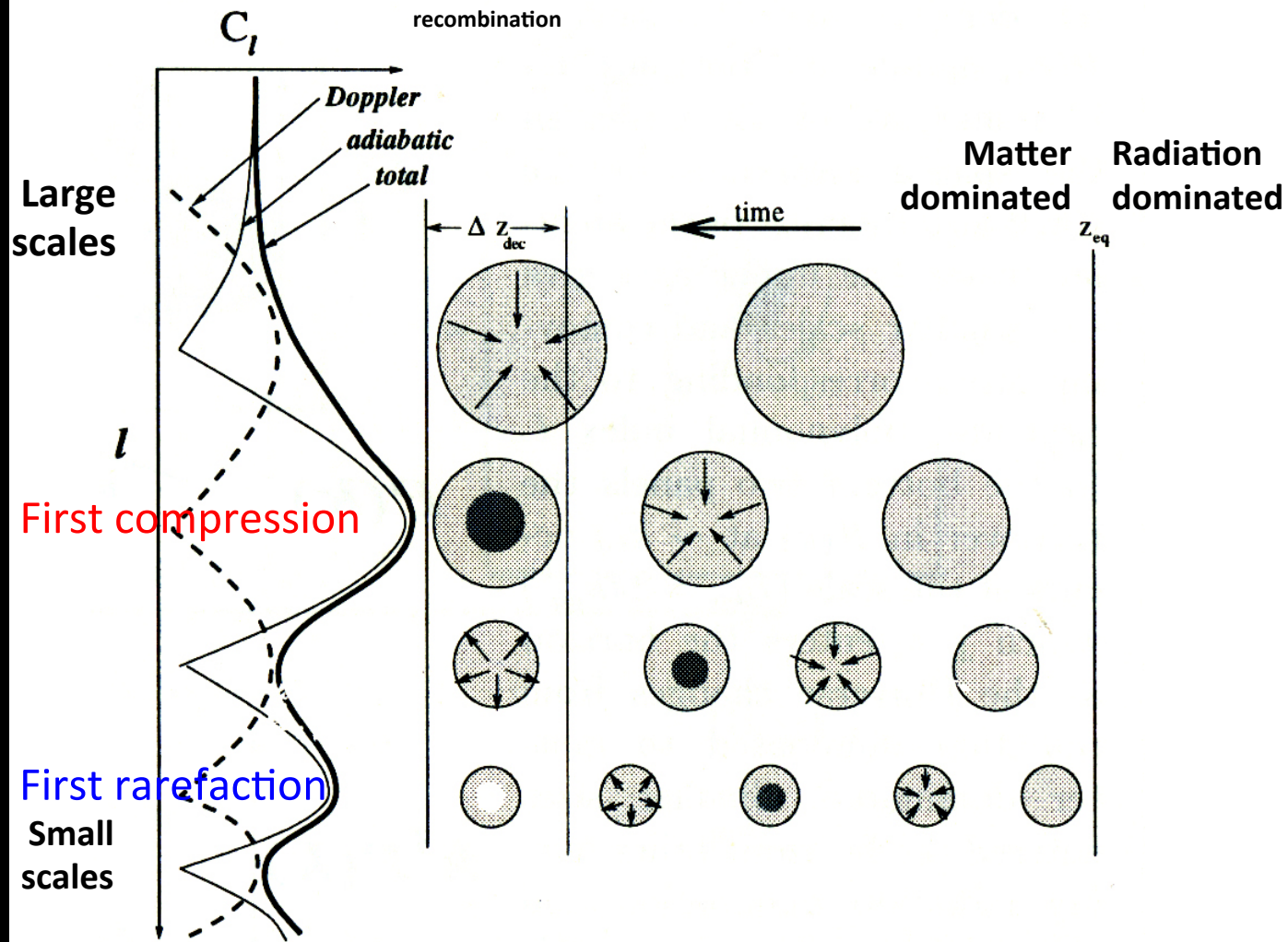
- Record for most distant QSO: $z=6.43$ (out of date!)
 - Estimate density fluctuation needed at recombination to form a quasar this soon...
 - Gravitational collapse begins when $\delta \sim 1$
 - In matter era, $\delta = \delta_i (t/t_i)^{2/3}$
 - $t = 2/(3H_0) a^{3/2}$ (\sim lower limit to age)
 - $H_0 = 70 \text{ km s}^{-1}\text{Mpc}^{-1} = 0.072 \text{ Gyr}^{-1}$
- $a = 1/(1+z) = 0.135$
- $t = 2/(3H_0) a^{3/2} = 462 \text{ Myr}$
- let $t = t_{\text{dec}} = 0.38 \text{ Myr}$ (WMAP)

$$\Rightarrow \delta = 0.009$$

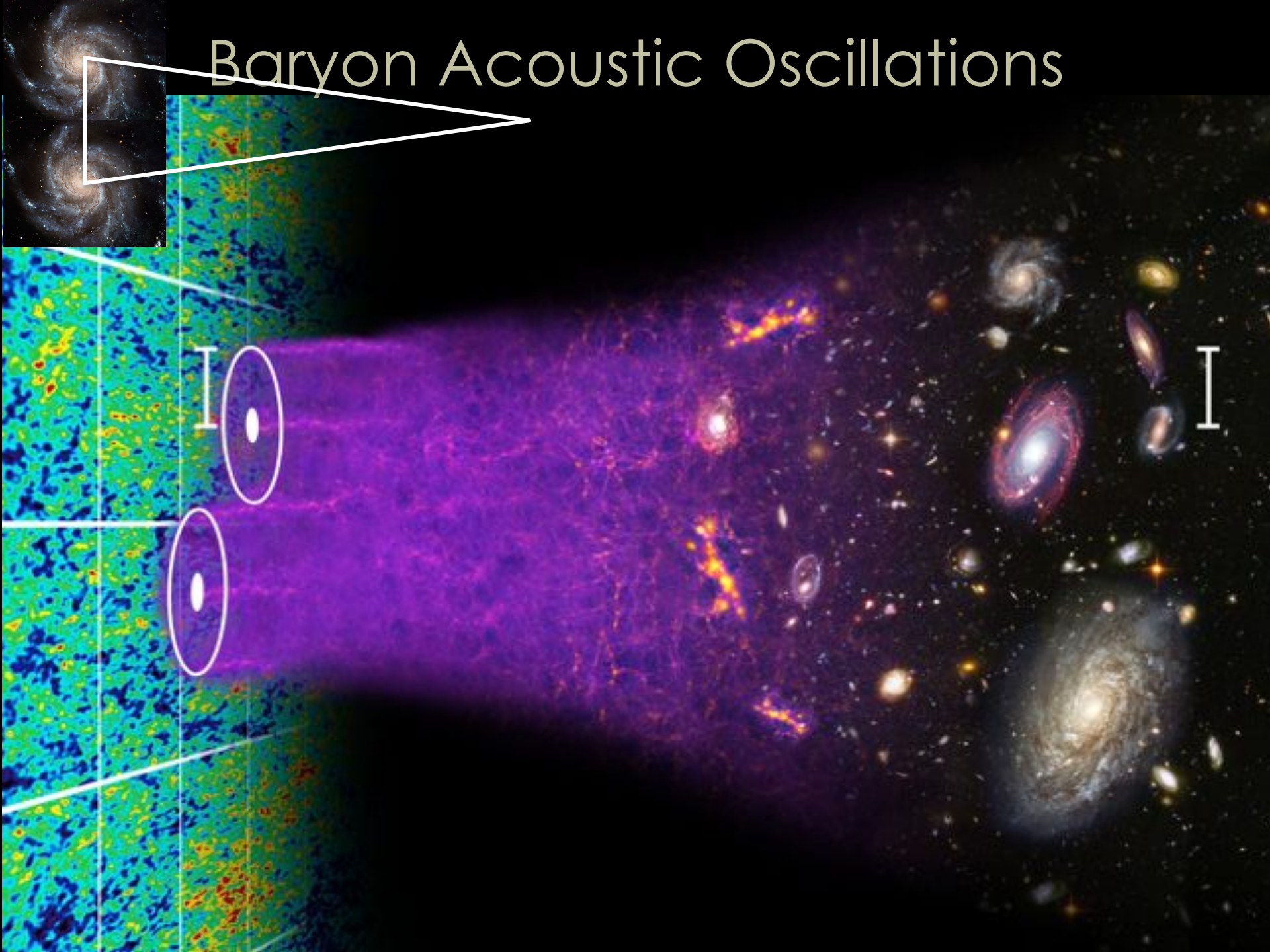
There must have been density fluctuations of several tenths of a per cent at decoupling.

But that doesn't match the CMB observations!



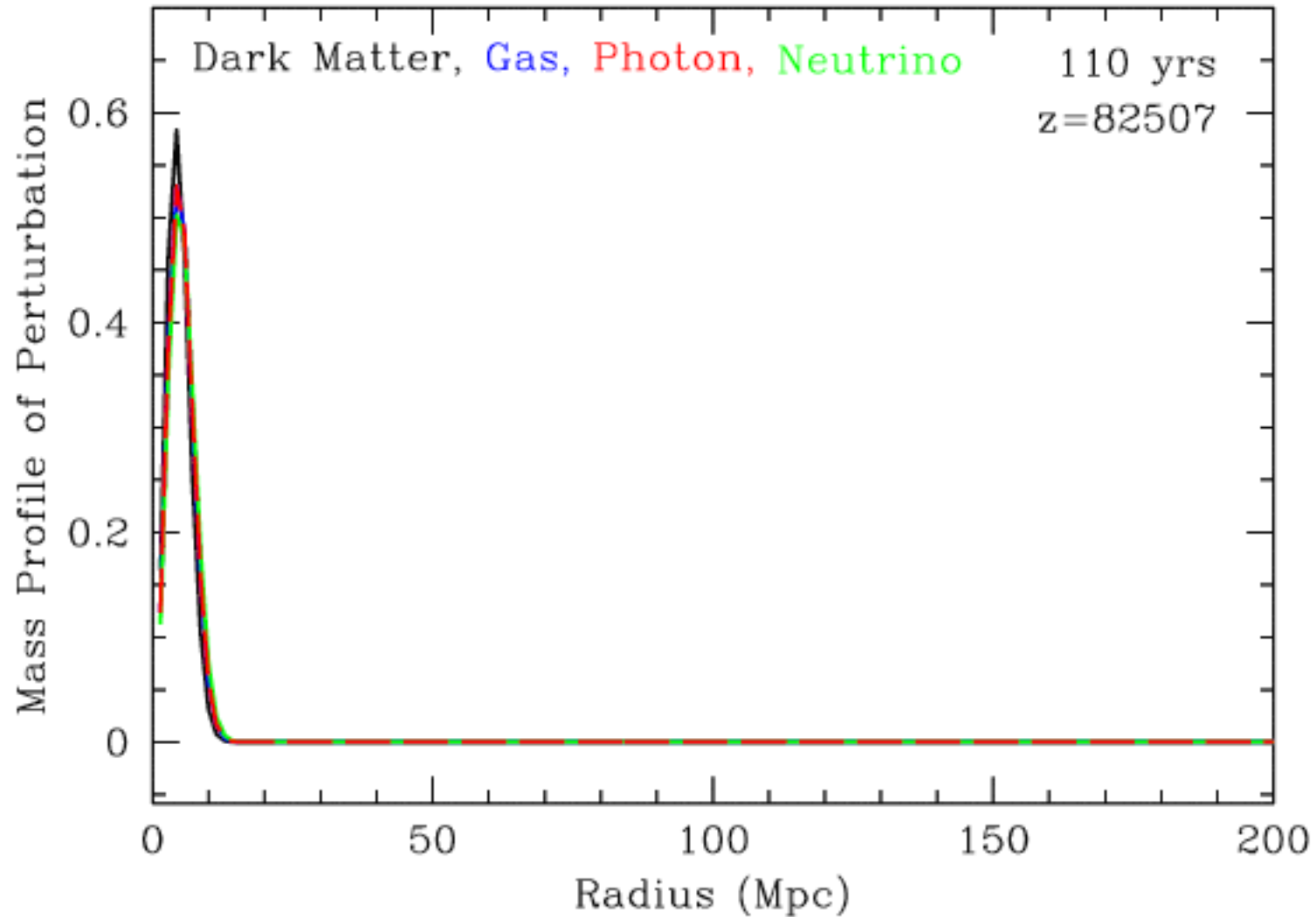


Baryon Acoustic Oscillations



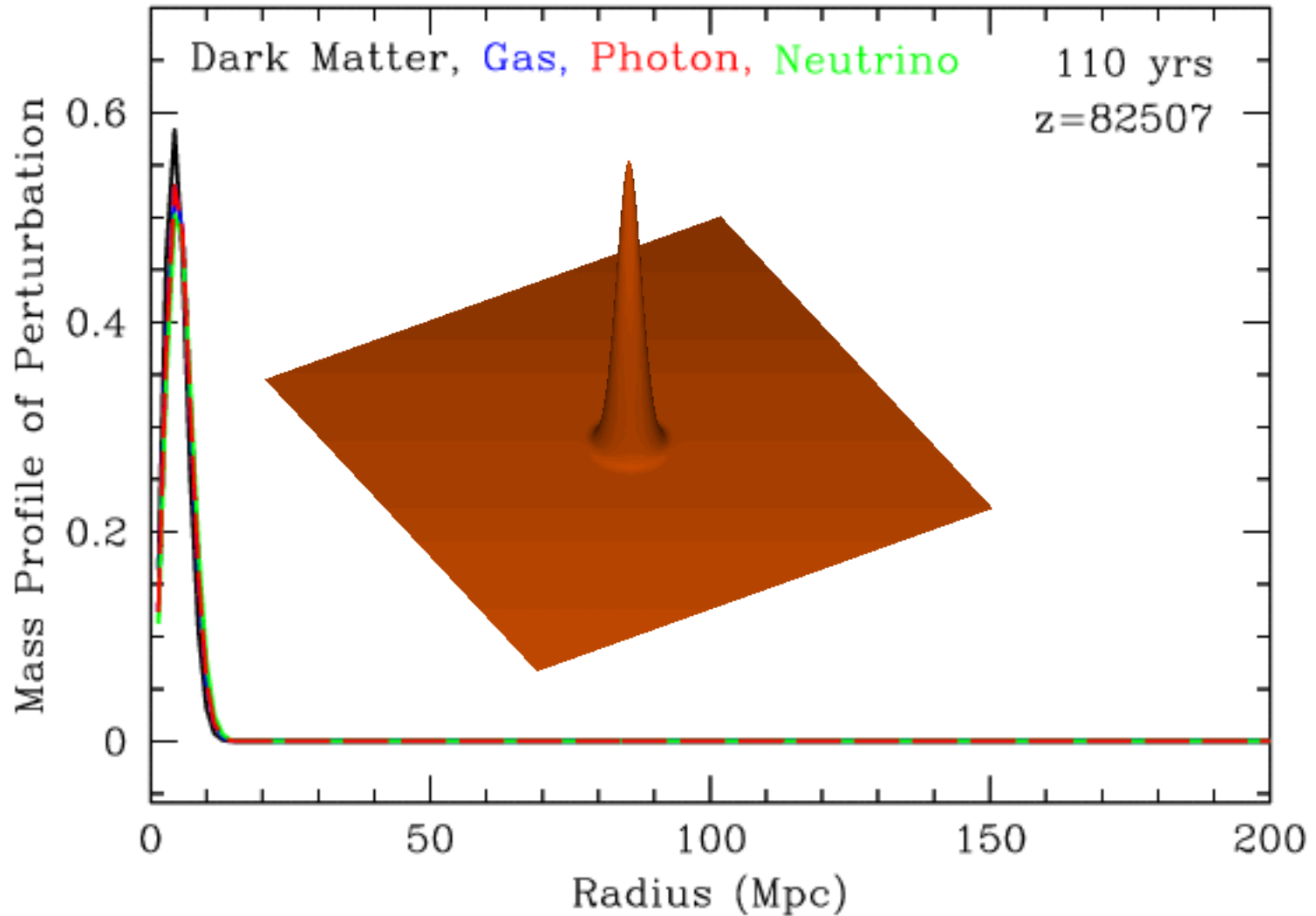
Formation of acoustic peak

- http://cmb.as.arizona.edu/~eisenste/acousticpeak/acoustic_physics.html



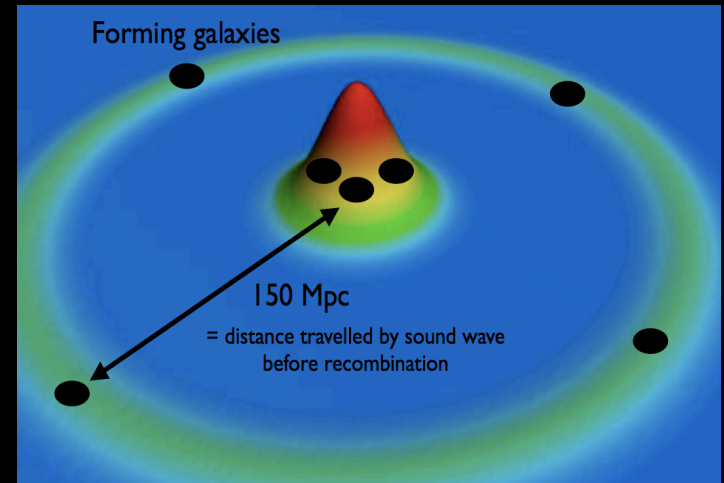
Formation of acoustic peak

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Calculate the BAO scale

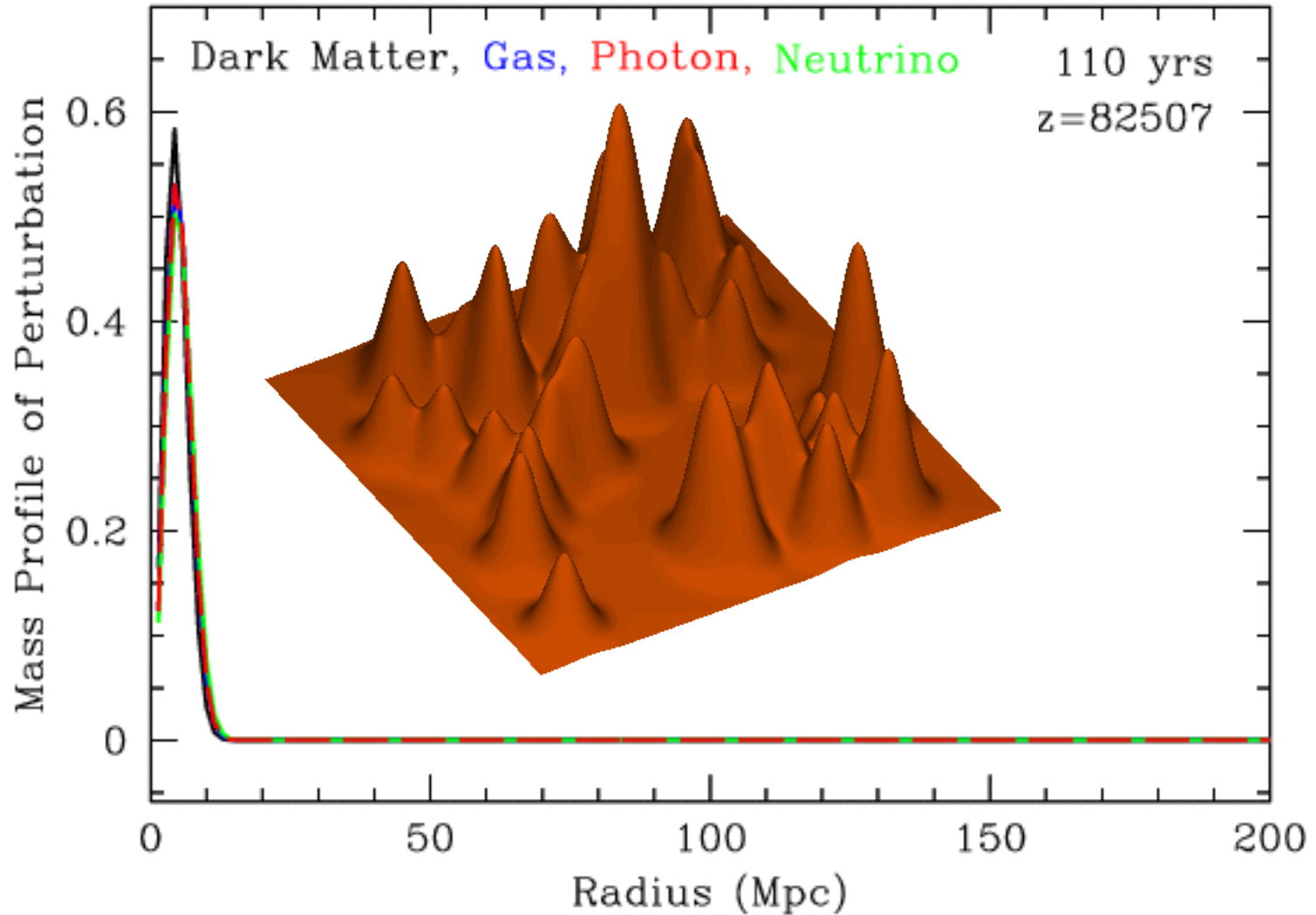
- Time between big bang and last scattering = 380,000yr
- Speed of sound wave = $0.58c$
- $1 \text{ pc} = 3.1 \times 10^{16} \text{ m}$
- $1 \text{ yr} = 3.56 \times 10^7 \text{ s}$
- $c = 3 \times 10^8 \text{ ms}^{-1}$



$$\begin{aligned} d &= c_s t \\ &= 0.58 \times 3 \times 10^8 \text{ ms}^{-1} \times 380,000 \text{ yr} \times \text{conversion factors} \\ &= 74 \text{ Mpc} \end{aligned}$$

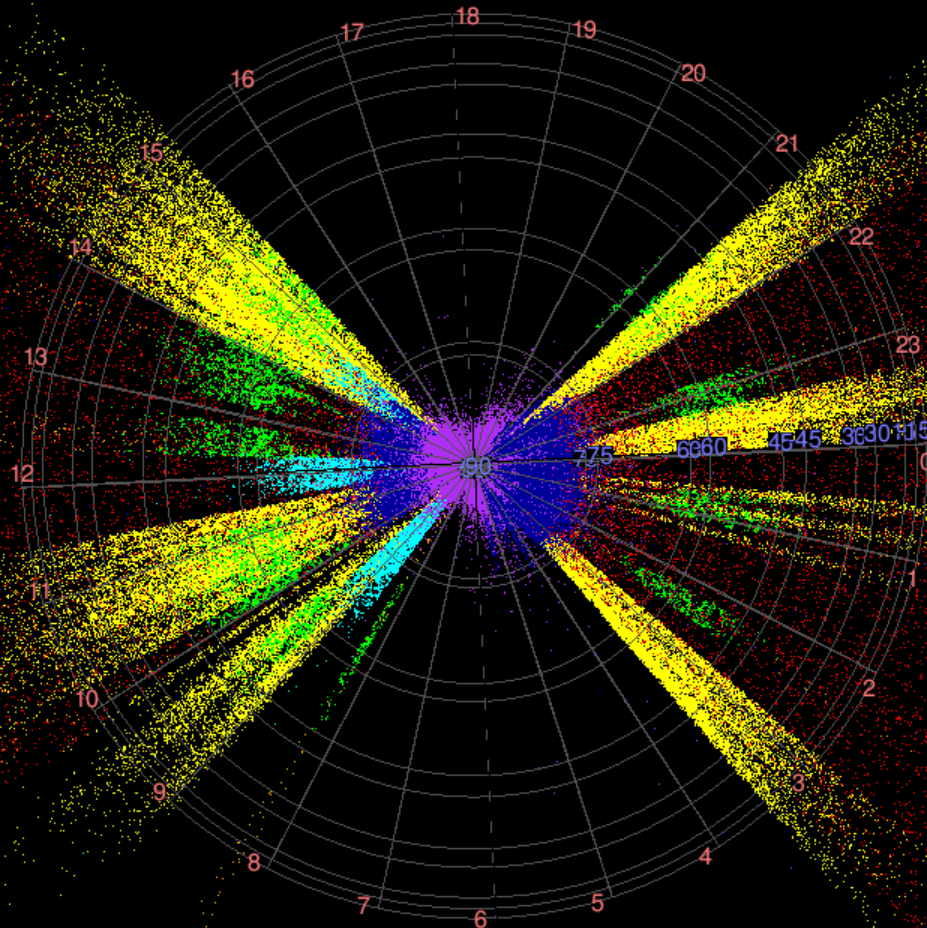
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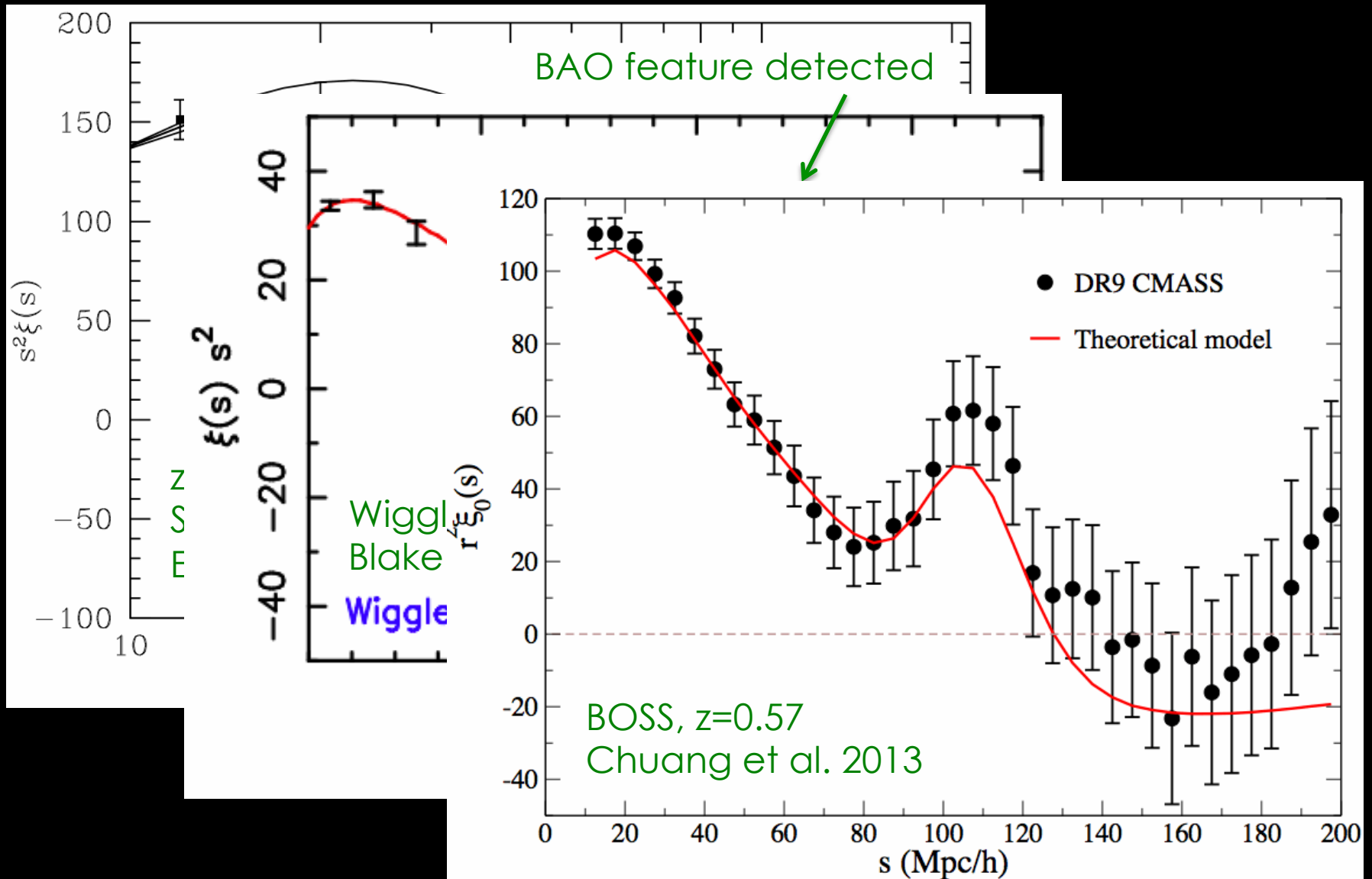
Redshift surveys (e.g. WiggleZ)

7 equatorial fields, each 100-200 deg²
>9° on side, ~3 x BAO scale at $z > 0.5$
Physical size ~ 1300 x 500 x 500 Mpc/h



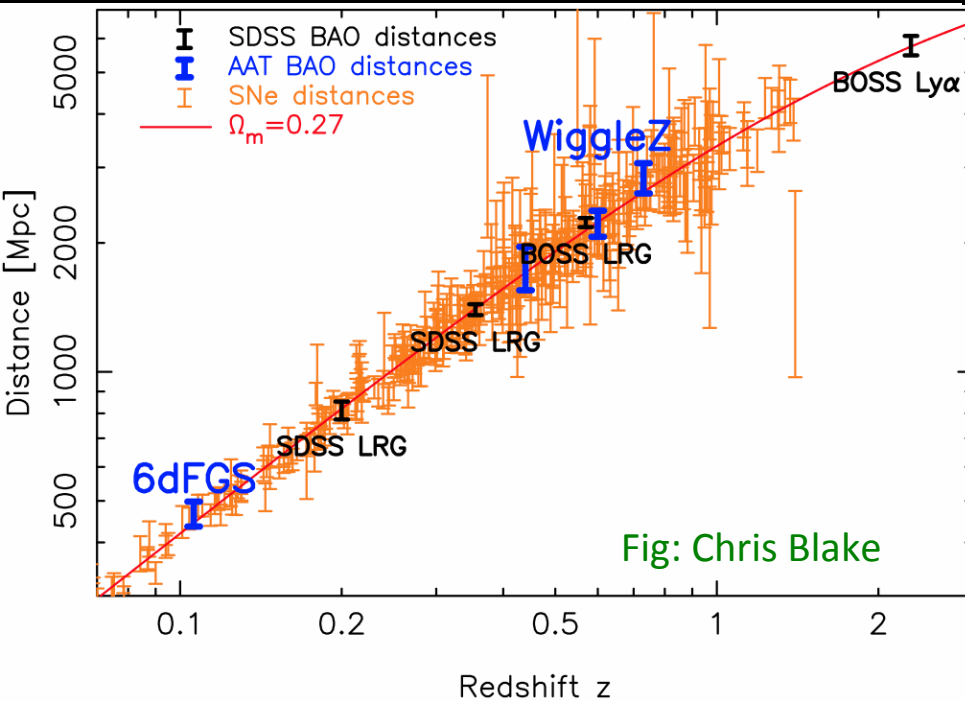
6dFGS (purple), 2dFGRS (blue), MGC (navy), GAMA (cyan), 2SLAQ-LRG (green),
WiggleZ (yellow), 2SLAQ-QSO (orange), 2QZ (red); the celestial sphere is at $z=1$.

Acoustic peak detections



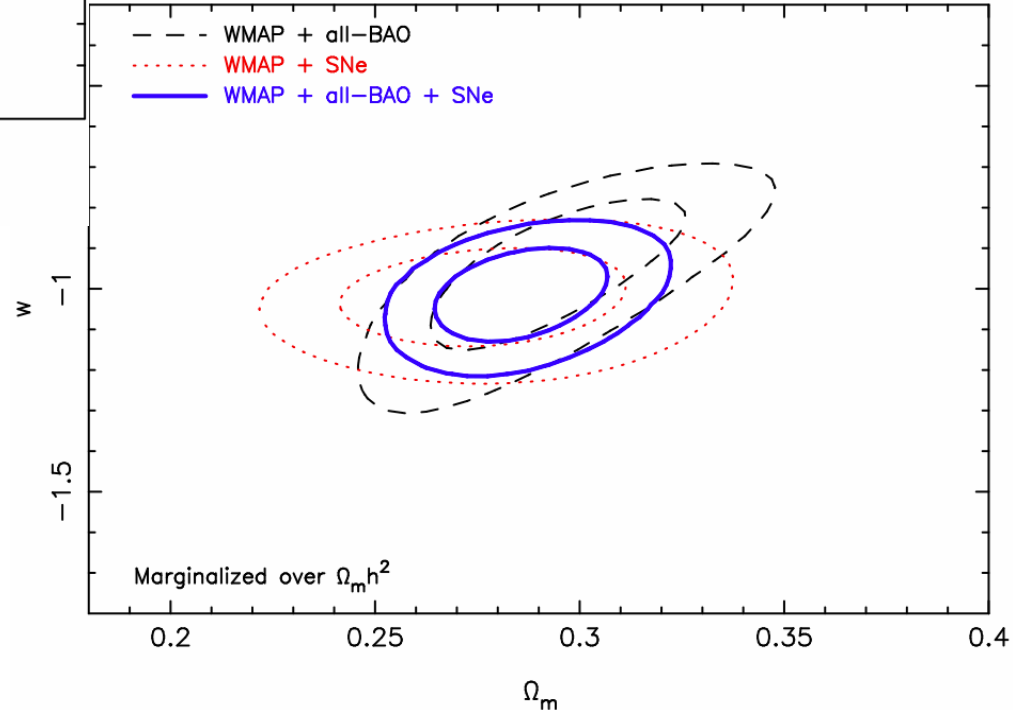
Baryon Acoustic Oscillations

Compared to supernovae

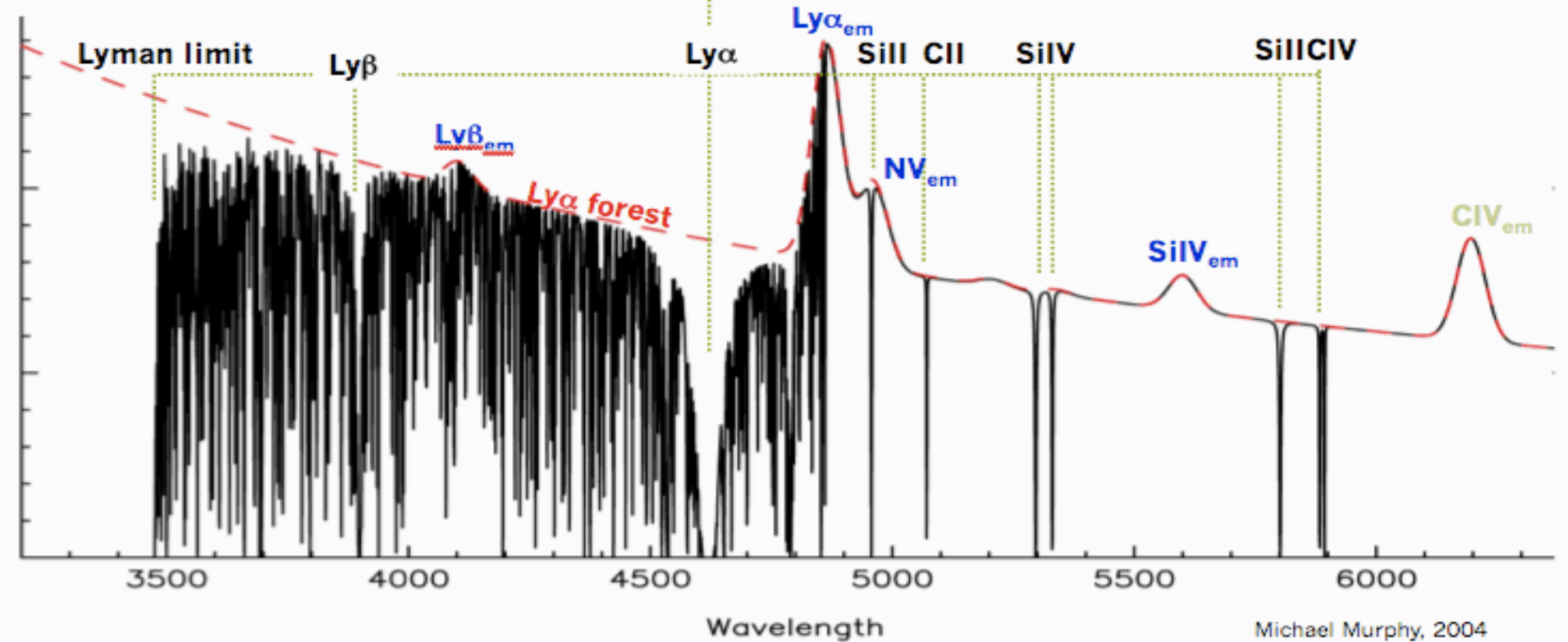
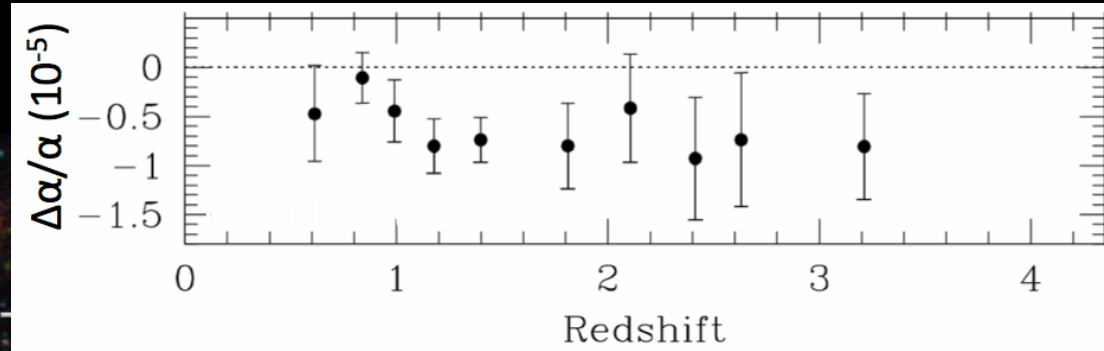
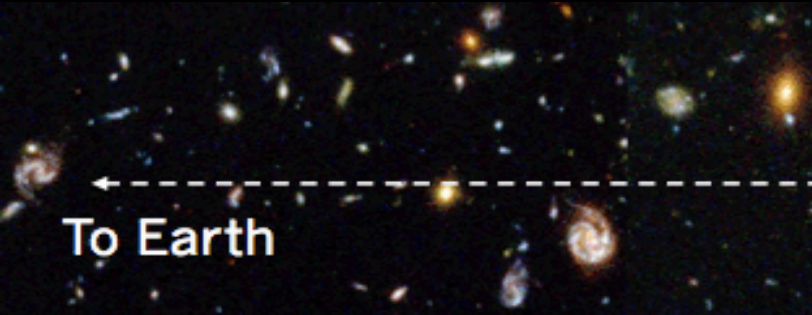


No longer need SNe!
 BAO distances alone now
 require acceleration!

W and SNe Combined with CMB

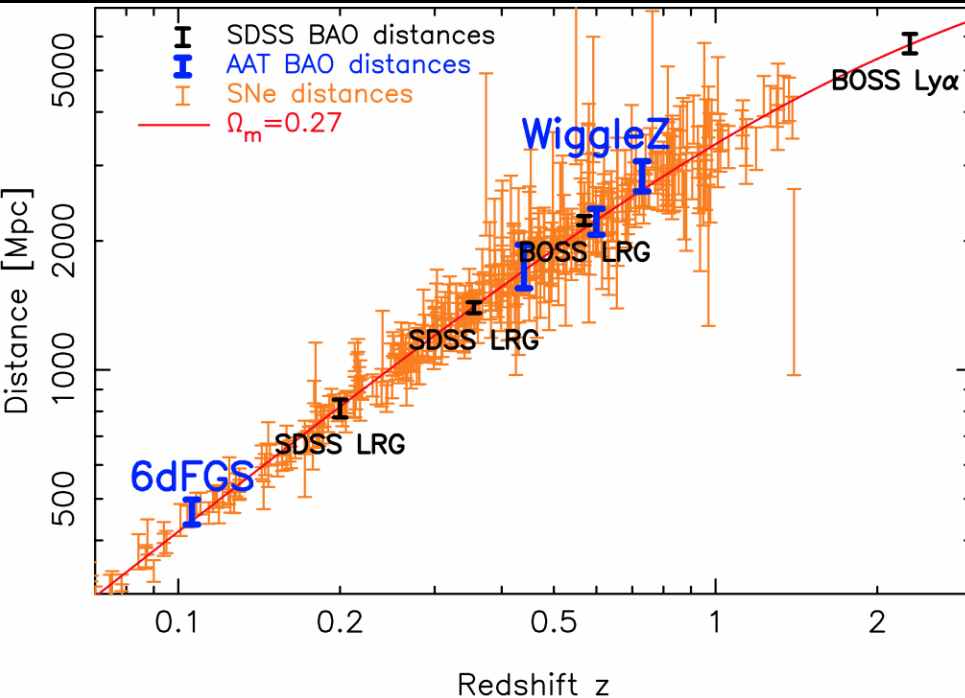


Lyman-alpha forest



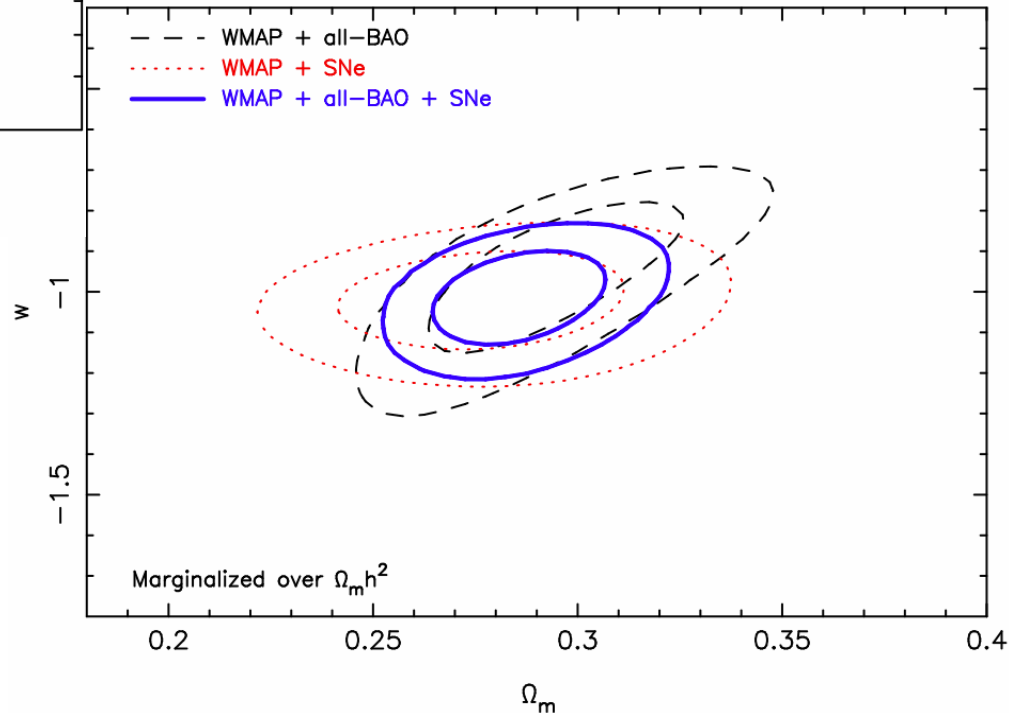
Baryon Acoustic Oscillations

Compared to supernovae



No longer need SNe!
BAO distances alone now
require acceleration!

Omega_m and SNe Combined with CMB



We don't know what is causing the acceleration

(And the leading candidate, vacuum energy, is 10^{54} too large)

WHAT IS DARK ENERGY?

Theoretical
ideas

Inhomogeneities

Massive void
Many voids

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

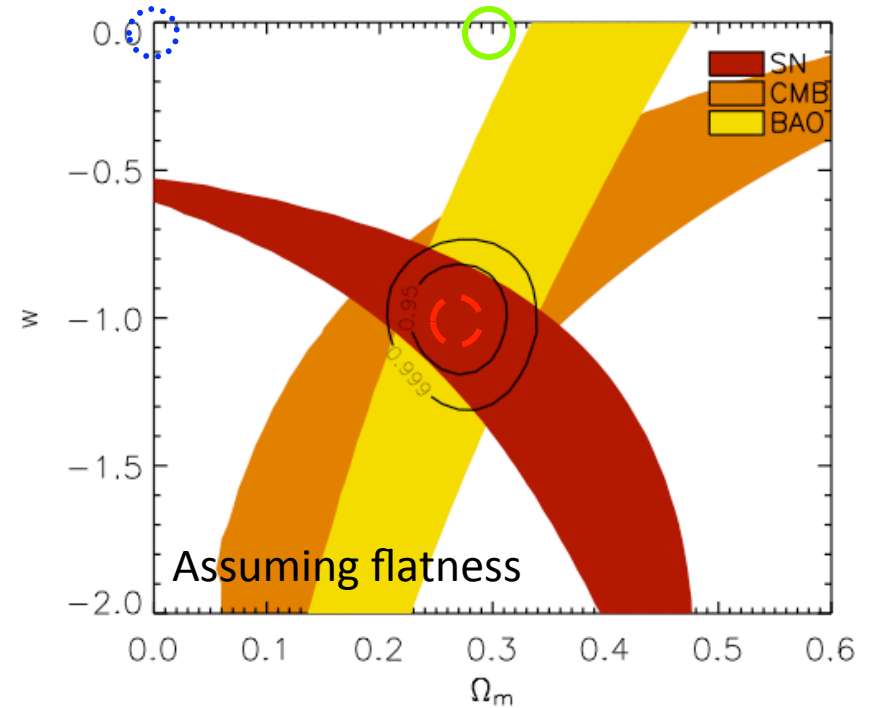
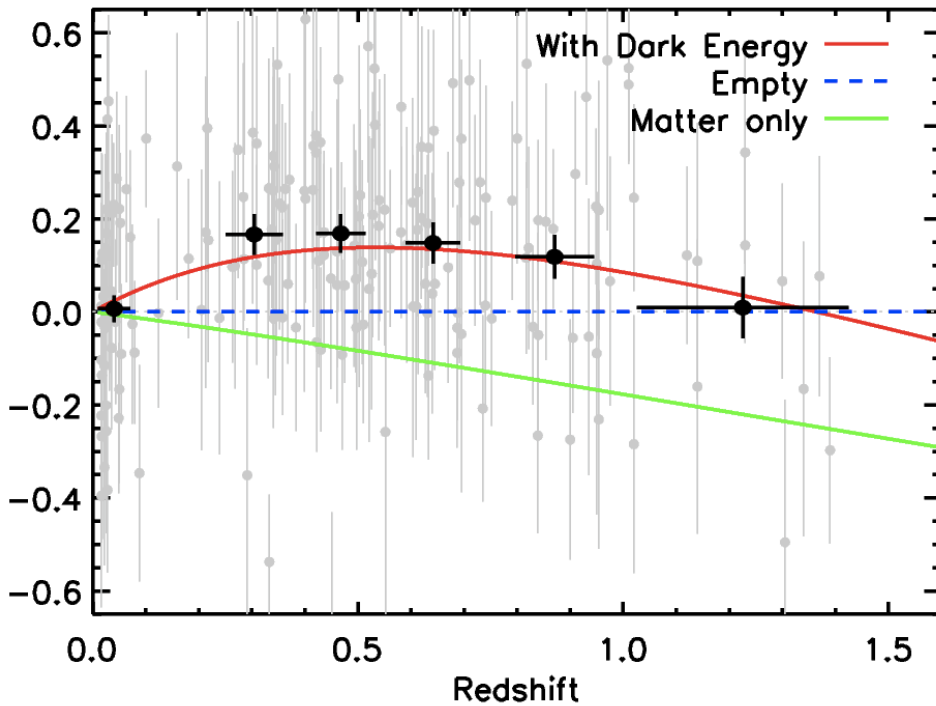
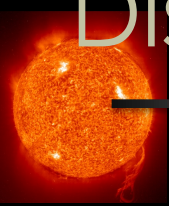
New Theory

$f(R)$
TeV \mathcal{S}
DGP

New Component

Vacuum energy
Quintessence
Other dark energy

Distance indicators show acceleration



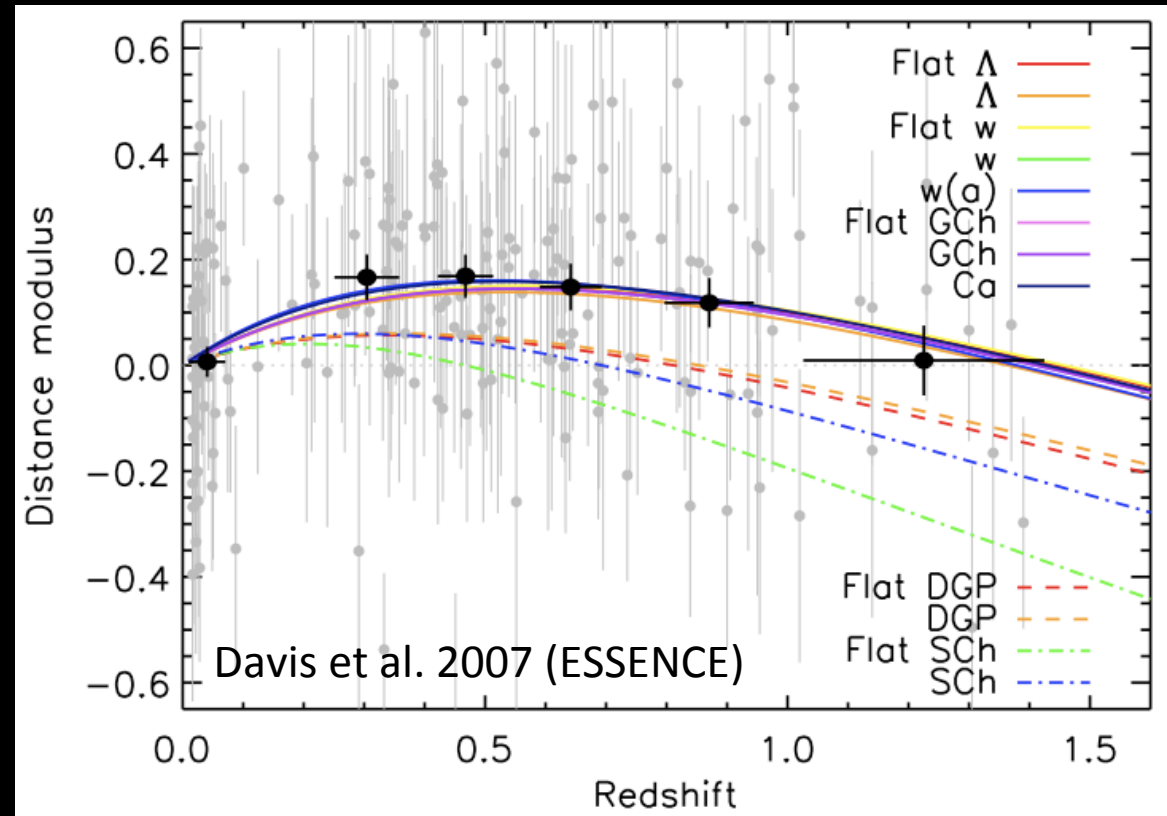
$$\frac{H^2}{H_0^2} = \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \frac{\Omega_x}{a^{3(1+w)}}$$

But distance indicators can't distinguish between some models

Every model that **fitted** the data could **mimic** Λ CDM for some values of its parameters

The **best fit** parameters **always** included Λ CDM

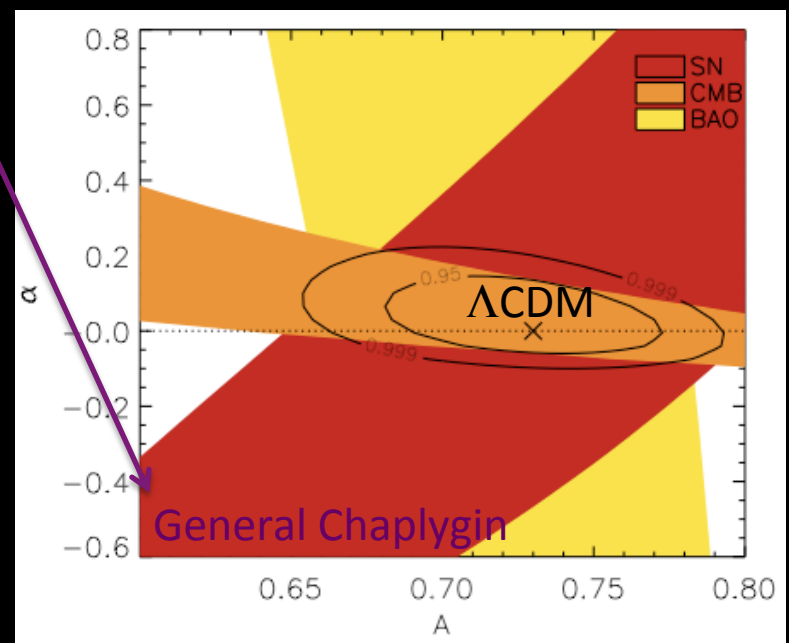
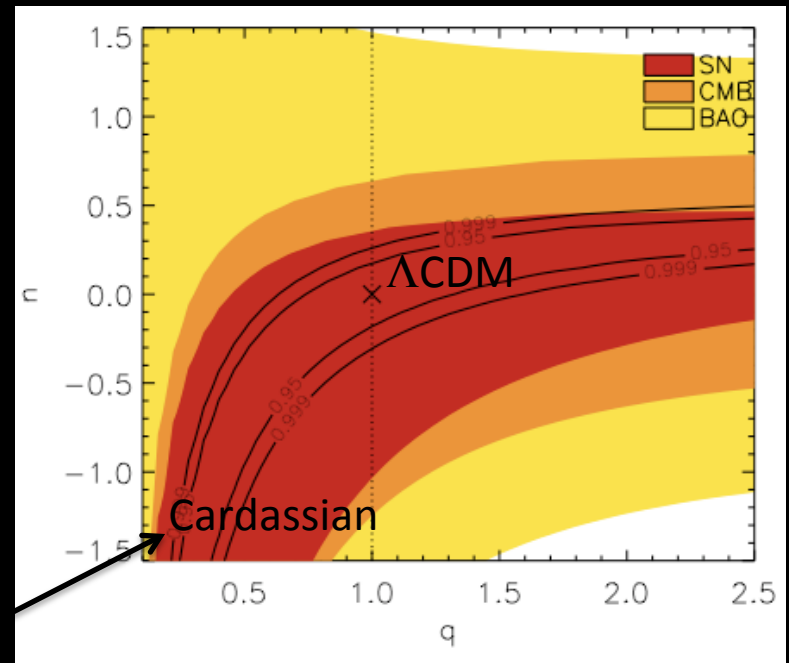
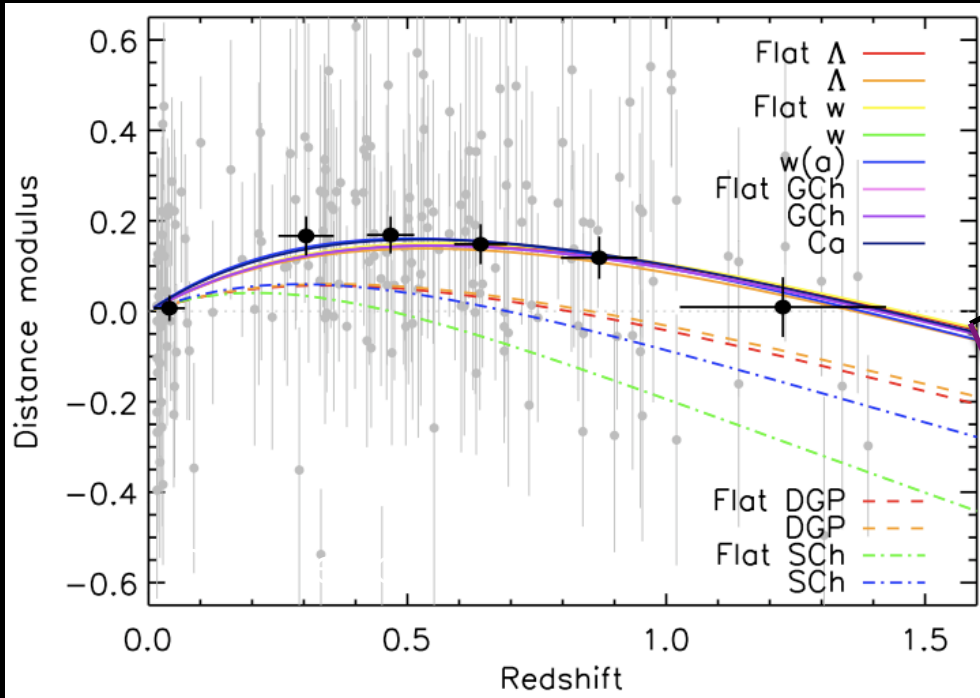
All models that could **not** **mimic** Λ CDM were **poor fits**



Measurements of the expansion **cannot IN PRINCIPLE** distinguish between some models, even in the limit of perfect data.

We need other **types** of tests.

But distance data can't distinguish between some models



Distinguishing dark energy models

GROWTH OF STRUCTURE

Other types of measurements needed

□ Growth

$$f = \frac{d(\ln \delta)}{d(\ln a)} \sim \Omega_M(z)^\gamma$$

overdensity

$\gamma = 6/11$ in Λ CDM
 $\gamma = 6/10$ in CDM
 $\gamma = 11/16$ in DGP

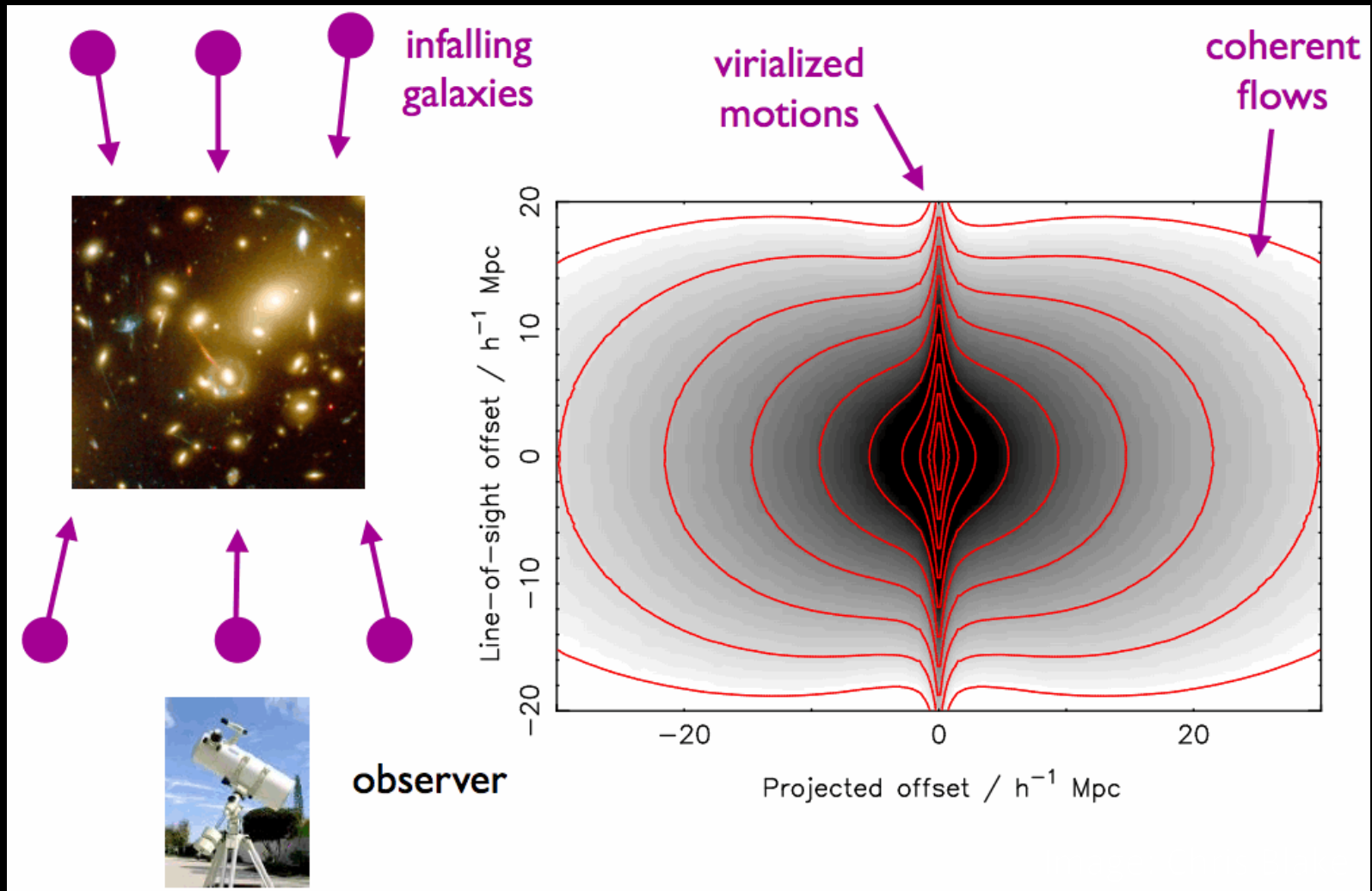
γ is different in different models

□ Amplitude of density fluctuations at present day

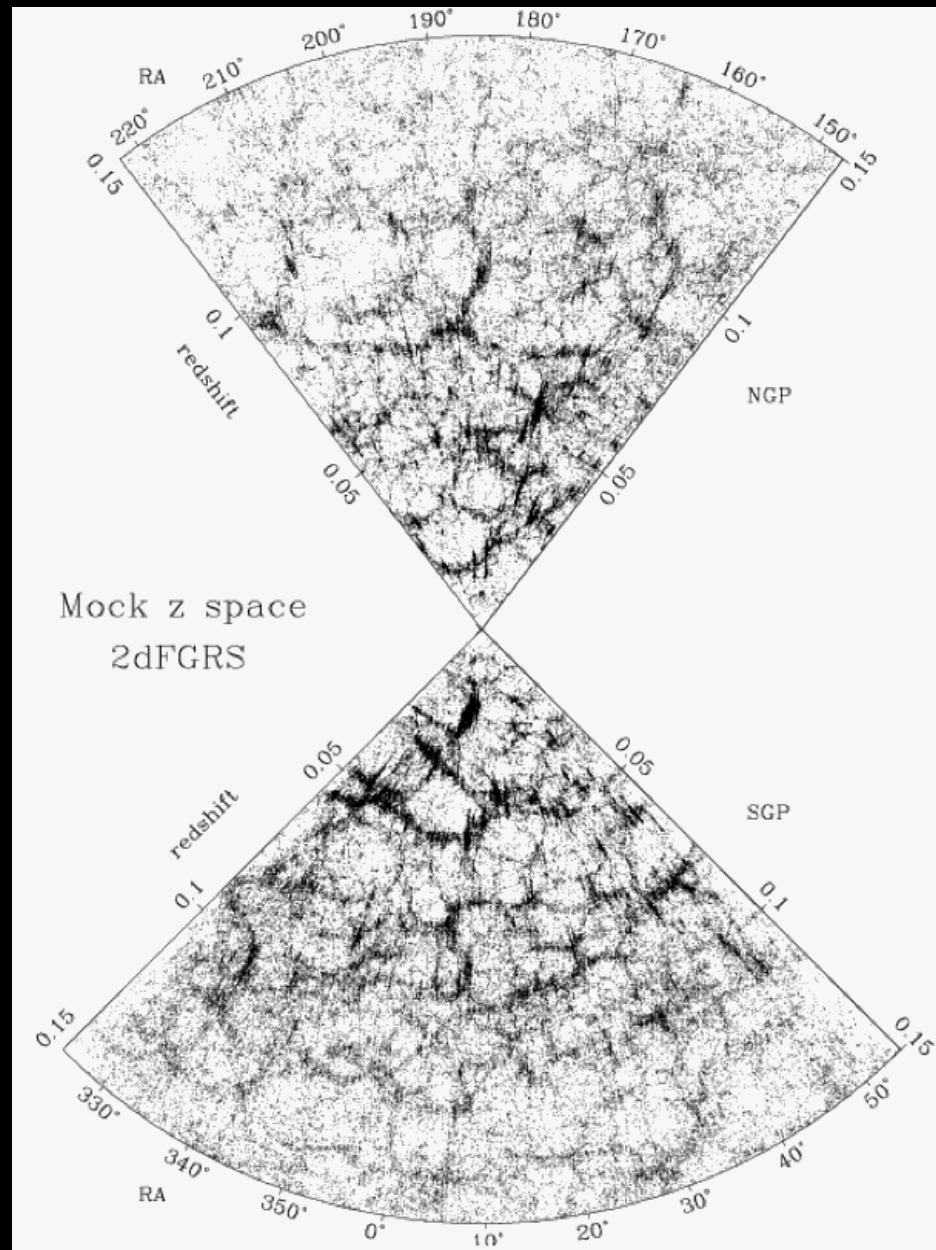
$$\sigma_8$$

1. measure density in spheres 8 Mpc in radius
2. calculate the dispersion

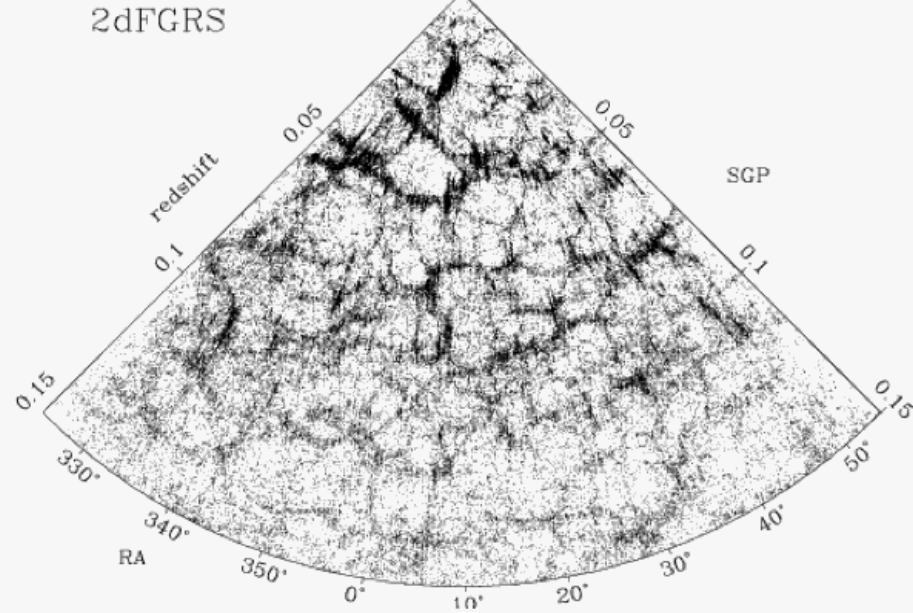
Growth of Structure



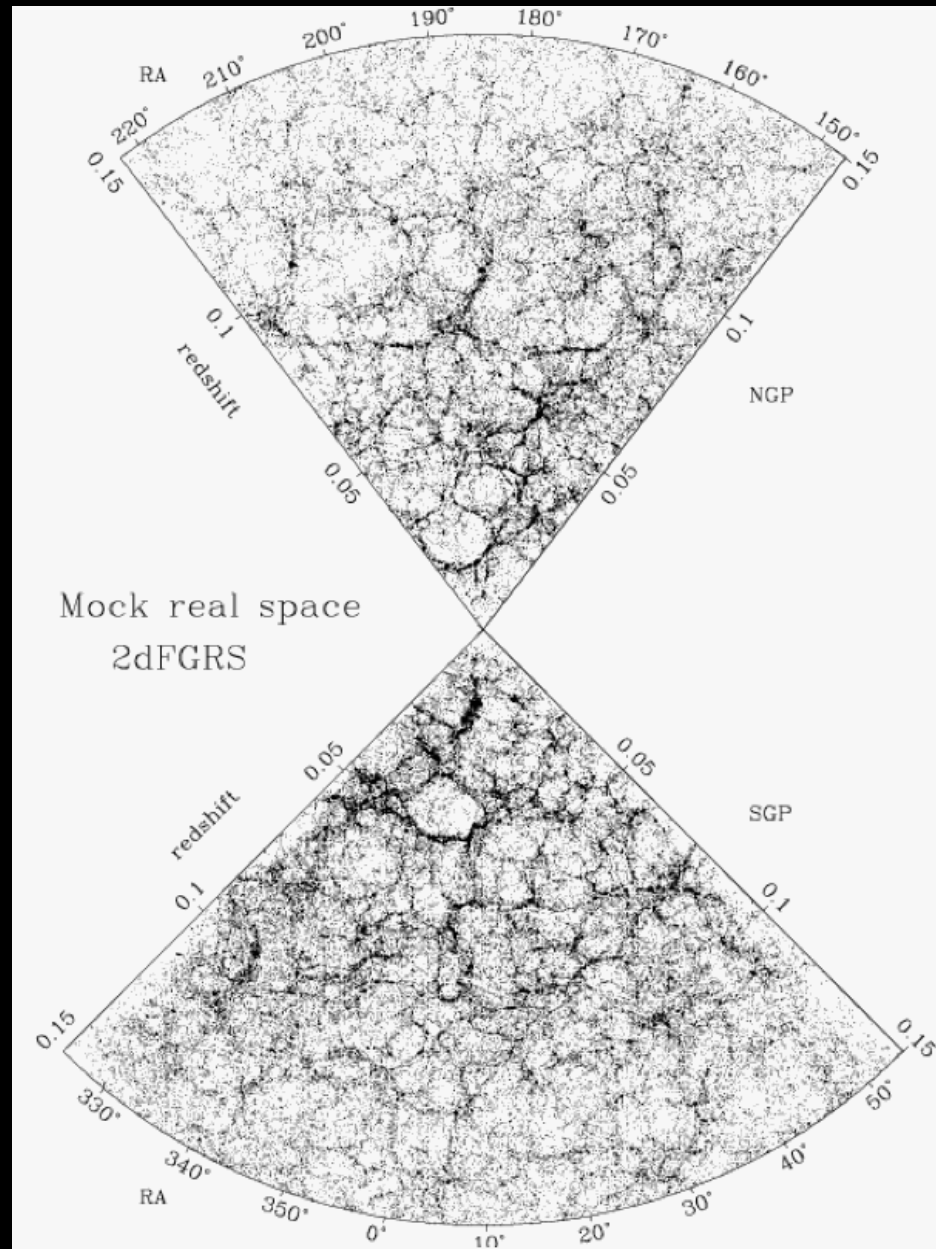
Fingers of God



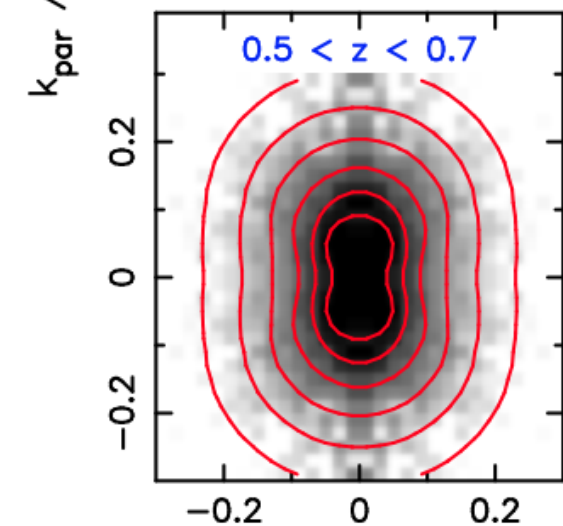
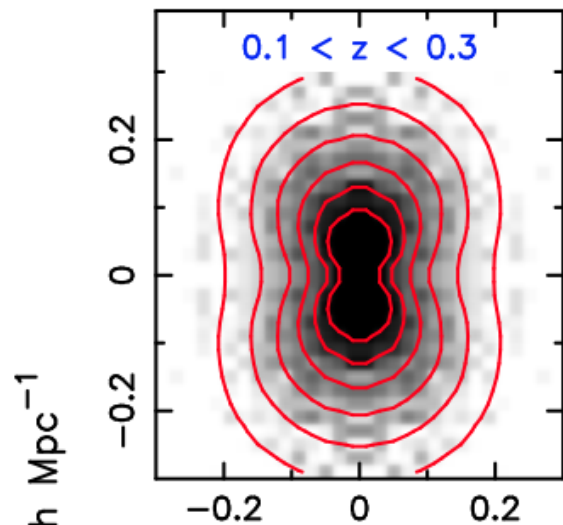
Mock z space
2dFGRS



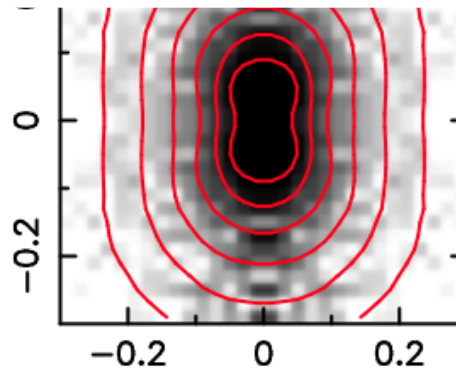
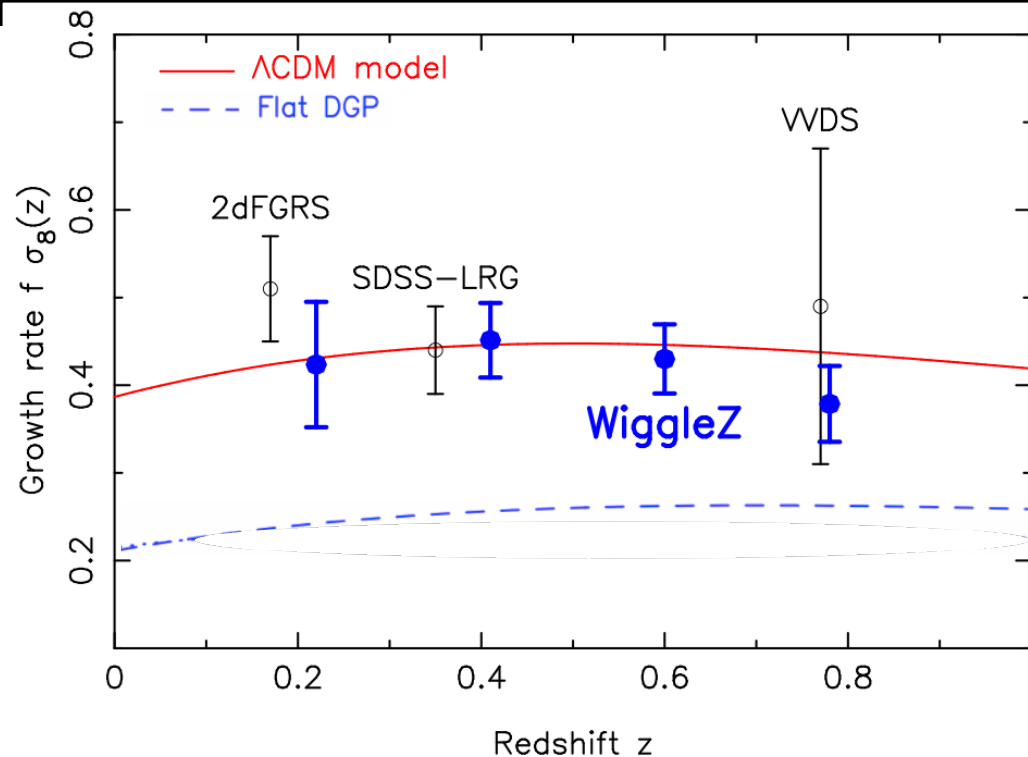
Fingers of God



Growth of Structure measured



$k_{\text{perp}} / h \text{ Mpc}^{-1}$



Power spec
2000
1000

Theoretical ideas

Inhomogeneities

~~Massive void~~

Many voids

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

New Theory

$f(R)$

TeV \mathcal{S}

DGP



New Component

Vacuum energy

Quintessence

Other dark energy

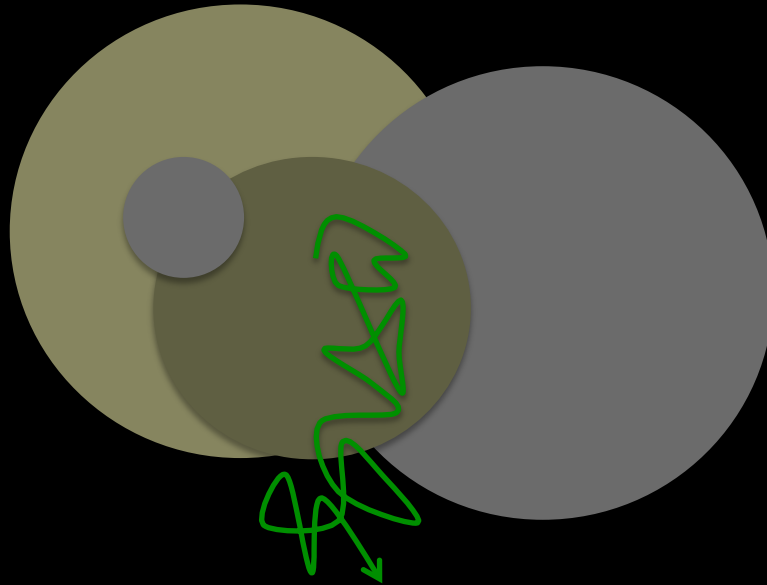
Summary

- Random initial conditions, constrained by horizons, expansion, and pressure, form patterns.
- We detect those patterns in the CMB
- We can also detect those patterns in the distribution of galaxies that formed from those initial fluctuations.
- We can use the patterns as standard rulers and spheres to measure the expansion history of the universe, and the growth of structure within it.

Silk Damping

- Photons leaked out of small regions

Small fluctuations did not survive



- Minimum mass to survive = mass within sphere of radius d
= about $8 \times 10^{13} M_{\odot}$

= about an elliptical cD galaxy or small galaxy cluster

Silk Damping scale

$$\sigma = 6.65e-24 m^2$$

$$\rho_b(z_{dec}) = 5.4e-19 kg m^{-3}$$

$$m_H = 1.7e-24 g$$

$$n = \rho_b / m_H = 3.2e8 m^{-3}$$

- Photons leaked out of small regions

Small fluctuations did not survive

Mean free path: $l = \frac{1}{n\sigma}$

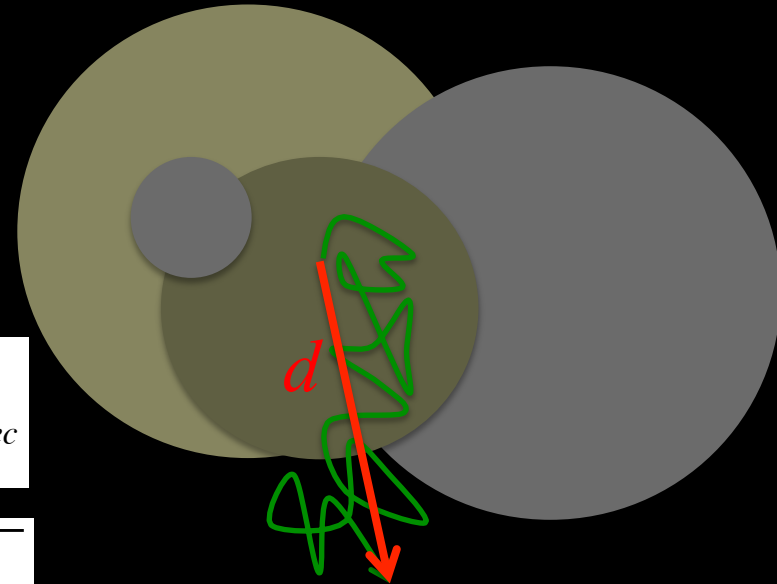
n σ e^- scattering cross-section

e^- #-density

Time between collisions: $t = l / c$

of collisions before decoupling: $N = \frac{t_{dec}}{t} = n\sigma ct_{dec}$

Distance travelled: $d = l\sqrt{N} = \sqrt{\frac{ct_{dec}}{n\sigma}} = 0.014 \text{ Mpc}$



- Minimum mass to survive = mass within sphere of radius d
= about $8 \times 10^{13} M_{\odot}$

= about an elliptical cD galaxy or small galaxy cluster

Characteristic expansion time

- Hubble time

$$t_H = \frac{1}{H} = \sqrt{\frac{3}{8\pi G\bar{\rho}}} = \sqrt{\frac{3}{2}} t_{dyn} \approx 1.22 t_{dyn}$$

Comparable to the dynamic time => both are important

$$\lambda_J = c_s t_{dyn} = c \sqrt{w} t_{dyn}$$

$$\lambda_J = \frac{c}{H} \frac{\sqrt{2}}{3}$$

Another way to calculate Jeans condition

$$(3/2)NkT \quad -(3/5)GM^2/r$$

- The virial theorem says:

- $2K + U = 0$ → a stable, gravitationally-bound system.

- $2K > 0$ → pressure dominates

- $U > 2K$ → gravity dominates (collapses)

- Estimate **Jeans mass** after Recombination

Mass above which a spherical system with constant density collapses

- $T=2970$ K, $\rho=5.4e-19$ kg m⁻³, $\mu=0.584$ for composition of Hydrogen: X=0.77, and Helium: Y=0.23 (see Carroll and Ostlie Eq. 10.16)

$$M_J \approx \left(\frac{5kT}{G\mu m_H} \right)^{3/2} \left(\frac{3}{4\pi\rho} \right)^{1/2} \approx 1.9 \times 10^6 M_\odot$$

- ~ Mass of globular star cluster

Two characteristic mass scales...

- Jeans mass after recombination $\sim 10^6 M_\odot$
 - Fluctuations $M > 10^6 M_\odot$ are amplified after recombination
- Recall you needed $M > 10^{13} M_\odot$ to survive Silk damping

$10^6 M_\odot$ to $10^{13} M_\odot$ neatly spans a range of structures
(globular cluster to massive elliptical galaxy)

- Could these higher-density regions have produced the structures we observe today?

Timing of Structure Formation

(see Carroll and Ostlie Example 30.2.1)

- Record for most distant QSO: $z=6.43$ (out of date!)
 - Estimate density fluctuation needed at recombination to form a quasar this soon...
 - Gravitational collapse begins when $\delta \sim 1$
 - In matter era, $\delta = \delta_i (t/t_i)^{2/3}$
 - $t = 2/(3H_0) a^{3/2}$ (\sim lower limit to age)
 - $H_0 = 70 \text{ km s}^{-1}\text{Mpc}^{-1} = 0.072 \text{ Gyr}^{-1}$
- $a = 1/(1+z) = 0.135$
- $t = 2/(3H_0) a^{3/2} = 462 \text{ Myr}$
- let $t = t_{\text{dec}} = 0.38 \text{ Myr}$ (WMAP)

$$\Rightarrow \delta = 0.009$$

There must have been density fluctuations of several tenths of a per cent at decoupling.

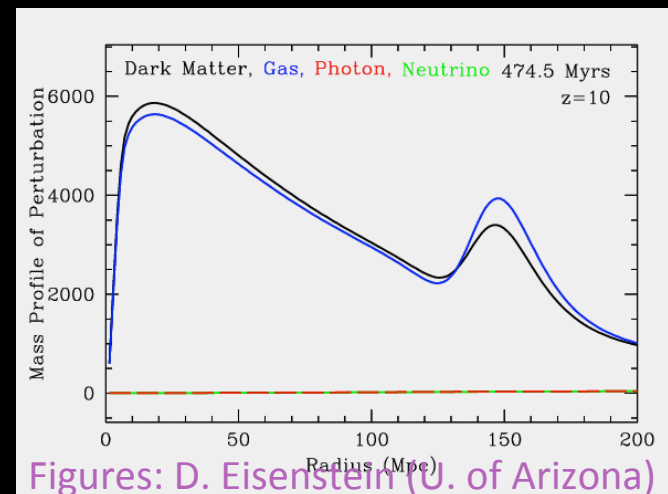
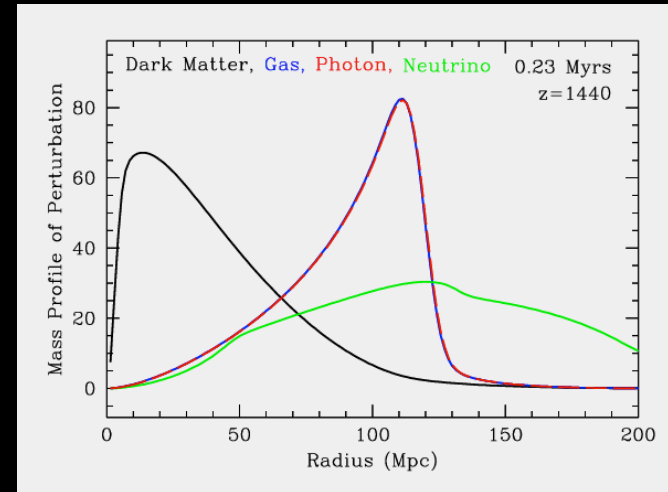
But that doesn't match the CMB observations!



Timing of Structure Formation

More evidence that dark matter is *not baryonic*

- CMB fluctuations
 - Observed fluctuations could not have grown fast enough to make galaxies and quasars!
- Solution: non-baryonic dark matter
 - Negligible interaction with radiation
 - Could start collapsing long before decoupling, at start of matter era
 - Could grow during matter era
 - Could reach required level by recombination with no trace in CMB signal
- After decoupling
 - Baryonic matter attracted to the large clumps of dark matter until they share the same density
 - They grow together to form objects we see today



Figures: D. Eisenstein (U. of Arizona)

Summary

There are a huge number of ways to use cosmological data to test fundamental physics

I haven't even mentioned galaxy evolution and formation studies, nor the study of star-forming galaxies at high redshift, etc...

