Density waves, protoplanetary disks and planets

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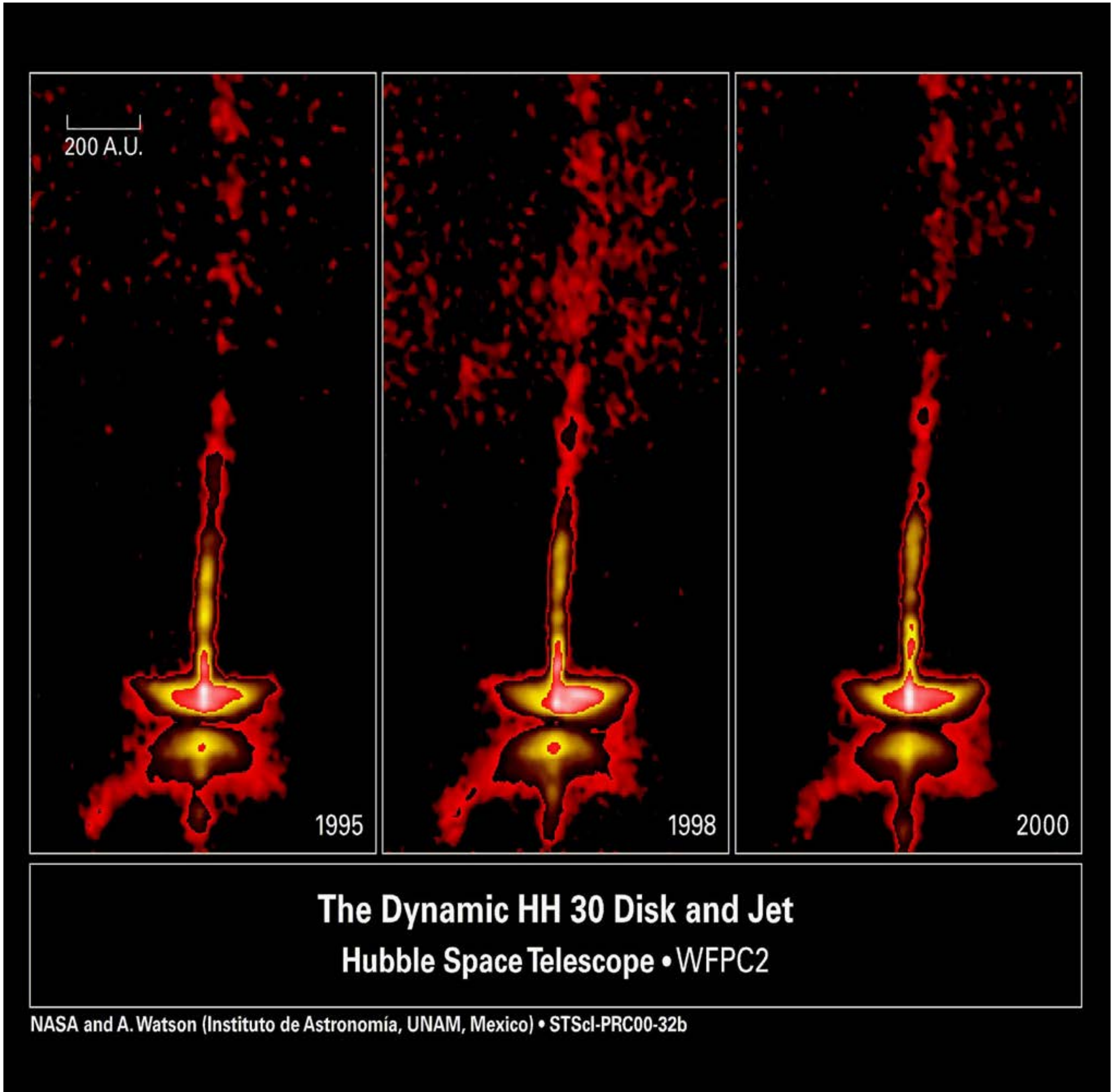
Sites of planet formation



Protoplanetary (accretion) disks

In Keplerian (differential rotation)
$\Omega^2 = GM/R^3$

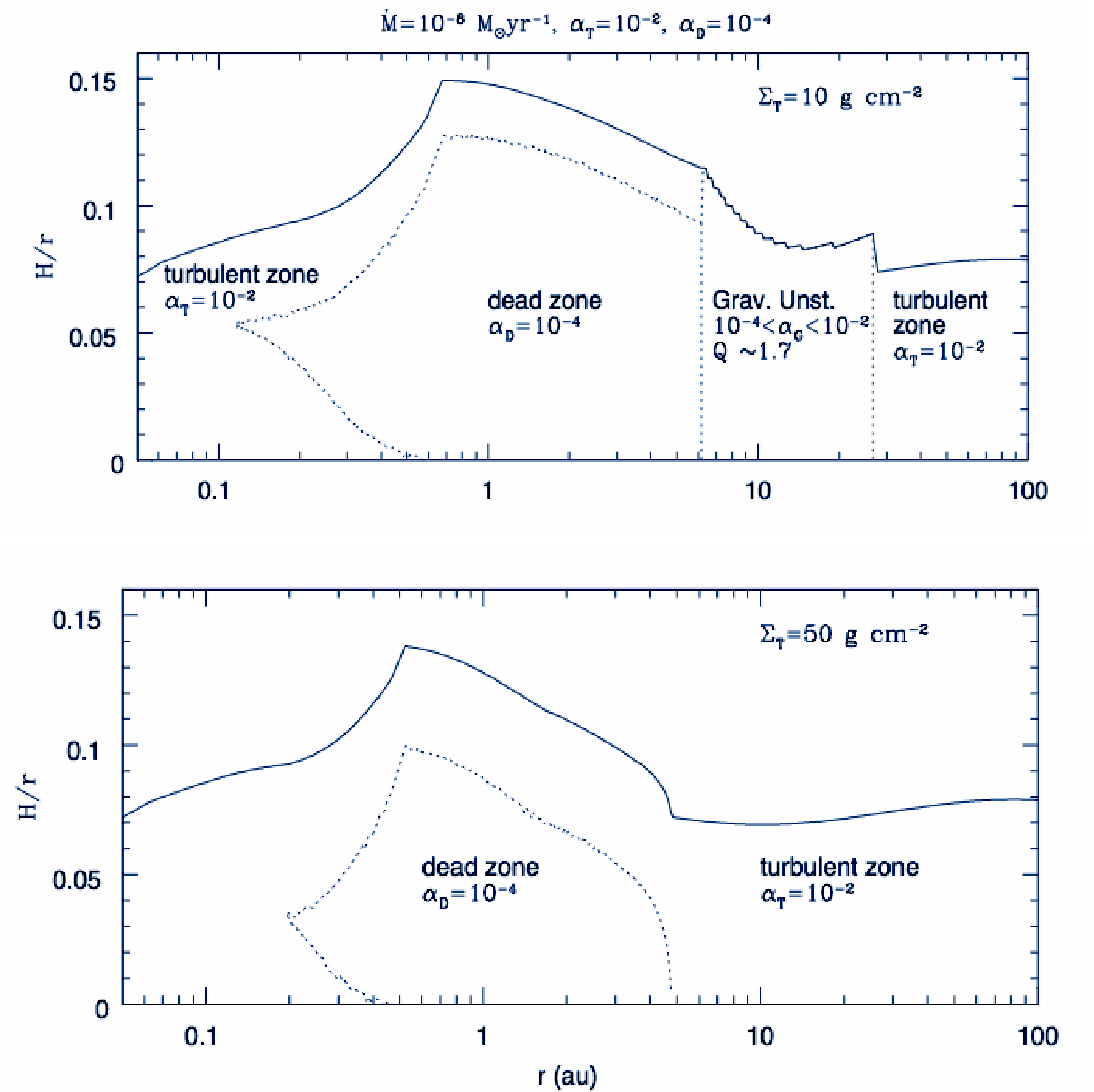
Aspect ratio
$HR << 1$

Hypersonic
$c = H\,\Omega << R\Omega$

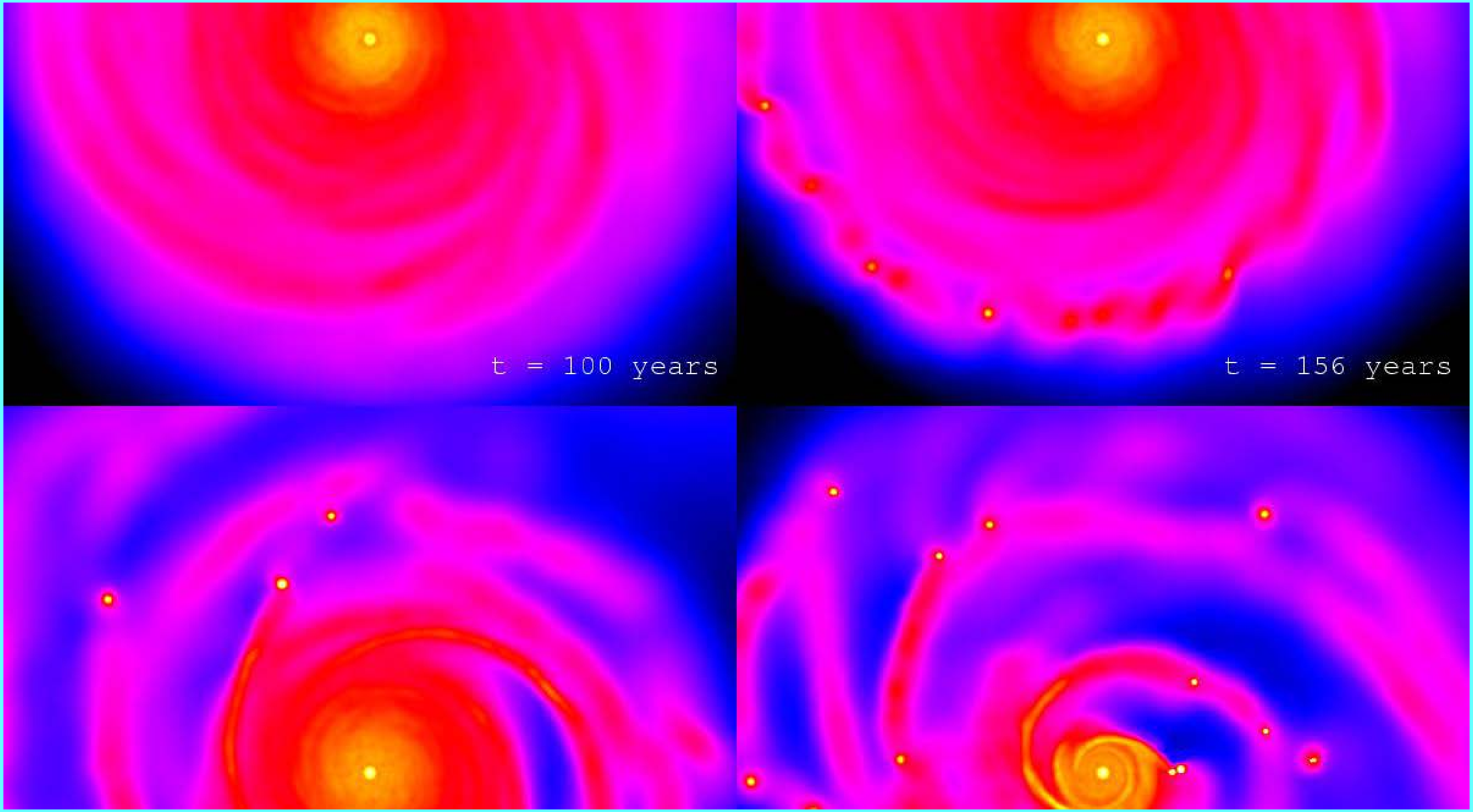
Schematic disc models (Terquem 2008)

Strength of self-gravity measured by Toomre parameter, Q , ~($M_d/M_*)(R_d/H)$.

Anomalous Kinematic viscosity $\nu \sim \alpha H^2\Omega$.
Spreading time $t \sim R^2/\nu$.



Two modes of planet formation
1) Gravitational Instability (top down):
spiral modes and fragmentation in disks with
$Q = \Omega c / \pi G \Sigma < \sim 1$



2) Planetary accumulation (bottom up):

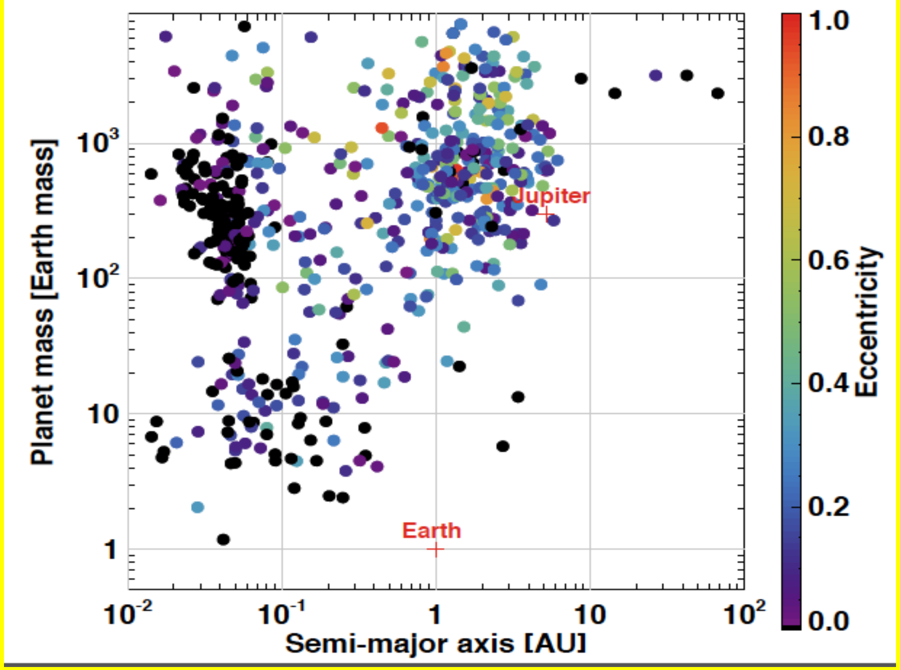
Many stages starting from dust grains sticking together leading to formation of planetesimals, growth by runaway accretion, followed by oligarchic growth of solid cores to several earth masses that can accrete gas.

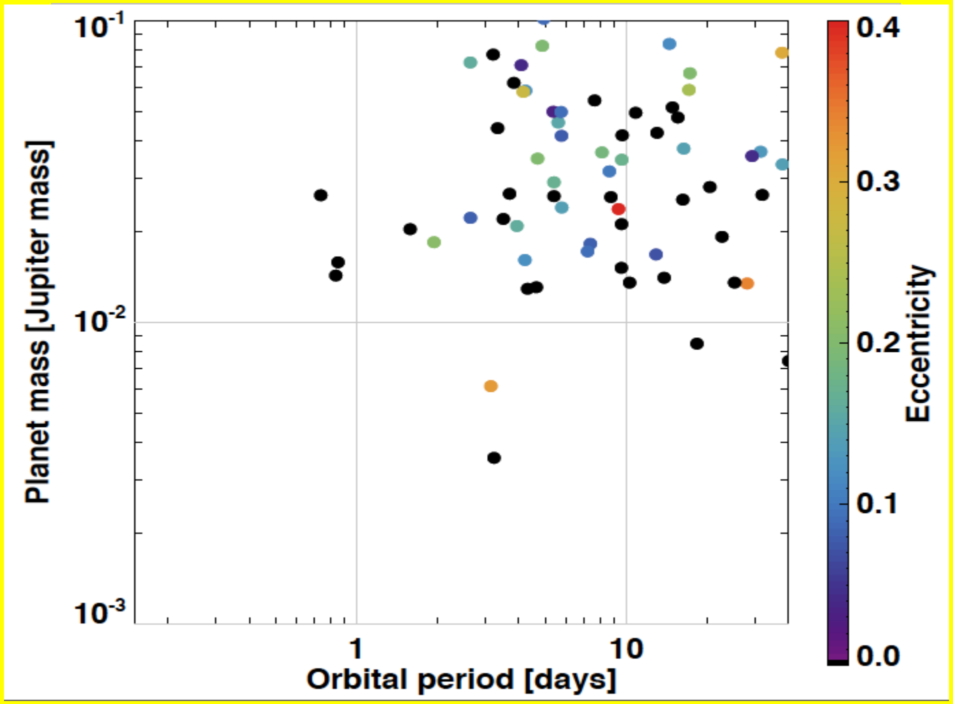
Long time scale ~ disk lifetimes 10^6 – 10^7 yr.
(for giant planet formation at a few AU).

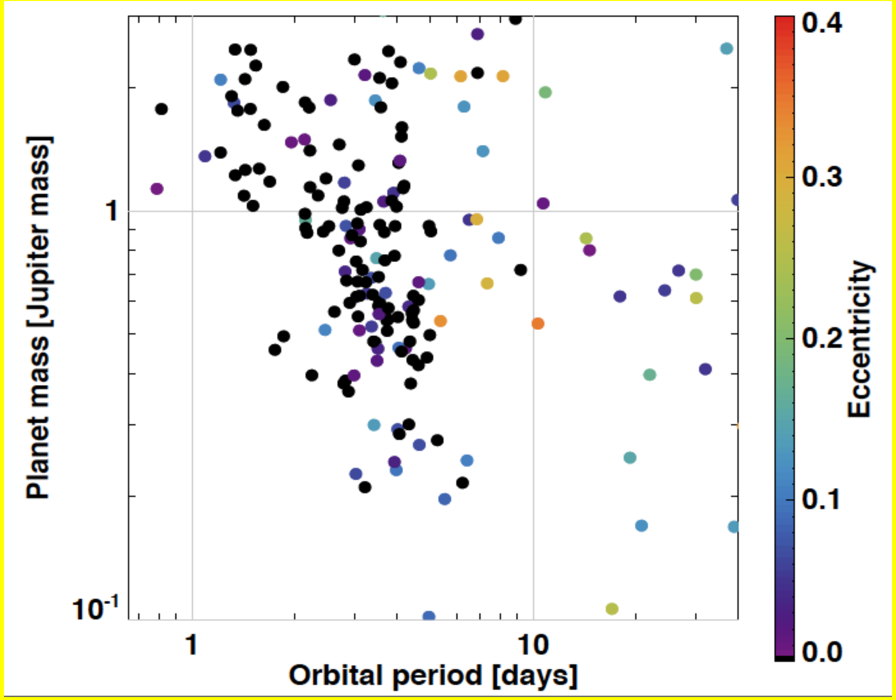
Gas free accumulation of terrestrial planets can take longer.
Difficulties at larger distances….gravitational instability leading to fragmentation favoured?

Dependence of planet mass on period (with eccentricities) for different mass ranges.

Orbital **migration?**







Tidal Interaction of a protogiant planet with a protoplanetary disc and orbital migration (Lindblad torques):

Slower material <-----------------------------------<

O Coorbital region

>-----------------------------------→ Faster material

Centre ↓

Density waves are excited and propagate into the surrounding disk.

The outer slower material drags the planet backward and the inner and faster material accelerates it. This frictional interaction causes orbital circularization and migration.

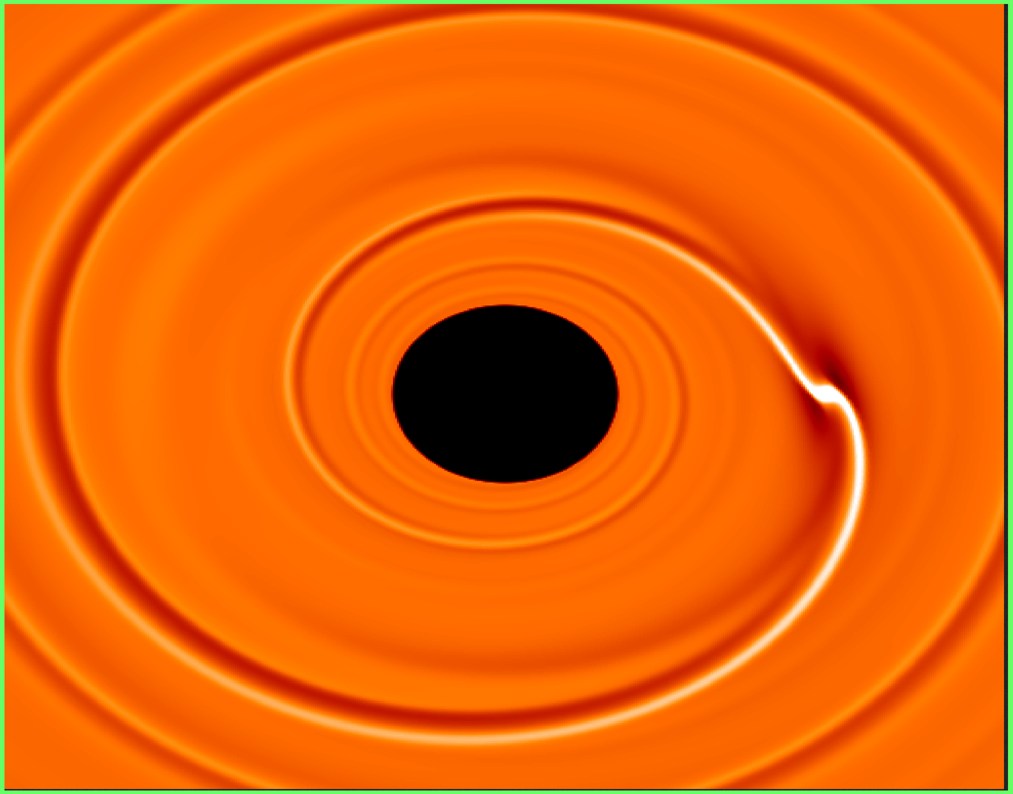
If the coorbital region is unimportant, the direction is controlled by which material has the stronger interaction. In particular if there is an inner cavity the outer and slower material wins leading to inward migration.

Type I migration of low mass protoplanet

Linear response comprising of density waves: Outer wake is stronger than inner wake leading to inward migration for

(locally) isothermal disks with $\Sigma \alpha r^{-3/2}$.

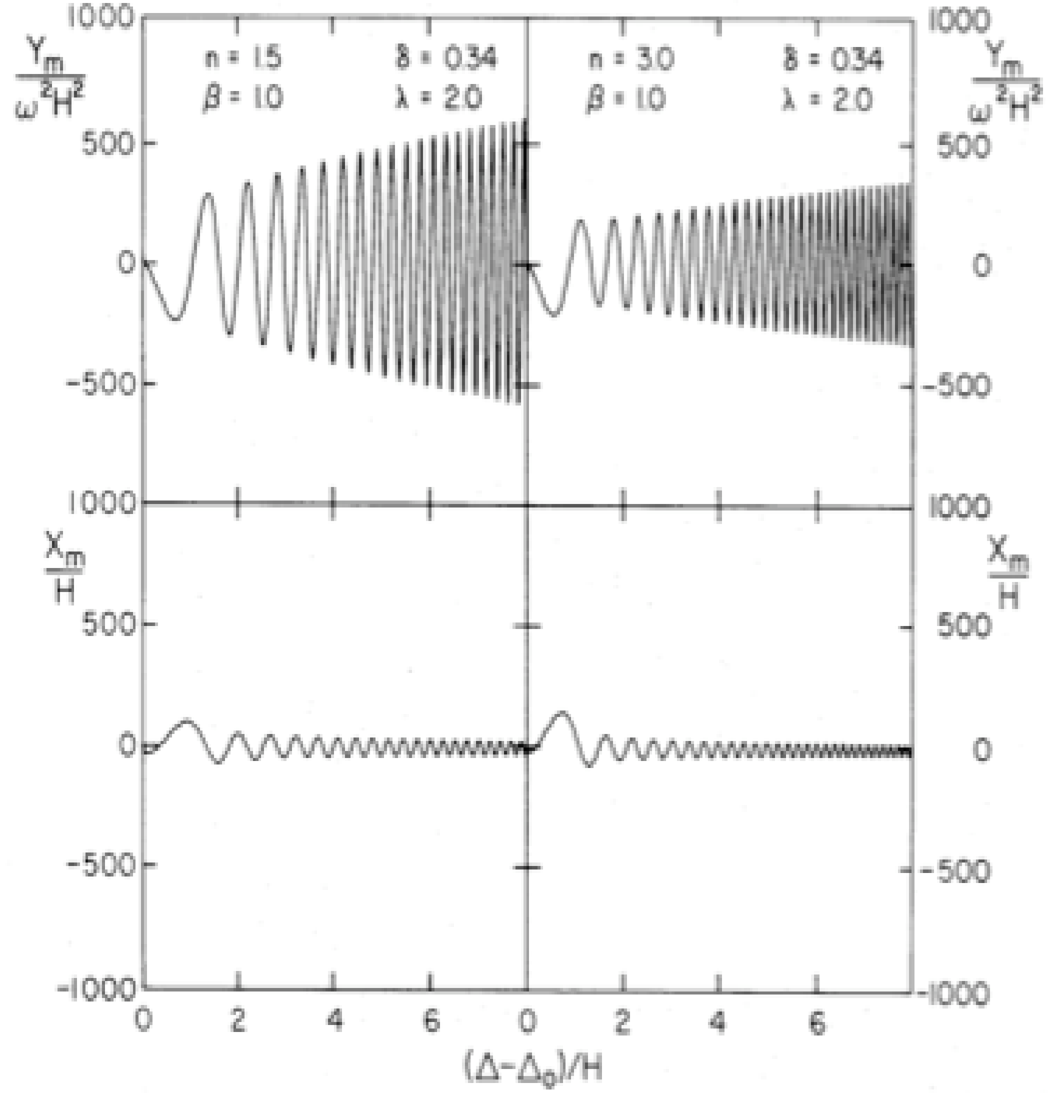
Mass distribution similar to minimum mass solar nebula with mass α $r^{1/2}$ with 2 M_J within 5 AU. Total mass ~ 0.02 solar masses.



Gap Formation

Linear response problem for disk forced by a protoplanet:

Calculate torque for disk edge a distance ~ H (one scale height) from the protoplanet by summing contributions from different m.



Gives torque

$$\dot{H}_1 = \text{sign}\,(\Delta_0) f \left(\frac{M_p}{M}\right)^2 \Sigma_n R_p^4 \omega^2 \left(\frac{R_p}{H_\infty}\right)^3 ,$$

CONDITIONS FOR GAP FORMATION

Viscous condition for gap formation (mass ratio > 40/Reynolds number)

$$\frac{M_p}{M_\star} > \frac{40\nu}{R^2\omega}.$$

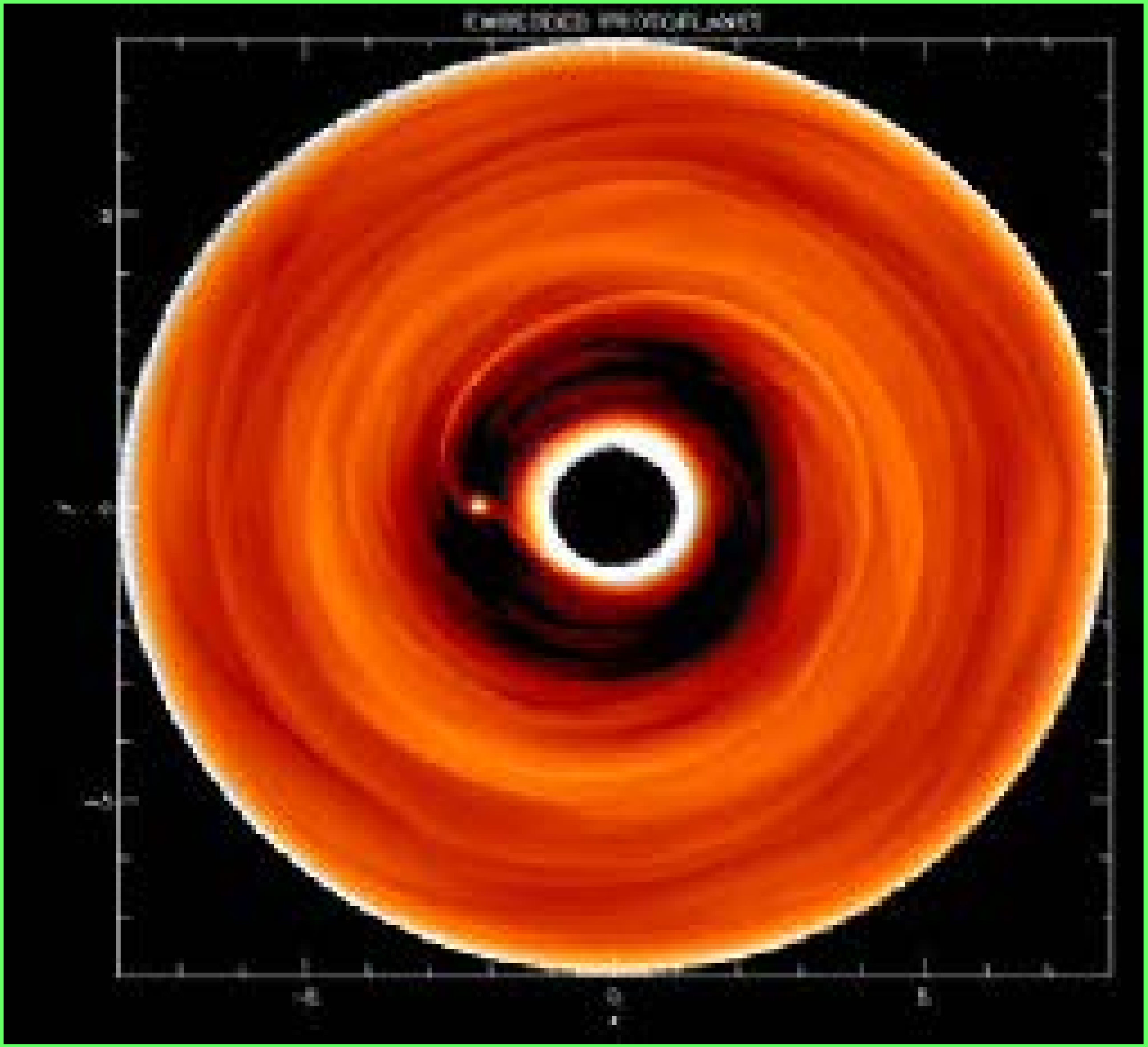
Thermal condition for gap formation (Hill radius > disk scale height

$$a\left(\frac{M_p}{3M_\star}\right)^{1/3} > H.$$

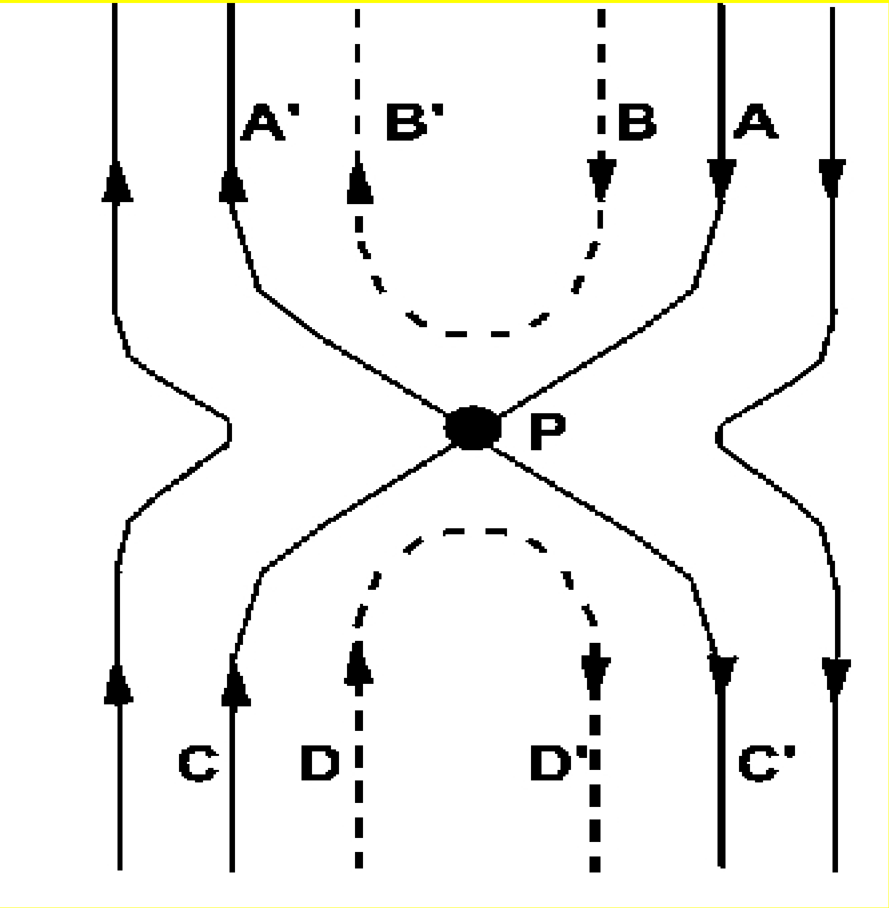
More recent study of Crida et al (2006) combines these conditions…. Has same content

$$\frac{3}{4}\frac{H}{R_H} + \frac{50}{q\mathcal{R}} \lesssim 1$$

Type II Migration when a gap forms occurs on the evolution time of the disk.



Coorbital or corotation torques Schematic illustration of coorbital flow for a low mass protoplanet



Due to horseshoe turns, if there is an underlying gradient of bsckground state variables (eg. in the surface density or entropy) , the surface density at A' will not be the same as that at C. Similarly the surface density at C' will not be same as that at A.
→ Coorbital torque (called Horseshoe drag).

Linear corotation torques:
For potential proportional to exp(imϕ)

$$\Gamma_{c,m} = -\frac{m\pi^2 \Phi'^2_m}{2d\Omega/dr} \frac{d}{dr}\left(\frac{\Sigma}{\omega}\right)$$

ω is vorticity α $r^{-3/2}$

Torque established on orbital time scale..
Not valid if horseshoe turns significant…

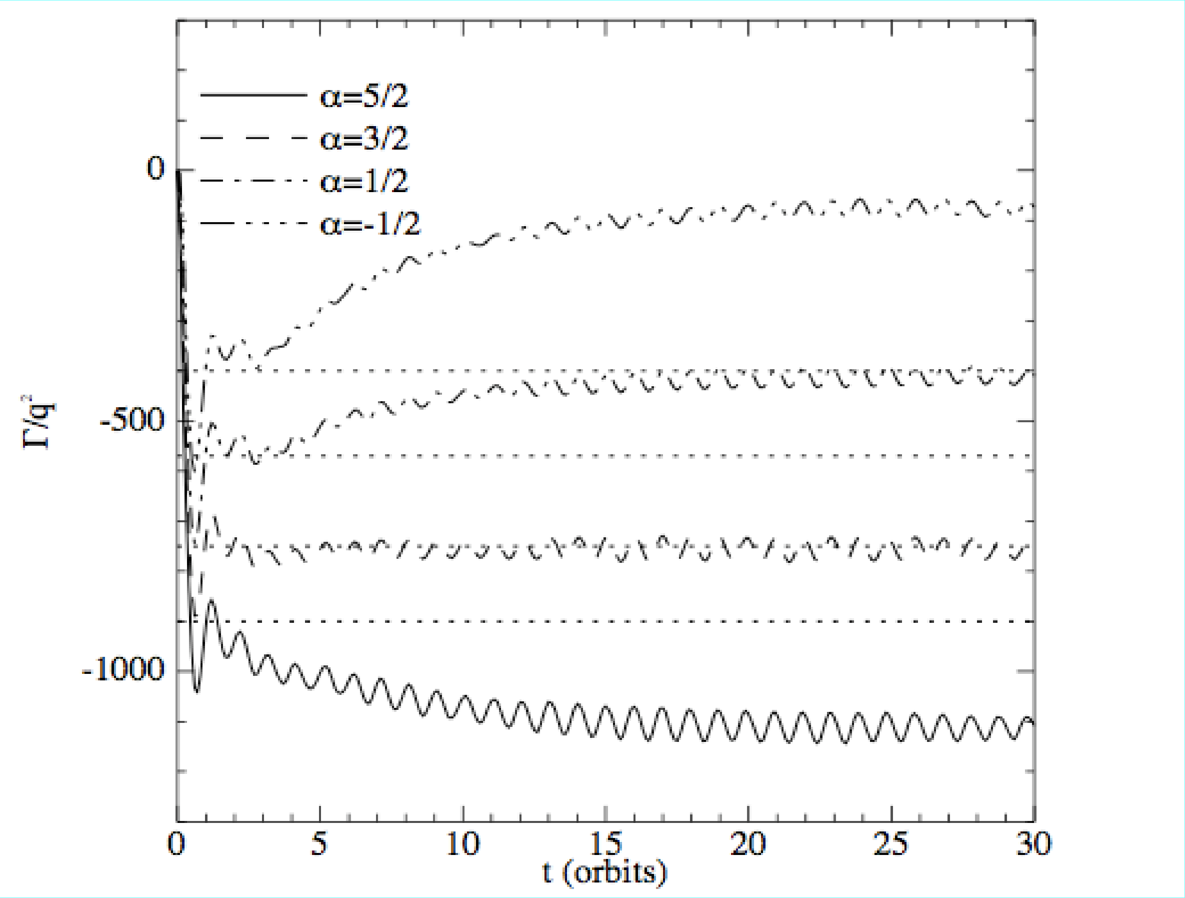
Three evolutionary phases

1) Linear phase

2) Initial horseshoe turns
establishment of
horseshoe drag

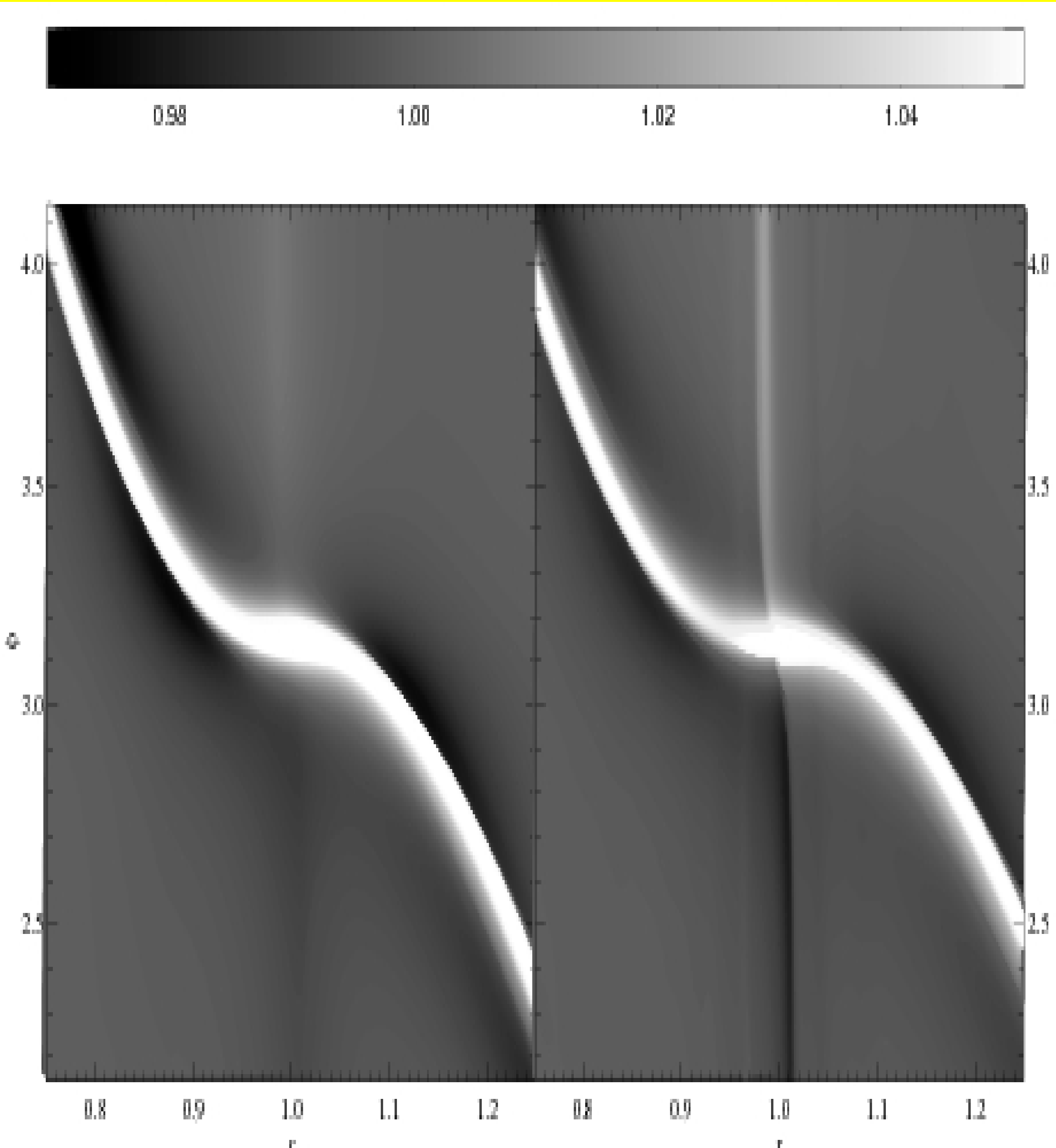
3) Material returns after
full libration: persistence of torque
depends on saturation

Linear corotation torques and horseshoe drag (pre- saturation)for the strictly isothermal (baratropic case) for different surface density profiles ($\Sigma \alpha\ r^{-\alpha}$)



Effect of a non barotropic equation of state

Migration can reverse depending on the entropy gradient



Surface density in the
oorbital region
or a 4 M_E protoplanet.

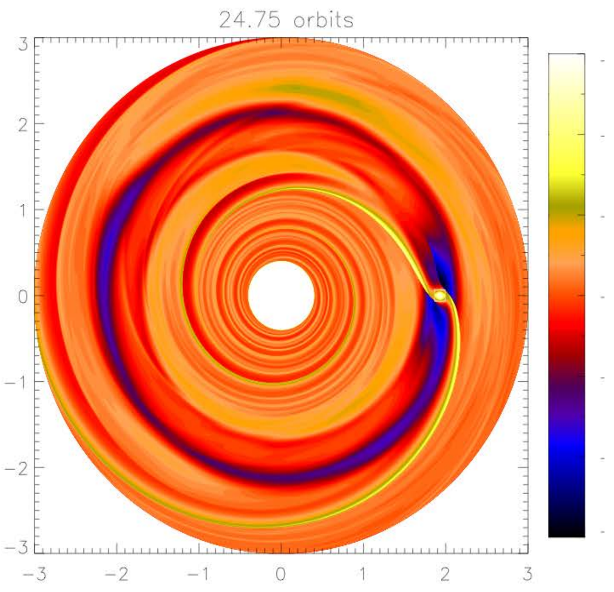
ocally isothermal
ase (left)

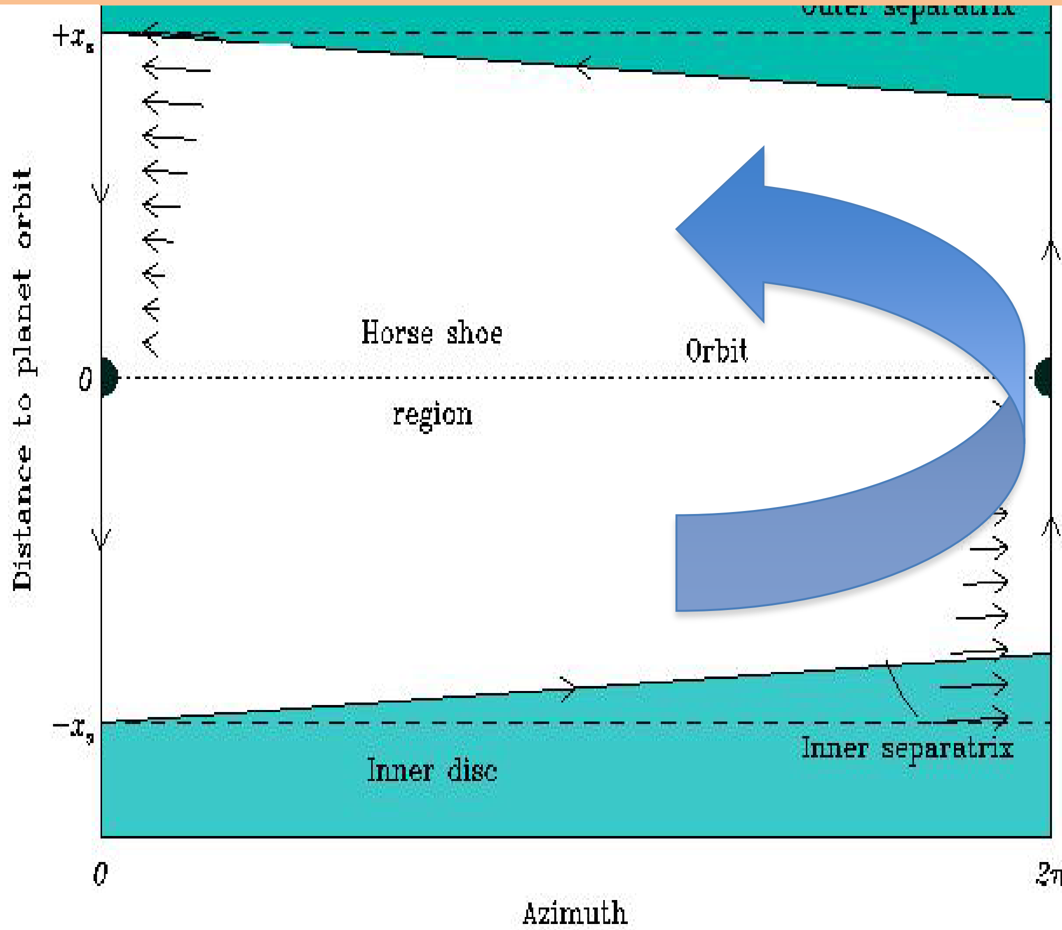
diabatic case
ith entropy increasing
utwards (right)

$_0$ α $r^{-1/2}$, T α r^{-1})

($\gamma = 1.4$)

Runaway (Type III) migration: Coorbital zone with partial gap



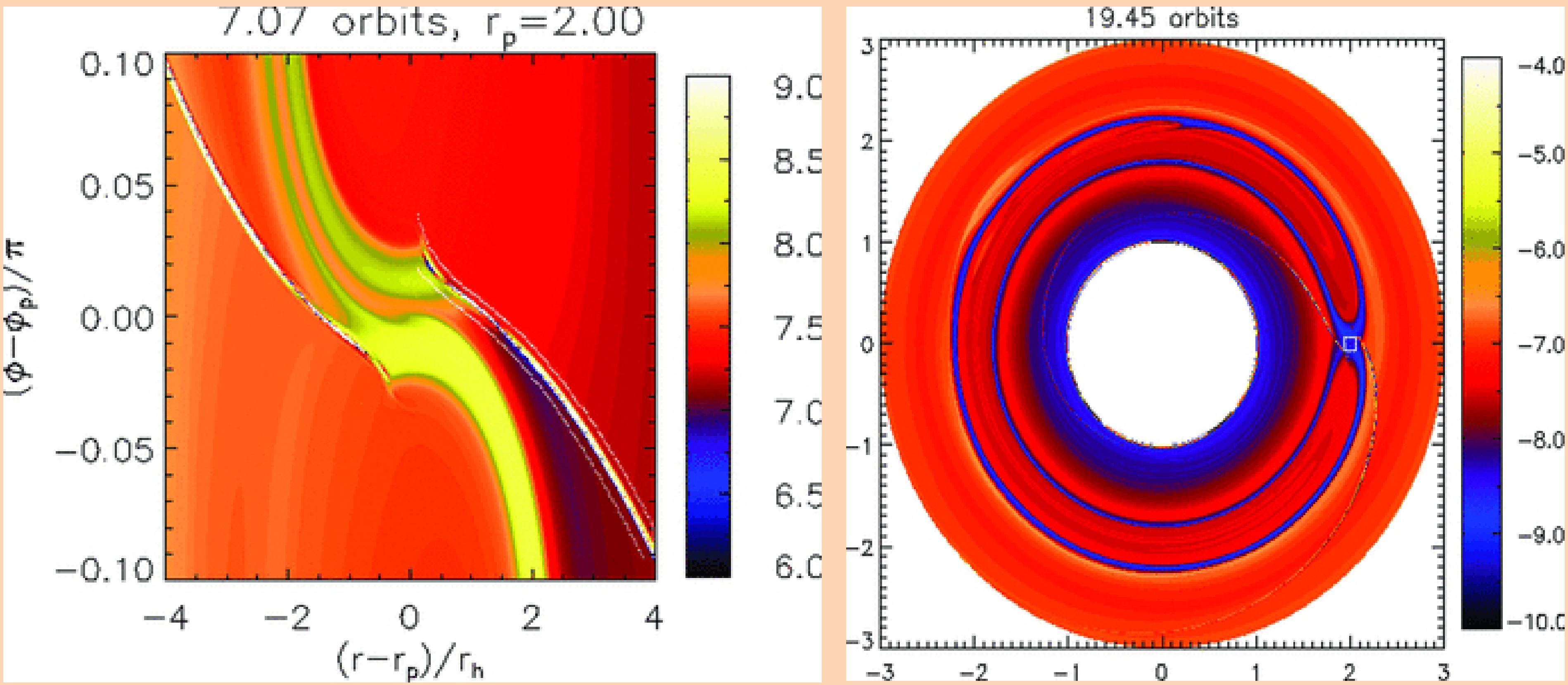


Other aspects:

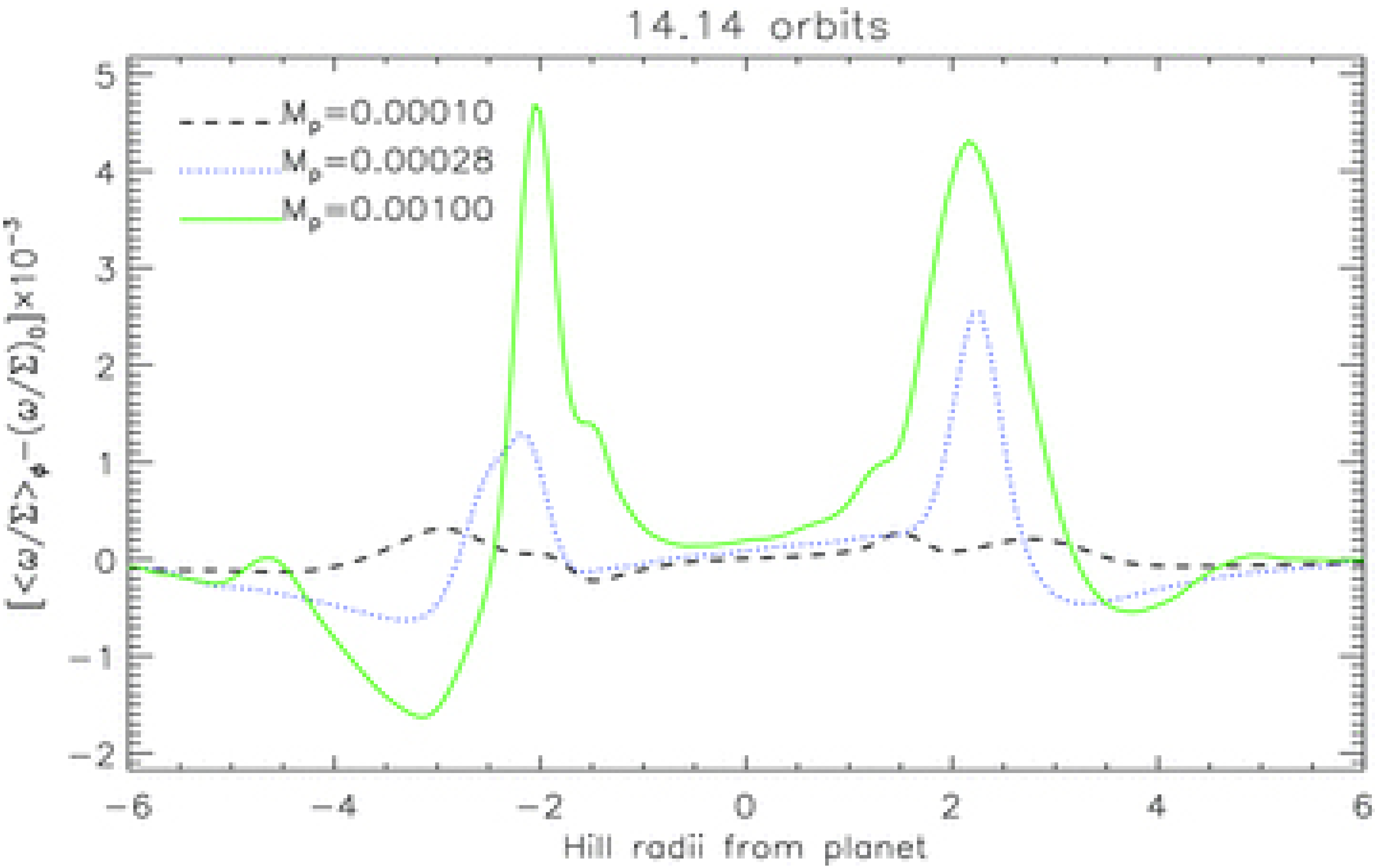
a) Stability of disk gap edges
notably in inviscid laminar disks
and for giant planets in the Saturn mass range
and larger.

b) Effects arising in multiplanet systems

Log vortensity (log ω/Σ) changes induced by shocks produced by a Saturn mass protoplanet in an inviscid disc. The broad high level regions are generated in the coorbital region

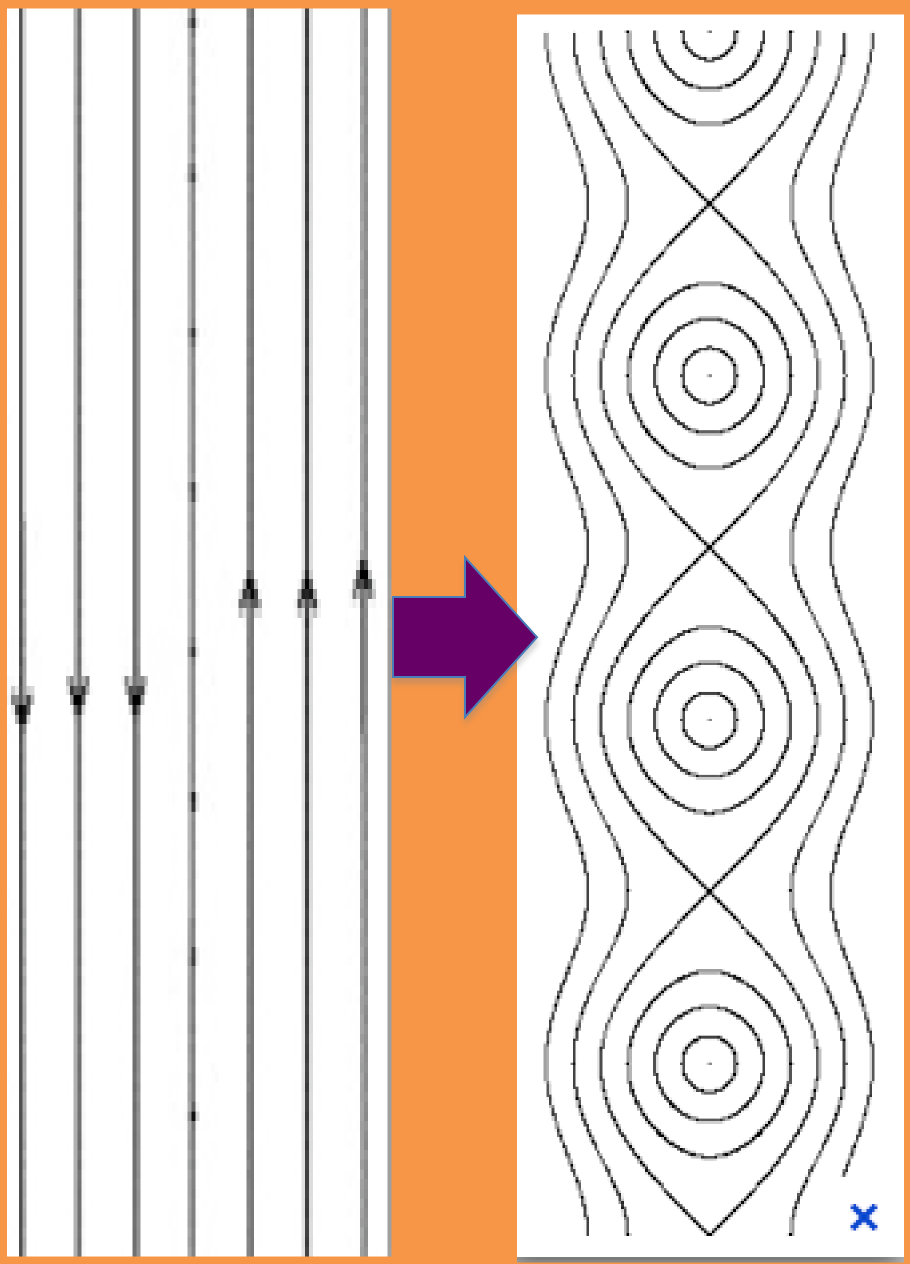


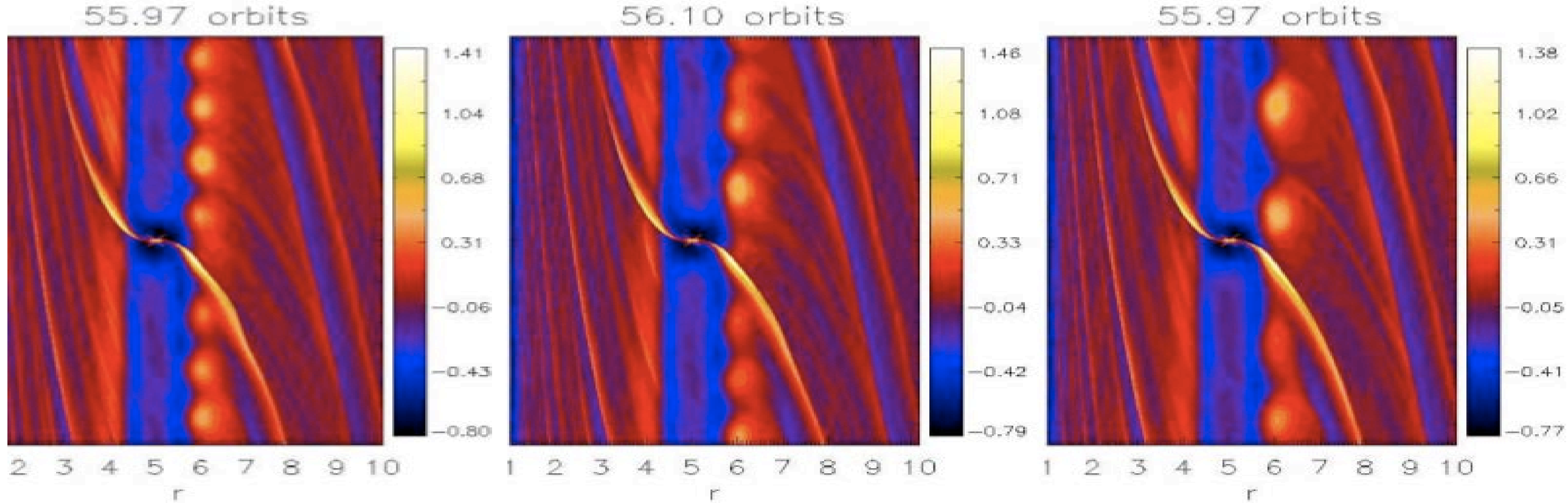
Induced vortensity perturbation for different planet masses showing inner and outer localized extrema (potential sites of instability)



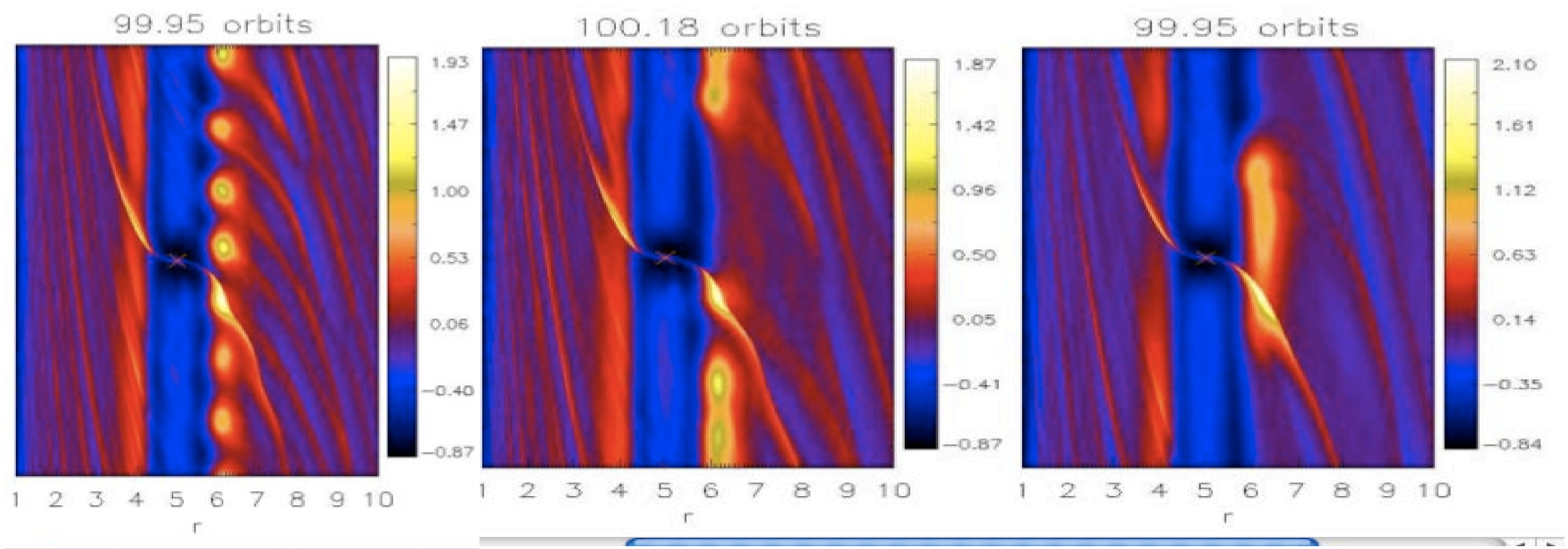
Condition for instability:

Conservation of specific vorticity ω/Σ on streamlines in 2D requires this to be an extremum for a linear instability (cf. C.C.Lin, 1945)

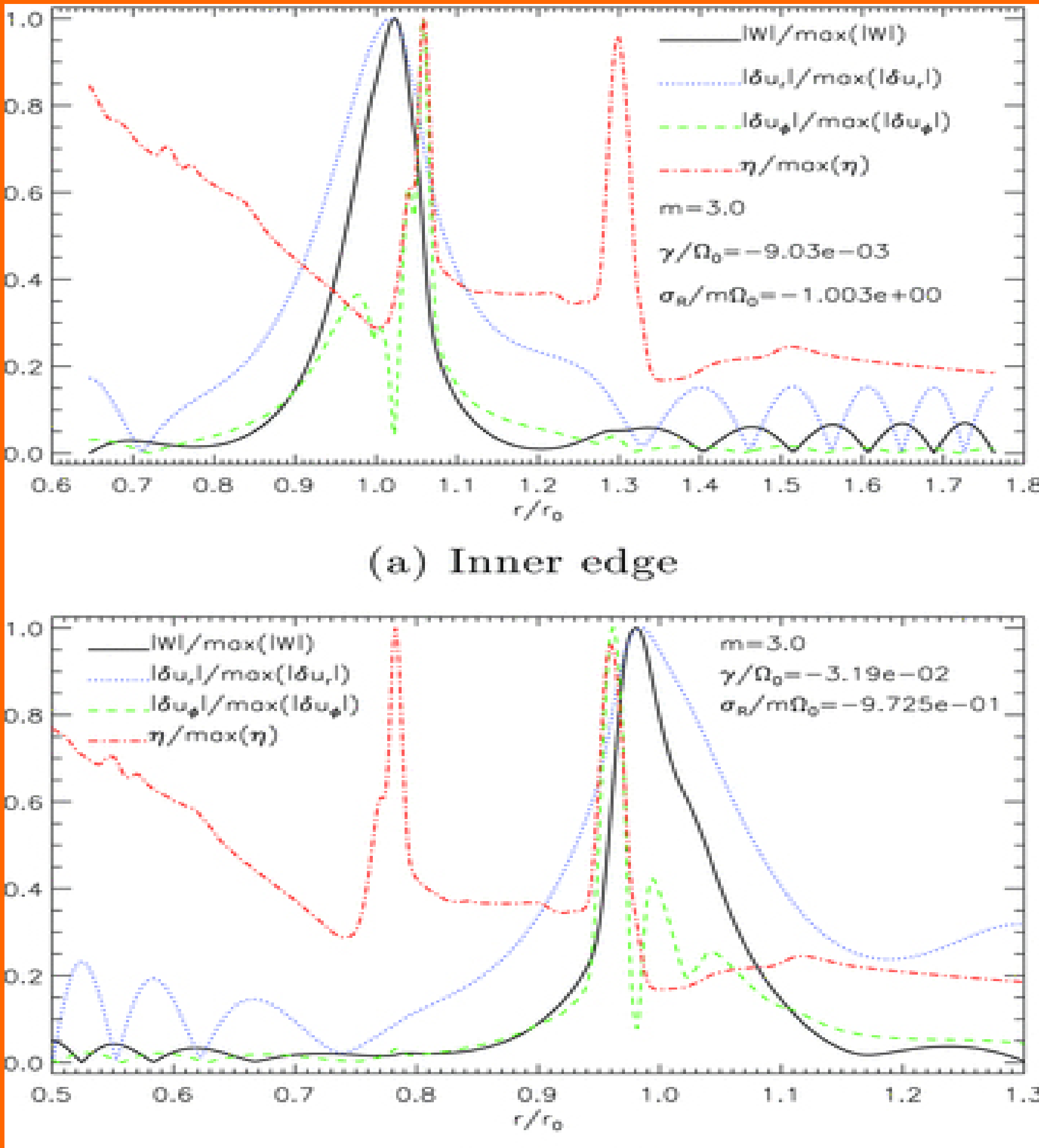




Surface density for Q =3,4,8: Top, linear phase, Botom, in the nonlinear regime.



Unstable modes associated with inner and outer rings: Modes with low m are localised near surface density (or inverse vortensity) maxima and decay away exponentially with some weak wave emission.



(a) Inner edge

(b) Outer edge

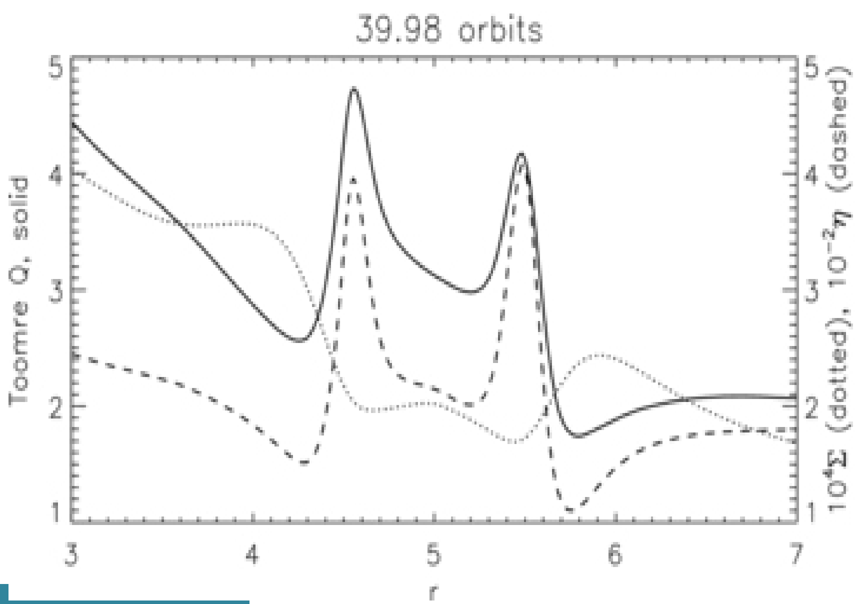
Efect of Self-gravity

Self-gravity becomes important when the Toomre parameter $Q = \Omega c / \pi G \Sigma$,
which has to exceed unity to be stable
to axisymmetric modes,
attains a value approaching unity over a substantial region of the disk.

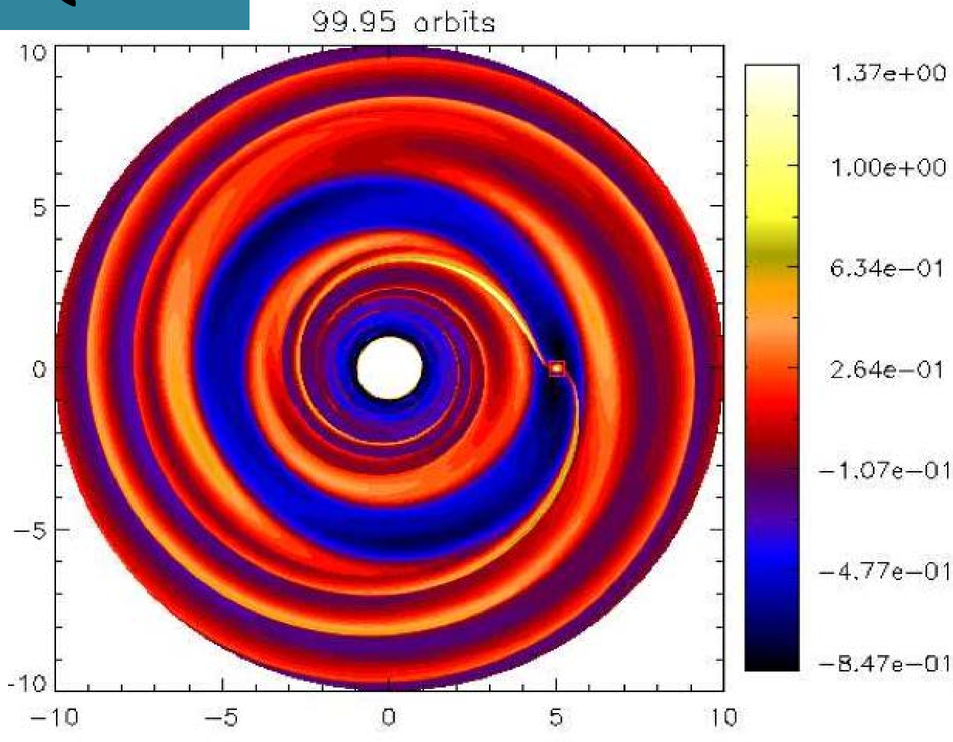
Vortices associated with vortensity minima then attain smaller azimuthal scales, and are replaced by large scale self-gravity dominated edge modes

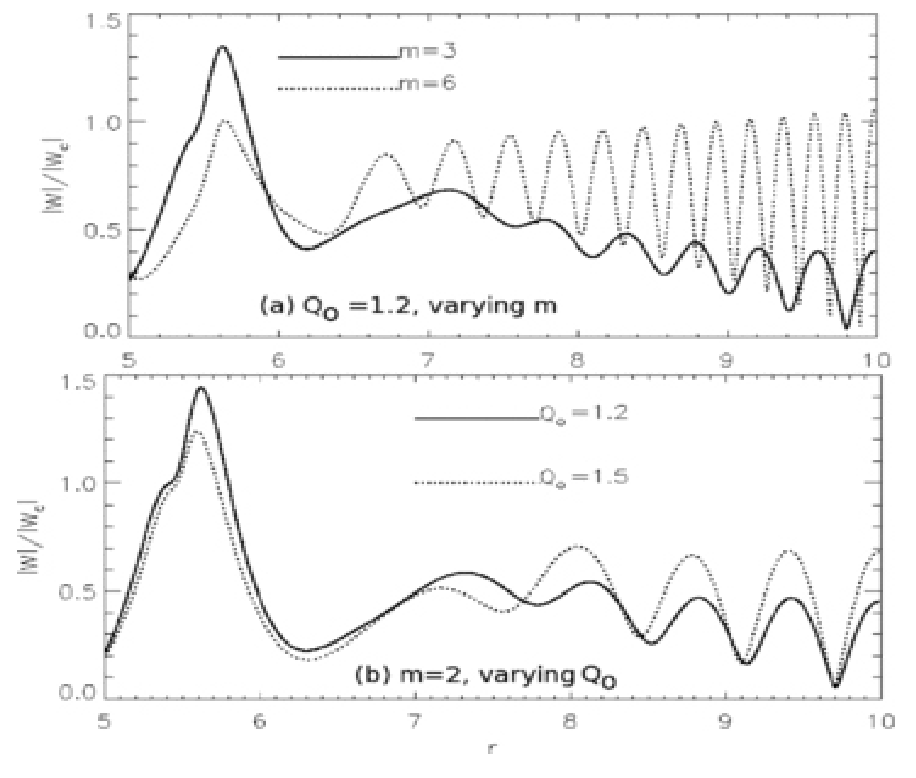
Associated with vortensity maxima.

Unstable edge modes peaked at outer vortensity maxima for Q= 1.2, 1.5

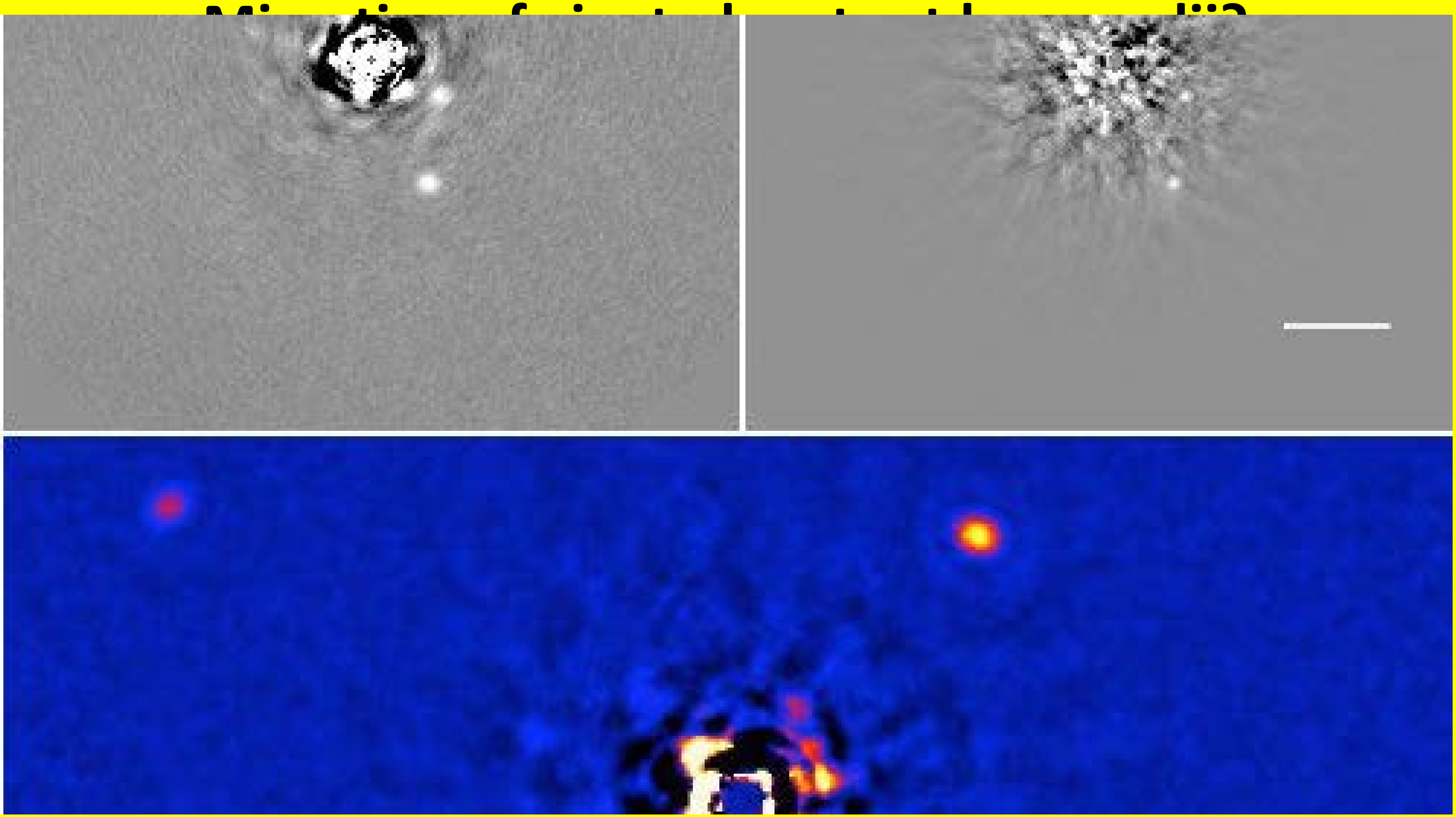


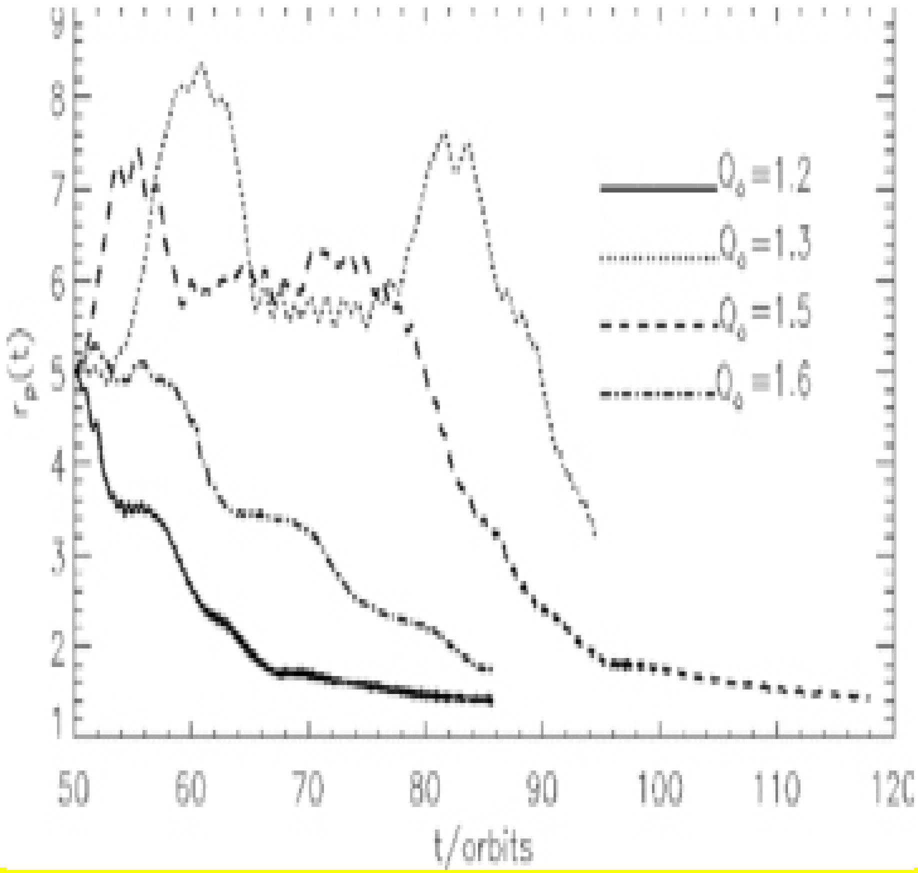
Q=1.5

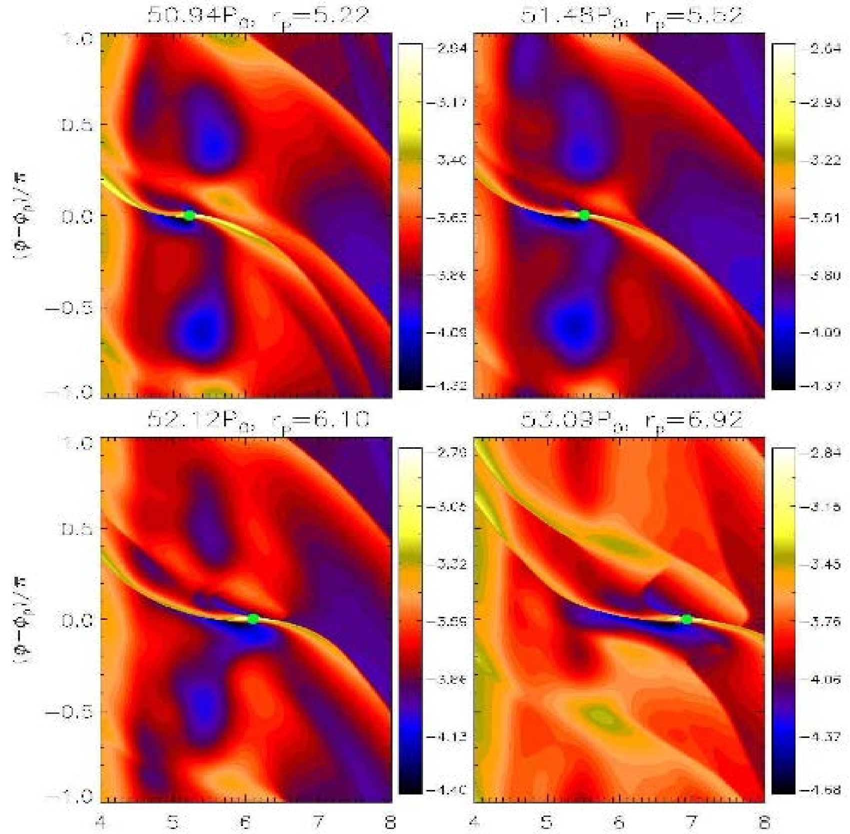




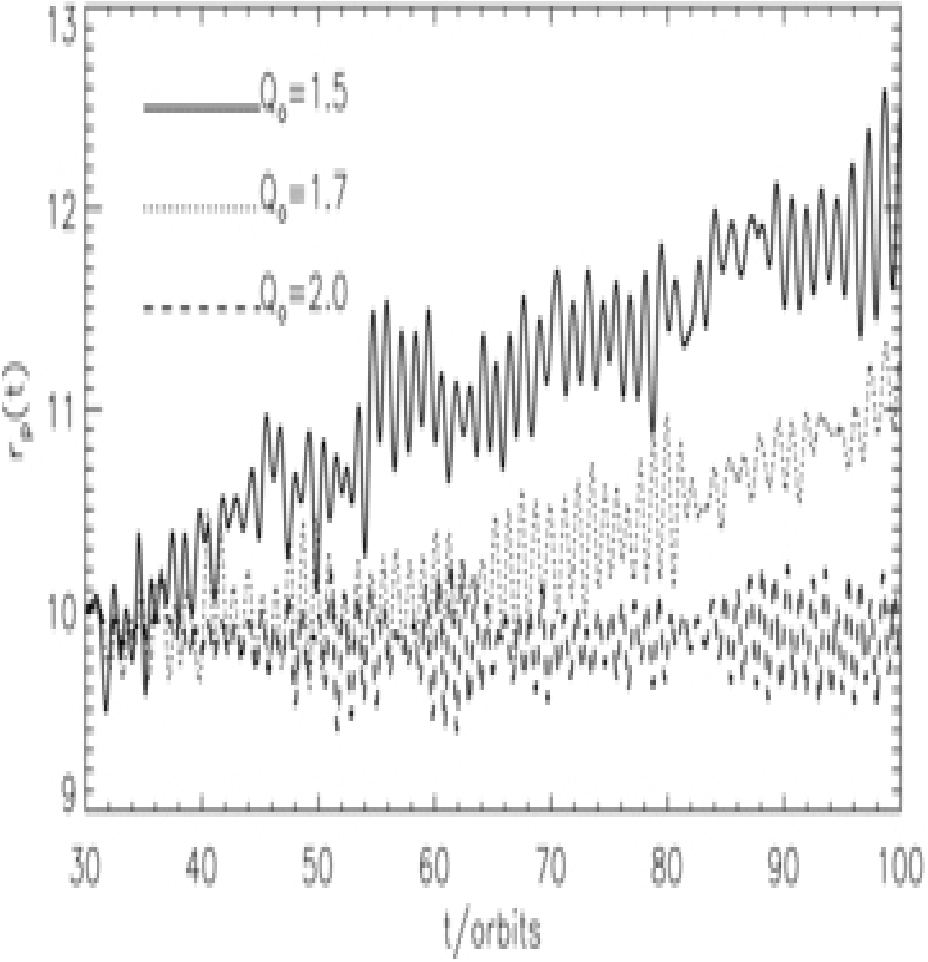
HR 8799 Four planets ~10 Jupiter masses imaged at 14.5 ,24, 38, 68 AU (Marois et al. 2010) (inner 3 planets possibly in a 1:2:4 Laplace resonance)

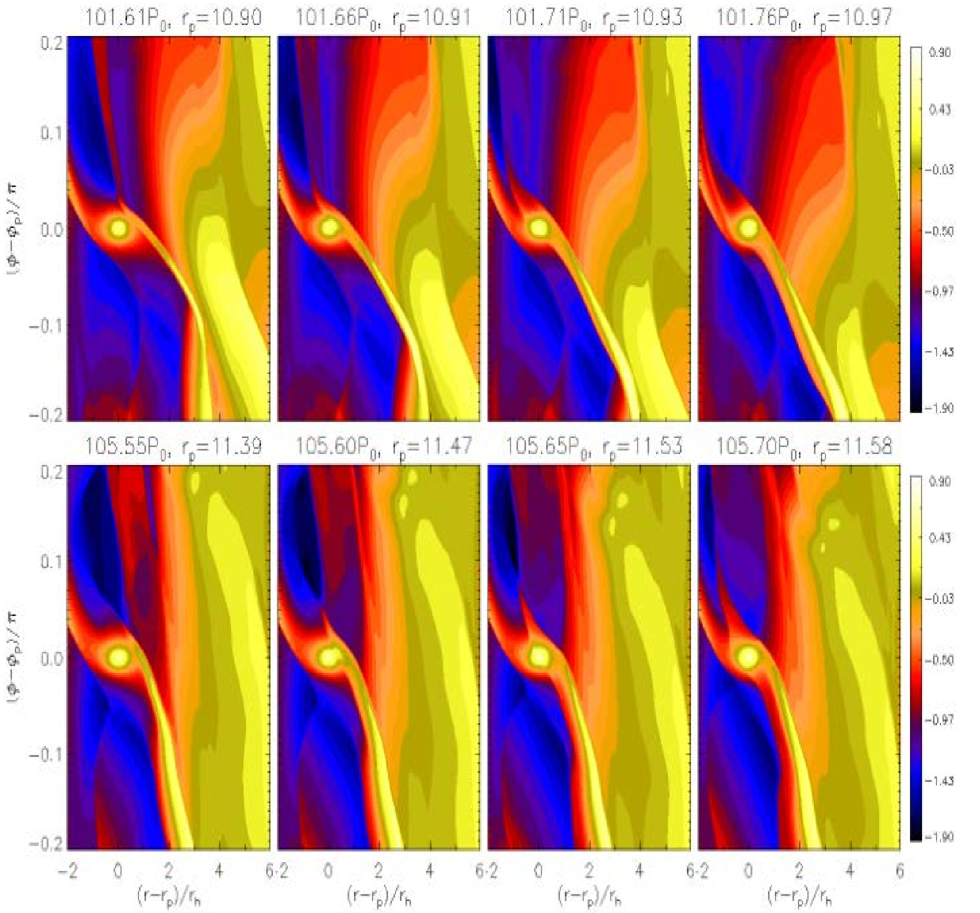






Migration of a Saturn mass planet under the influence of outer gap edge modes in disks with 1.2 < Q < 2 (0.047 < M_d / M_* < 0.079). Material entering the horseshoe region causes the planet to be scattered outwards. (M-K Lin & Papaloizou 2011)



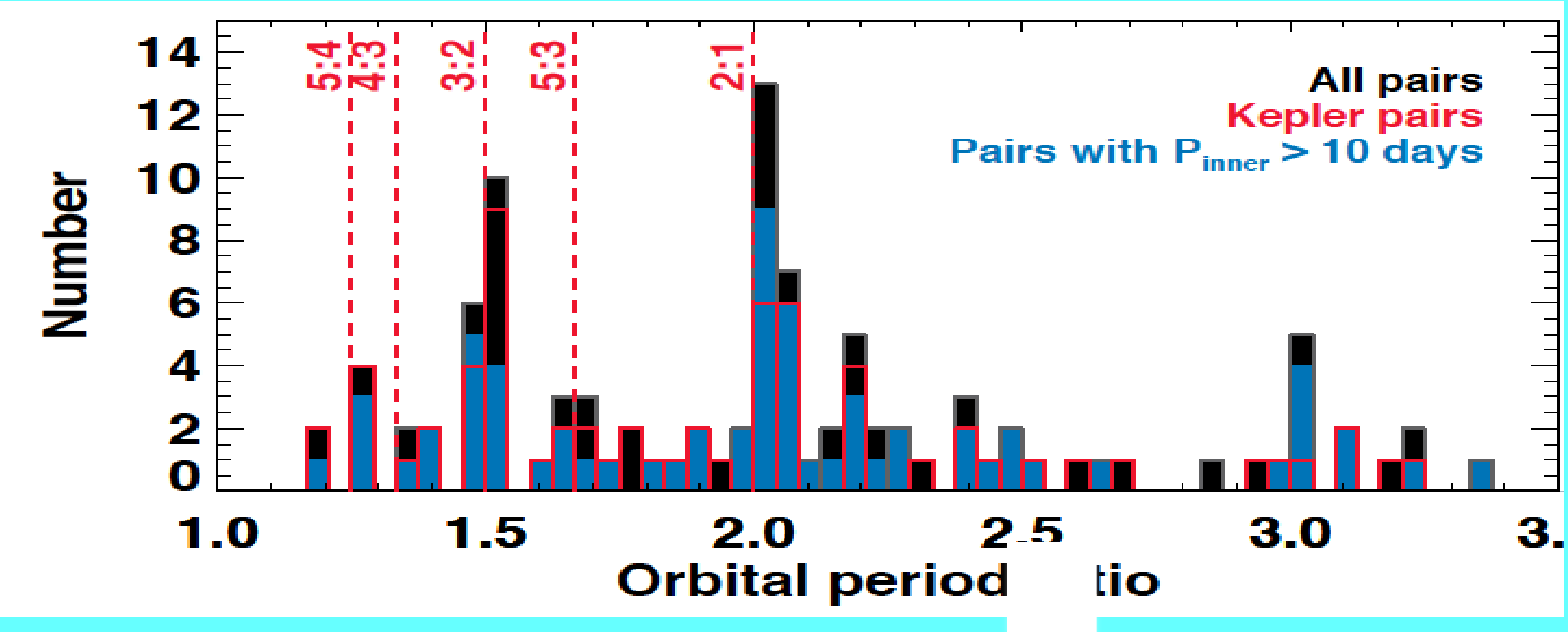


Migration of a 2 Jupiter mass planet in disks with $1.5 < Q < 2$. For $Q < 1.7$ edge modes occur (outward migration).

For larger values vortex modes occur (weak migration) . (M-K Lin & Papaloizou 2012)

Convergent migration of planet pairs: Formation of commensurabilities

Period ratios for confirmed planet pairs



GJ 876 a system with a three planet Laplace Resonance

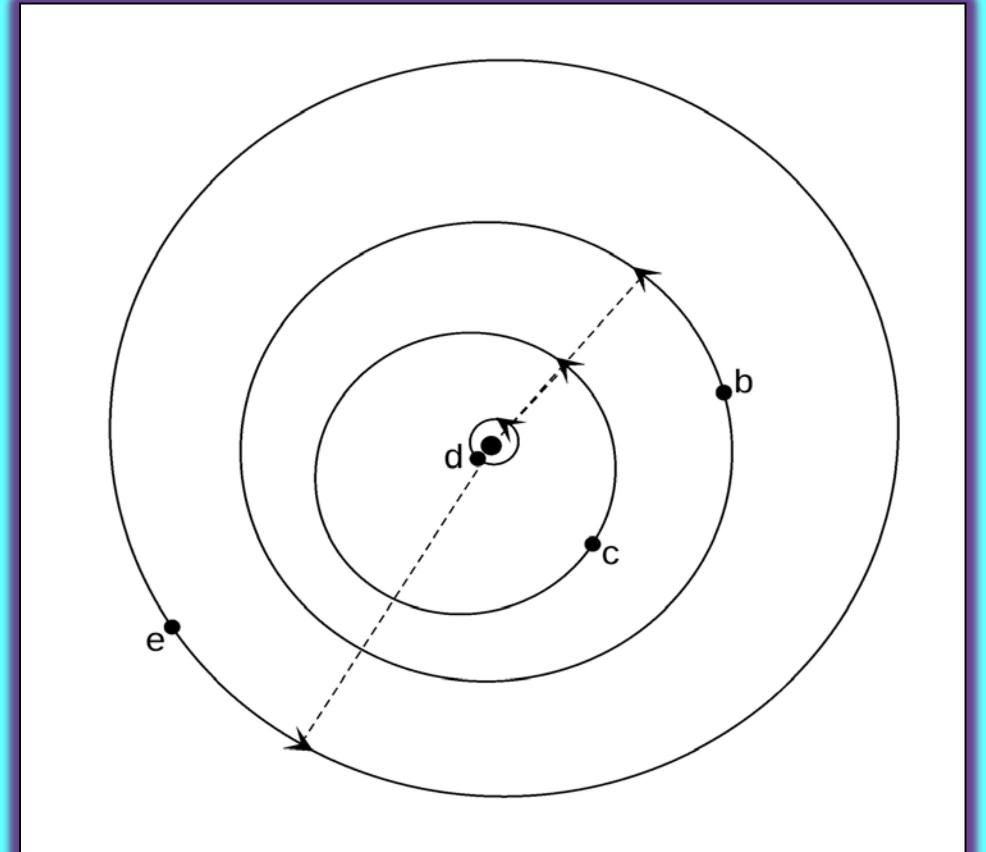
$$2n_2 - n_1 \sim 0,$$

$$2n_3 - n_2 \sim 0,$$

$$2n_3 - 3n_2 + n_1 = 0 .$$

The Gliese 876 system

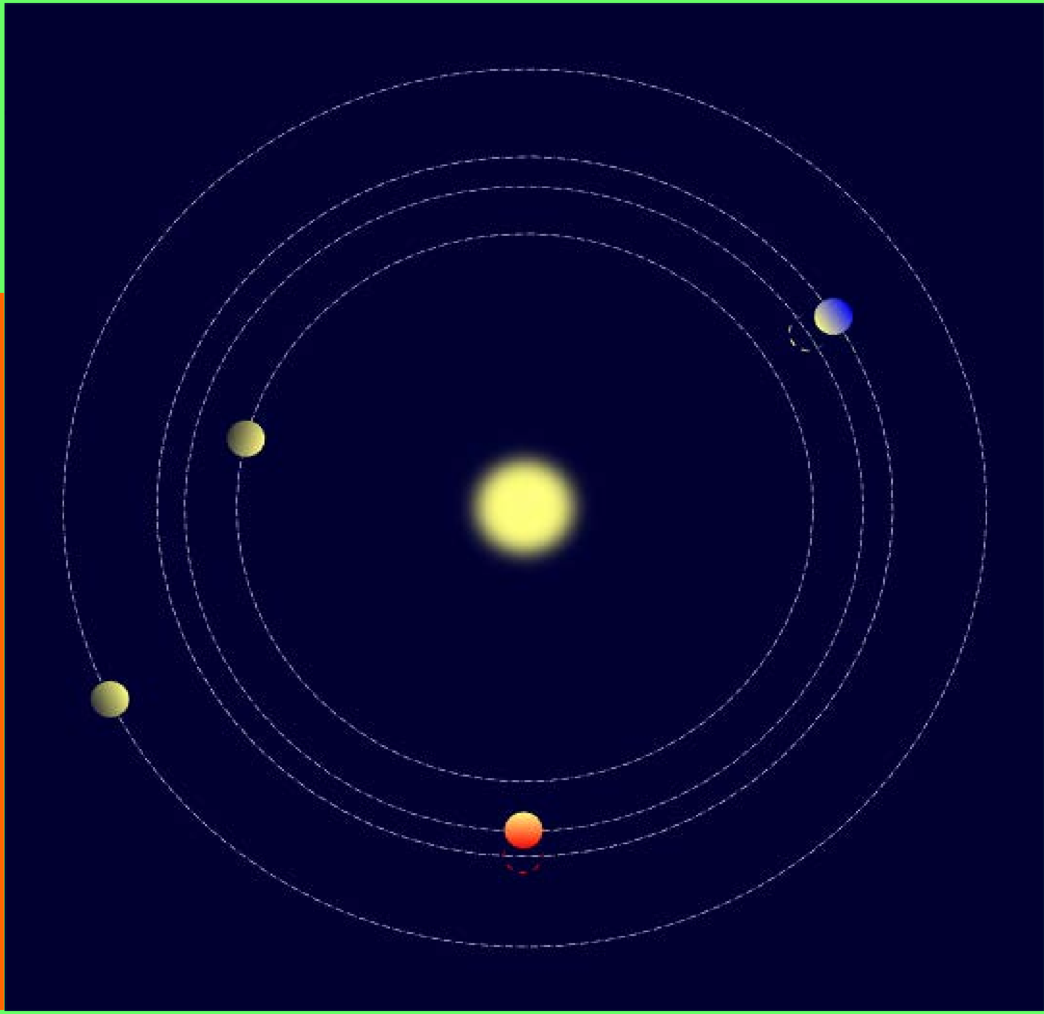
	Mass	A (AU)	Period days	Eccentricity
d	6.83 M⊕	0.0208	1.93	0.207
c	0.714 MJ	0.1295	30.00	0.255
b	2.275 MJ	0.208	61.11	0.032
e	14.6 M⊕	0.334	124.26	0.055



KOI 730

System of 4 (super-Earth) planets with 4:3 , 3:2, 4:3 resonances

P_1 **= 7.3831d.**
P_2 **= 9.8499d.**
P_3 **= 14.7903d.**
P_4 **= 19.7216d.**

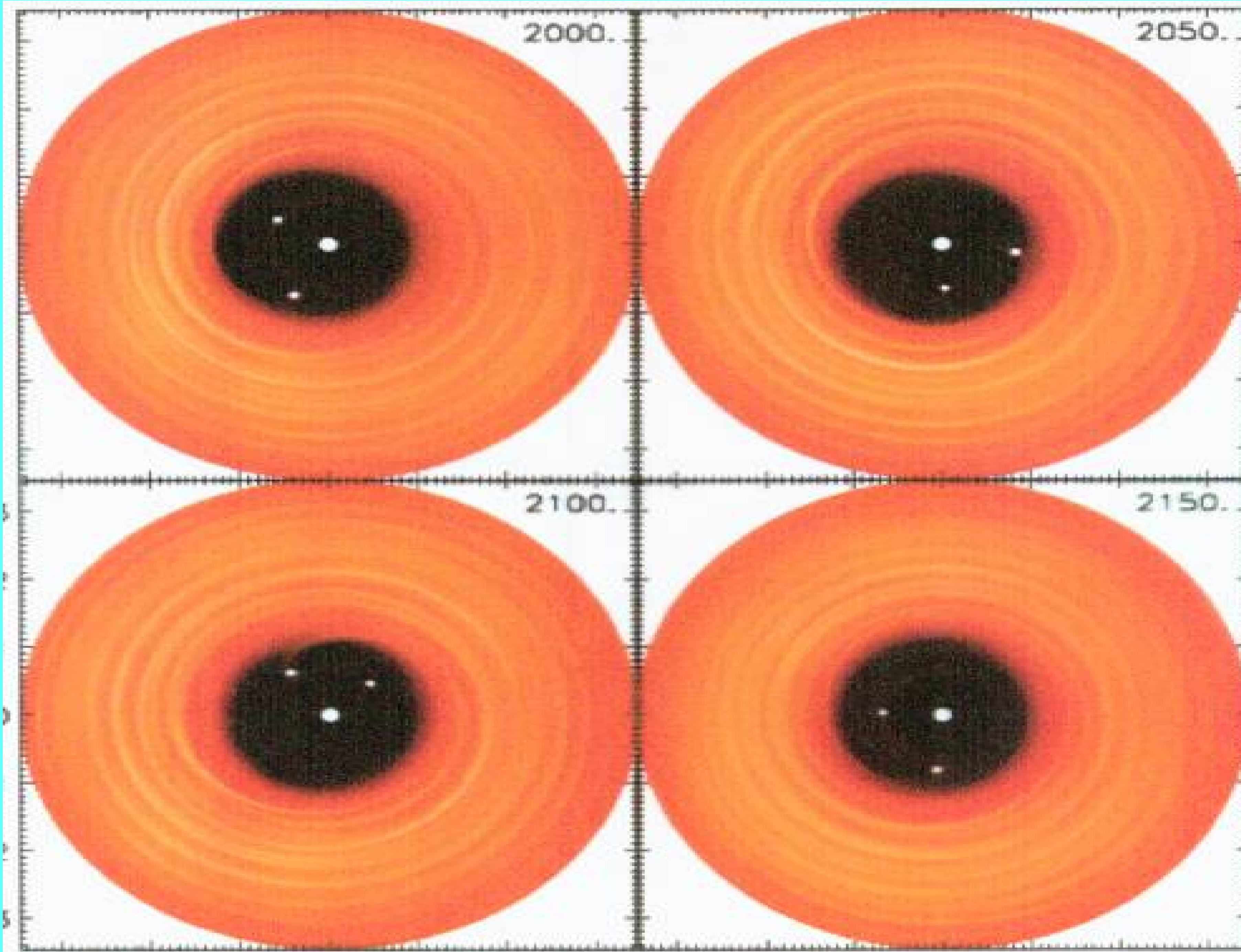


Resonant coupling of migrating planets Example of the two giant planets in GJ876

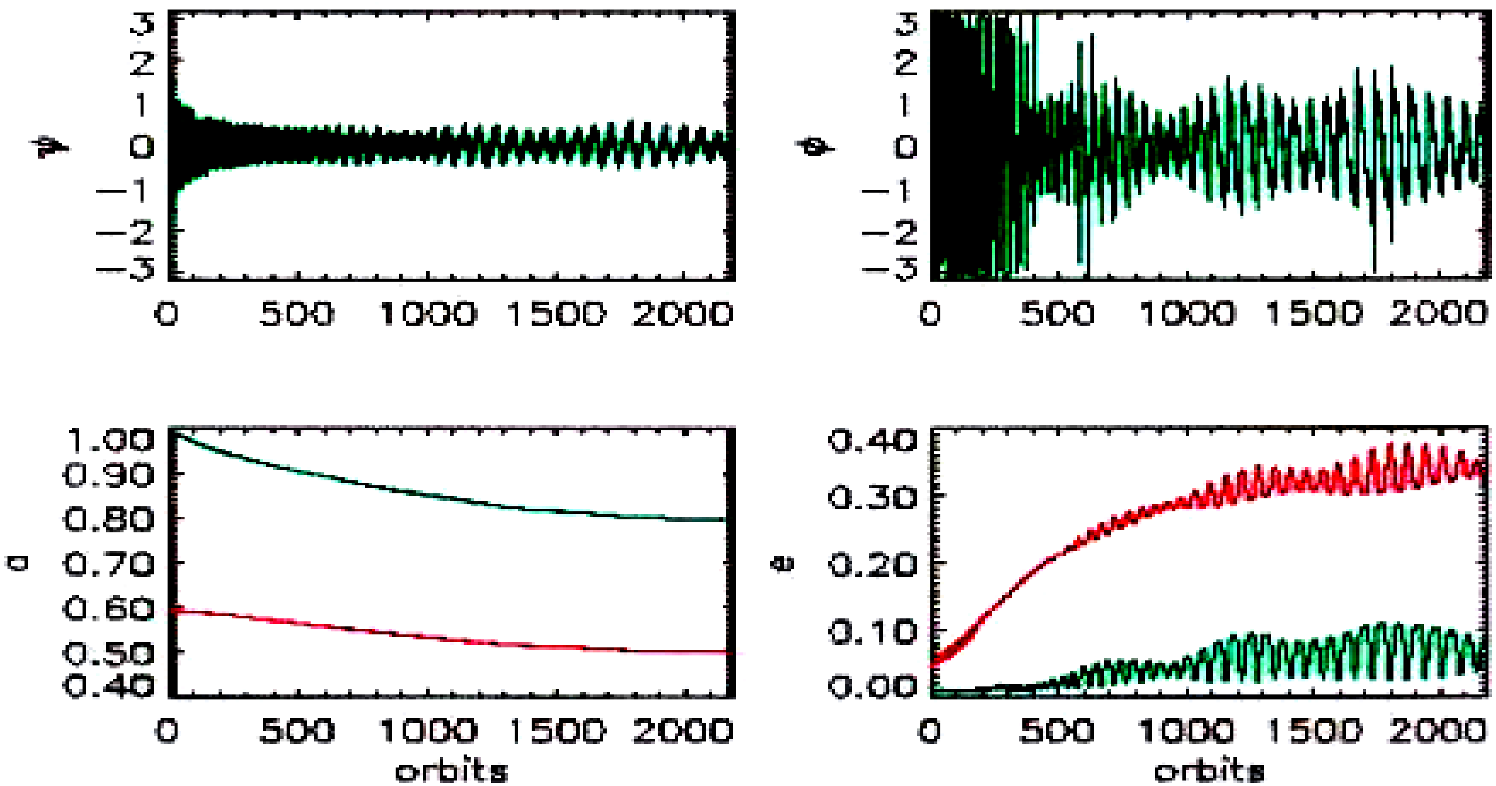
- First planet orbits in inner disc cavity.

- Second planet forms in outer disc. Material between them is cleared by tidal interactions resulting in both orbiting inside the cavity.

- Second planet is driven inwards due to disc interaction until commensurability is attained. This is subsequently maintained with two planets migrating together.


2000.
2050.
2100.
2150.

Resonant angles $2\lambda_1 \square \lambda_2 \square \omega_1$, $2\lambda_1 \square \lambda_2 \square \omega_2$ Semi-major axes and eccentricities for GJ876



Resonantly coupled lower mass exoplanets

Gas giants in low order commensurabilities: GJ876 and HD 82943 may be formed by convergent type II migration. This may also occur for low mass planets undergoing type I migration. Studies with 2D hydrodynamic simulations together with N body codes.

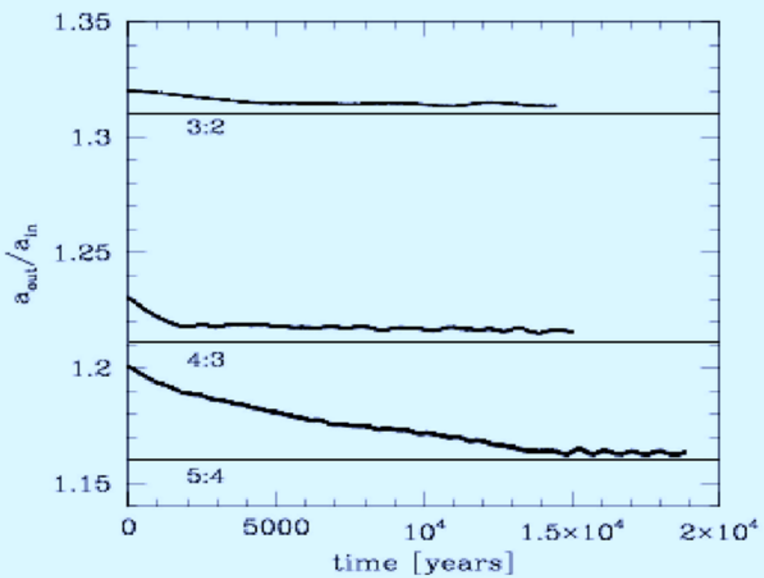
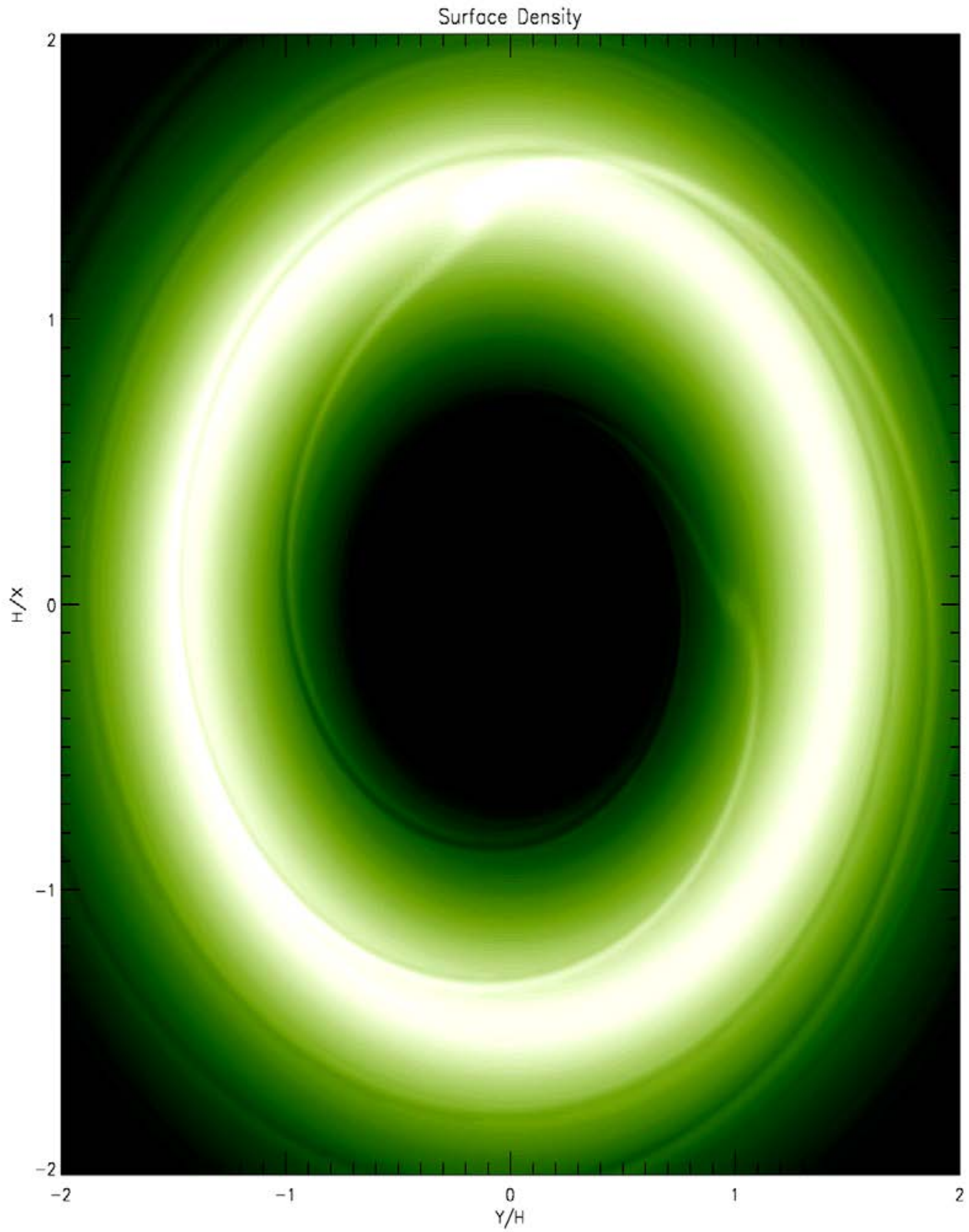
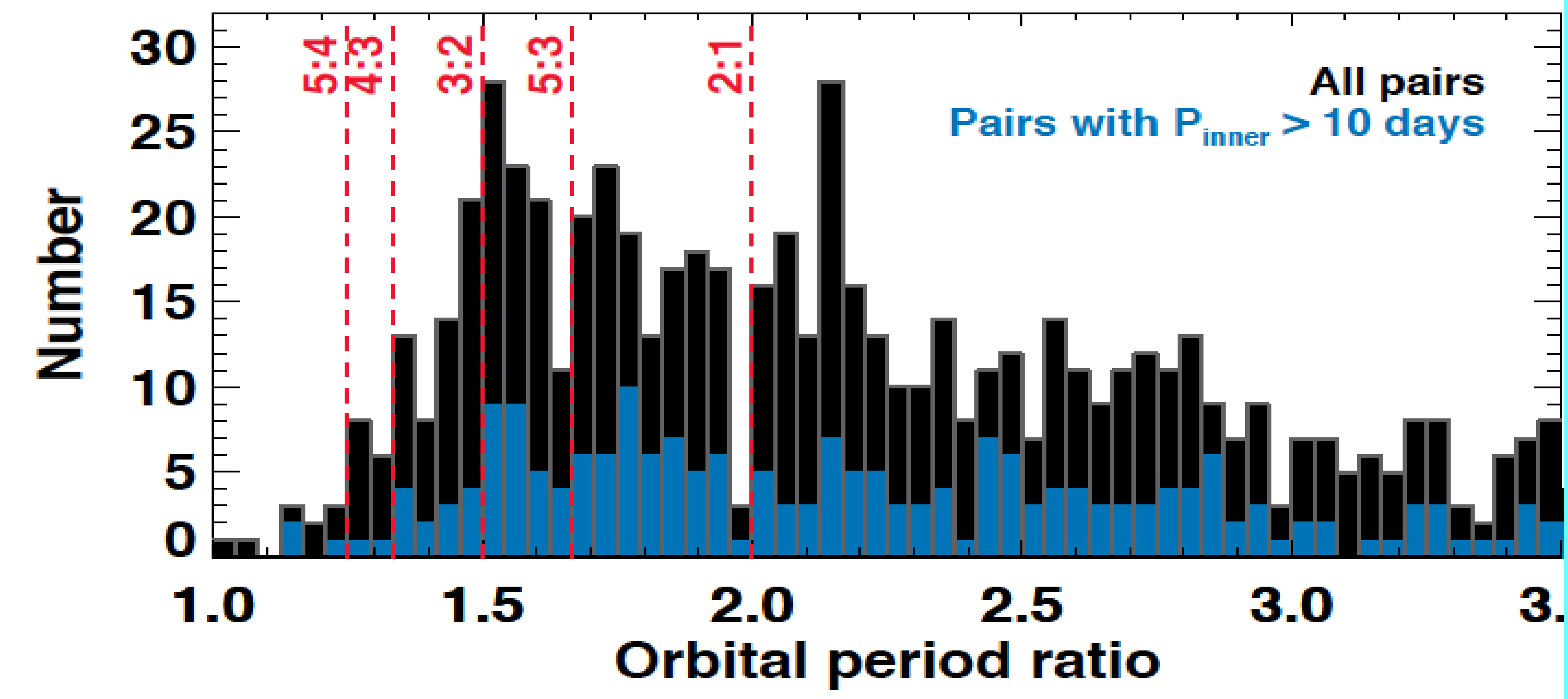
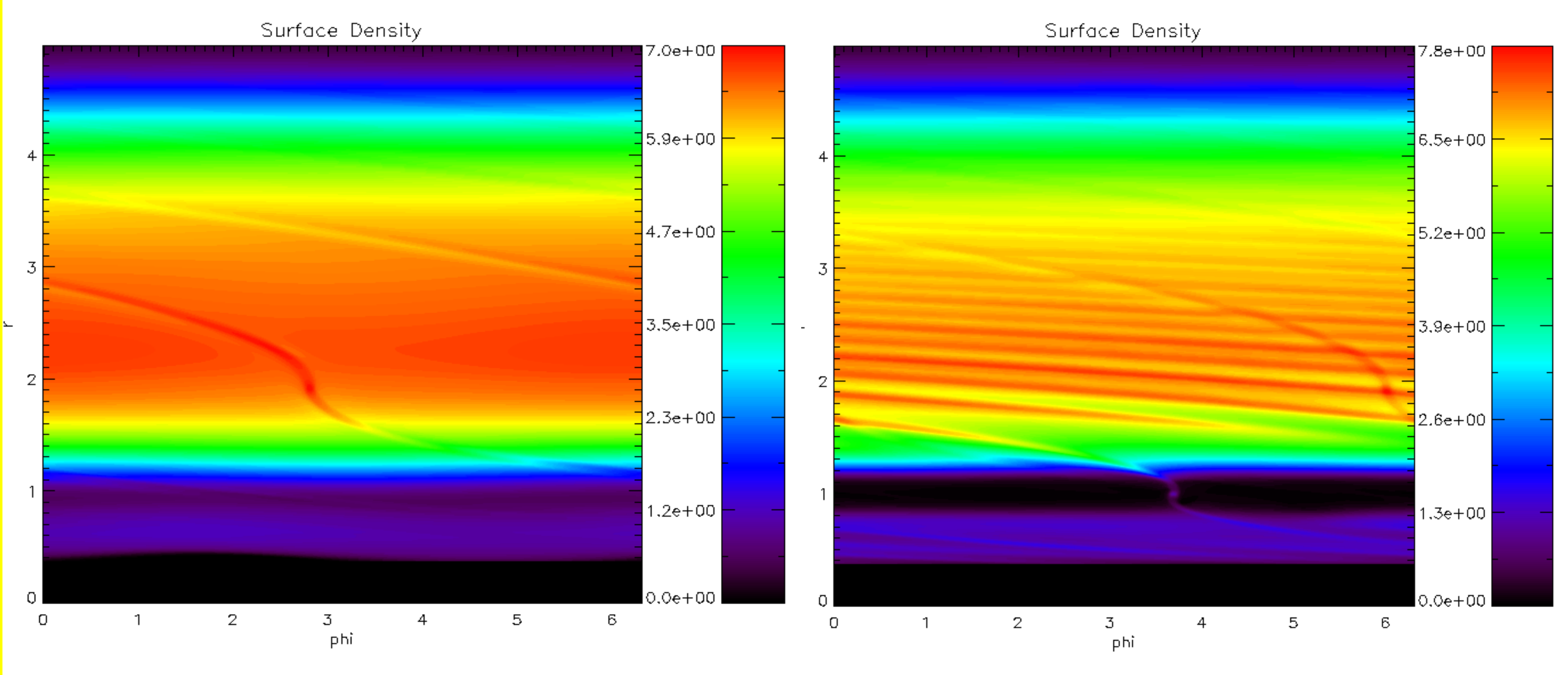


Figure 12. The semi-major axis ratio when $m_1 = m_2 = 4M_\oplus$ with $\Sigma_0 = \Sigma_1$ (uppermost curve) and with $\Sigma_0 = \Sigma_4$ (two lower curves)

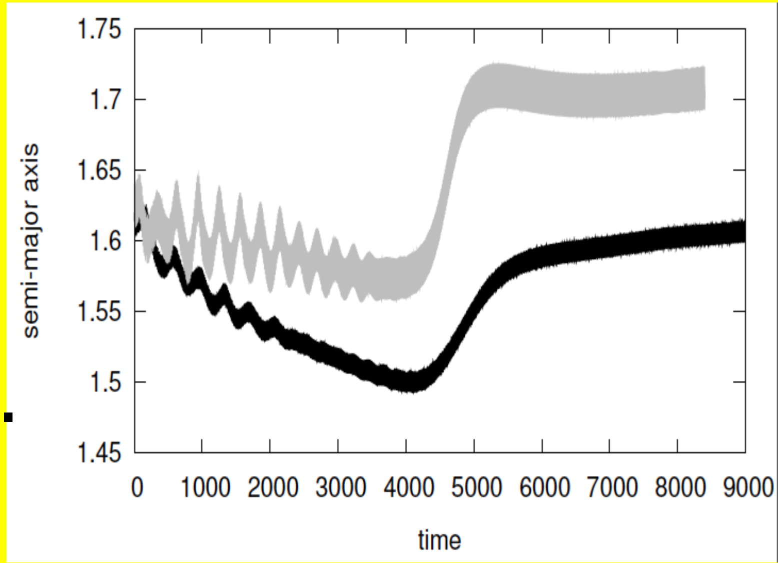


Period ratios for confirmed planet pairs including KOI
Excess slightly exceeding resonant values
Mechanisms for pushing them apart: Tidal circularization?
Wake-planet interaction?



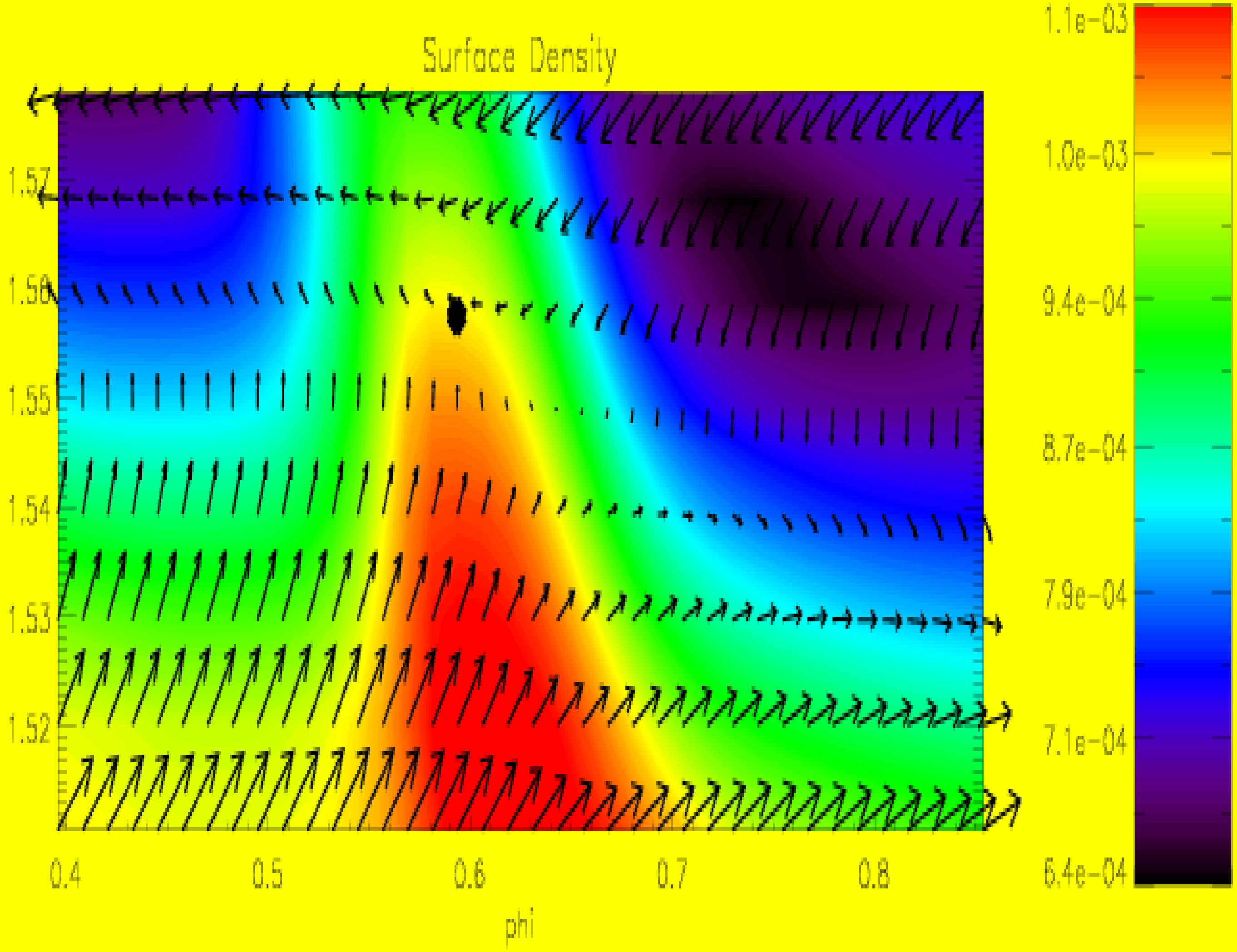


Jovian mass planet with exterior 5.5 earth mass super-earth. (Podlewska-Gaca et al. 2012)

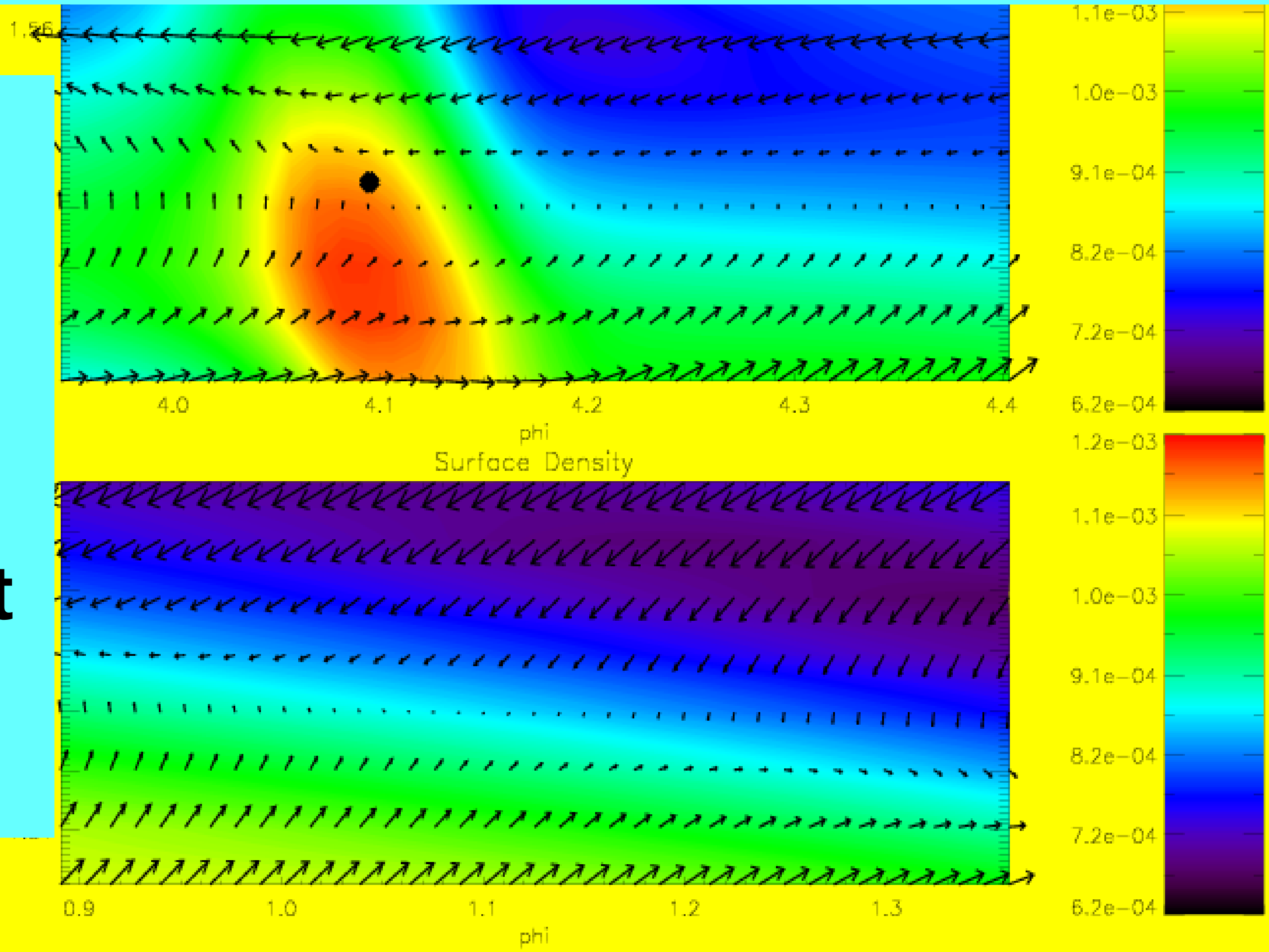


Coorbital region of the super-earth perturbed by giant planet a particular time.

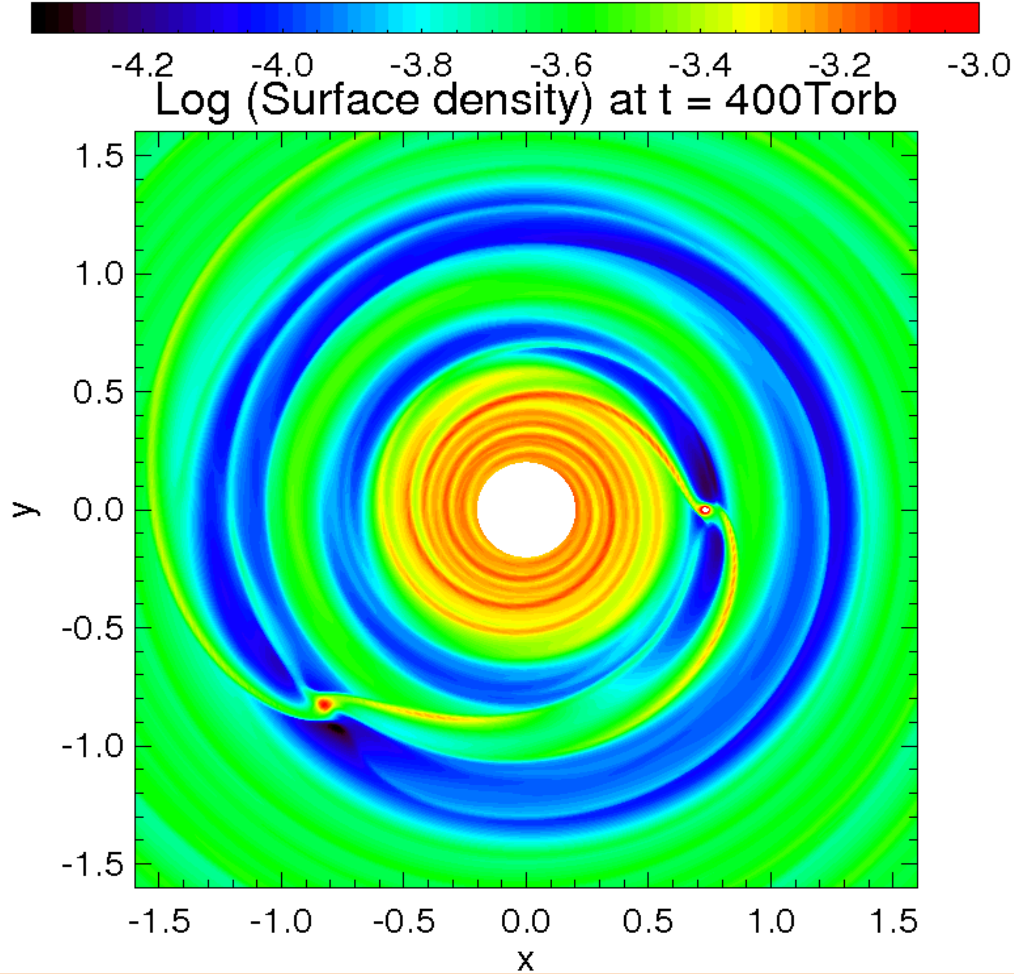
Coorbital region of super-earth has distorted horseshoe like streamlines

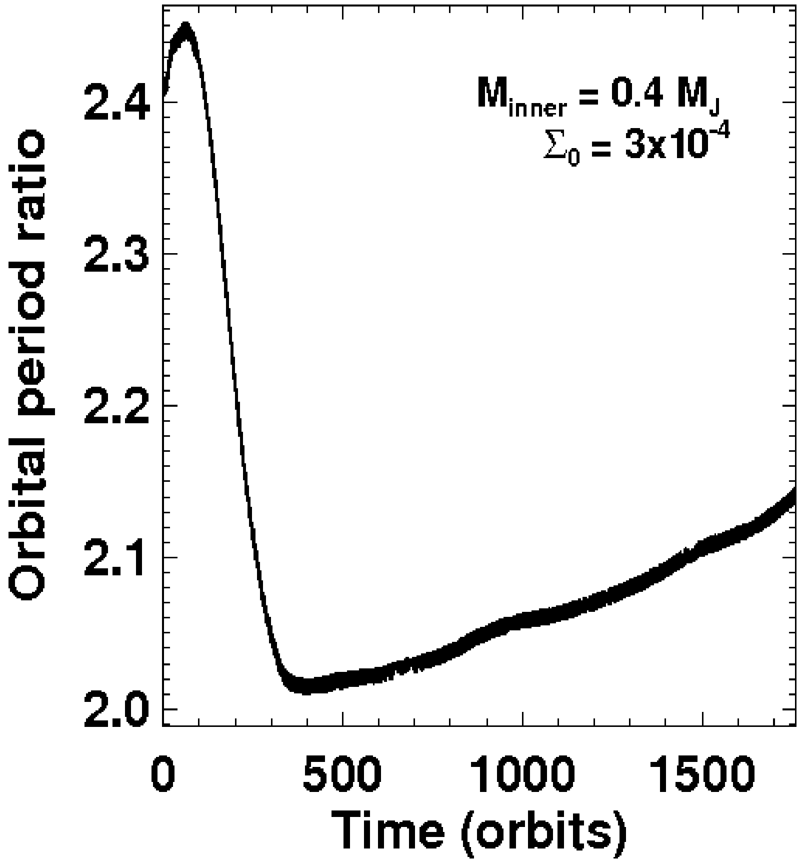


Coorbital region of the super-earth perturbed by giant planet at different phases at another time.



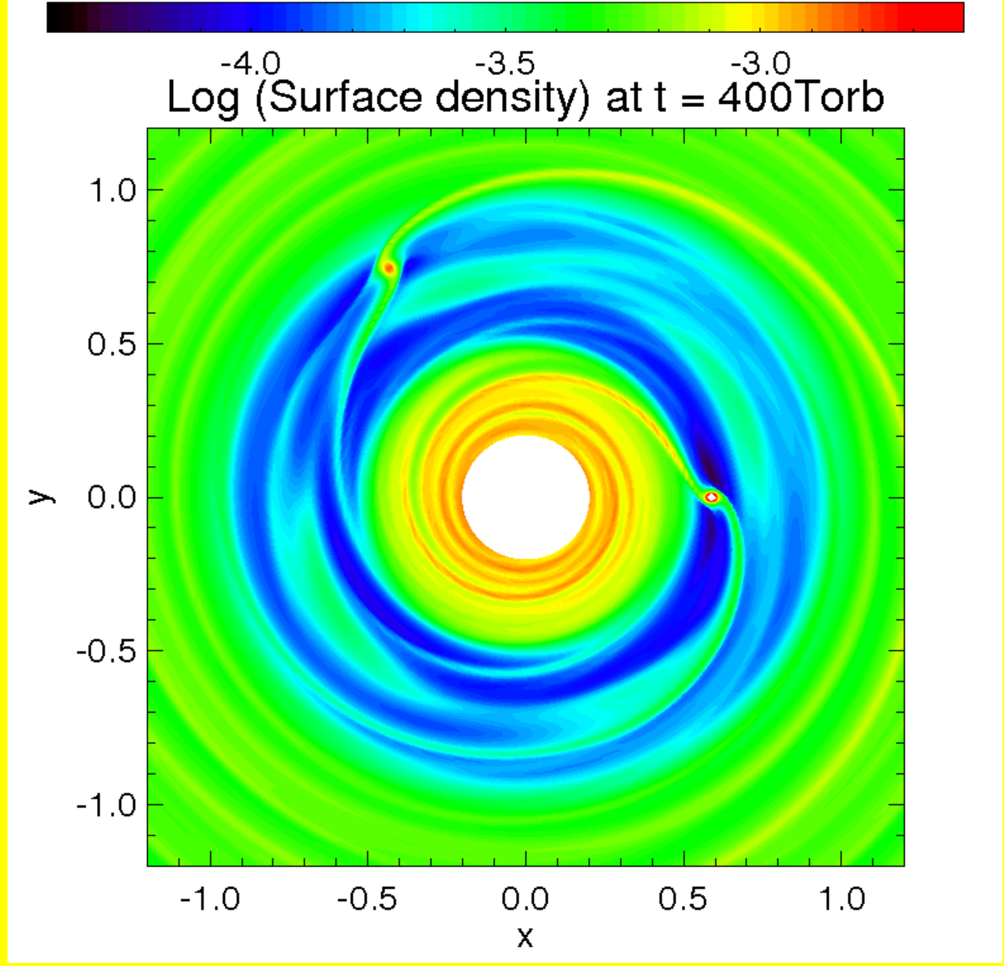
Angular momentum transferred to coorbital region is communicated to the giant planet

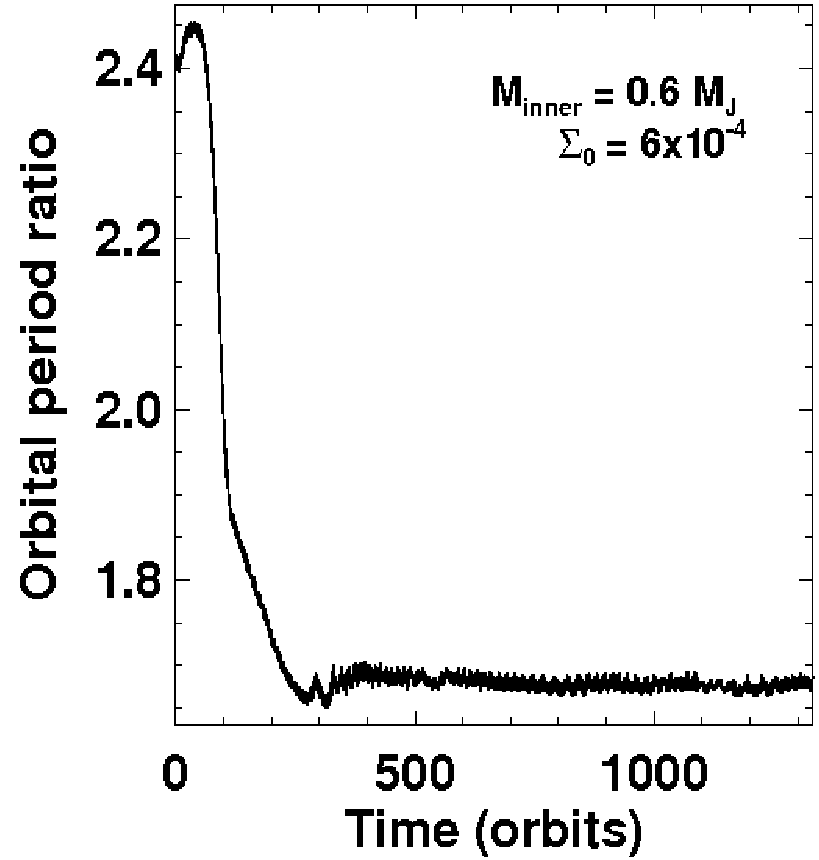




Migration of two planets (0.37 & 0.4 Jupiter masses) showing convergent migration towards a 2:1 commensurability followed by repulsion due to wake planet interactions.

(Baruteau & Papaloizou 2013)





Migration of two planets (0.37 & 0.4 Jupiter masses) showing convergent migration towards a 3:2 commensurability in a disk with higher surface density. Repulsion due to wake planet interactions is not very effective. (Baruteau & Papaloizou 2013)

Remarks

1) Disk planet interactions inevitably lead to important effects such as migration and gap formation through the excitation of density waves.

2) A complex underlying disk structure and effects arising from planet-disk-planet interactions result in a rich range of behaviour that should contribute to understanding planet formation and evolution

T Tauri phase of protoplanetary disks in which Solid core growth occurs has lifetime ~ 10^6 - 10^7 yr.

Mass distribution similar to minimum mass solar nebula with mass α $r^{1/2}$ with 2 M_J within 5 AU.
Total mass ~ 0.02 solar masses.

Necessity of and need for orbital migration

Cores of giant planets need to be formed beyond 'snow line' at ~ 2AU.
Gravitational instabiliy occurs > few AU.
But giant planets seen at 0.04 AU and
The mass distribution of observed systems
Is over concentrated at small radii.