

# 3D ASPECTS OF DENSITY WAVES

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# COLLABORATORS

- General properties of 3D waves with low  $m$  (Ogilvie, Pringle)
- Buoyancy torques, 3D and high  $m$  (Zhu)

# LIN & SHU (1964)

- 2D gas (star) dynamics
- Long Waves (self-gravity)
- Free waves
- Importance of resonances



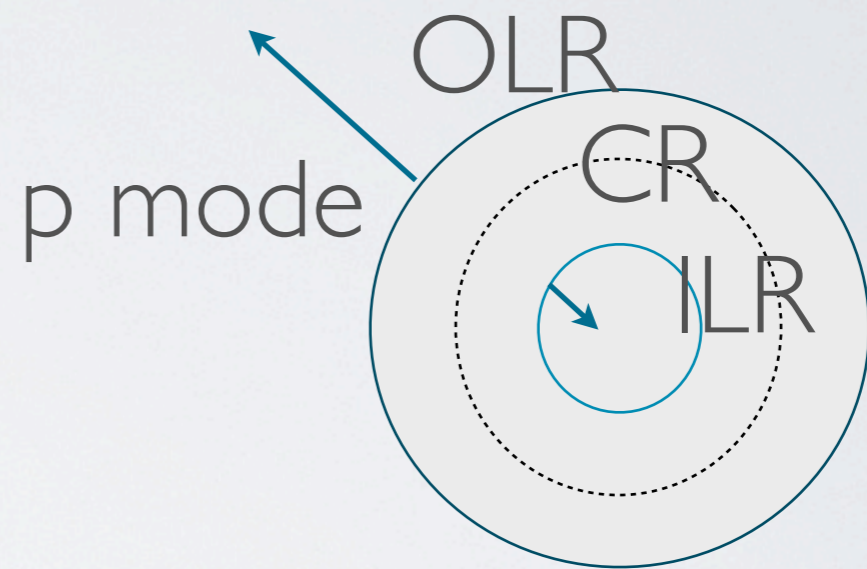
# FREE/DRIVEN WAVES

- Waves driven by external forcing Feldman & Lin (1973)
- Carry energy/angular momentum
- Change disk structure where waves damp
  - Bar - arms in ISM of galaxies
  - Star - truncate disks
  - Planet - open gaps, migration, wakes
  - Satellite - gaps, waves in planetary rings

# 2D WAVES NO SELF-GRAVITY

- Modified pressure waves: p modes propagate away from corotation

$$m^2(\Omega - \Omega_p)^2 = \kappa^2 + k^2 c^2$$



# 3D WAVES

- Various collective effects possible, in addition to self-gravity.
- f mode wave 3D (pressure, vertical buoyancy)
- p mode waves 3D (pressure forces)
- r mode waves 3D (rotation and inertial forces)
- g mode waves 3D (vertical buoyancy forces)

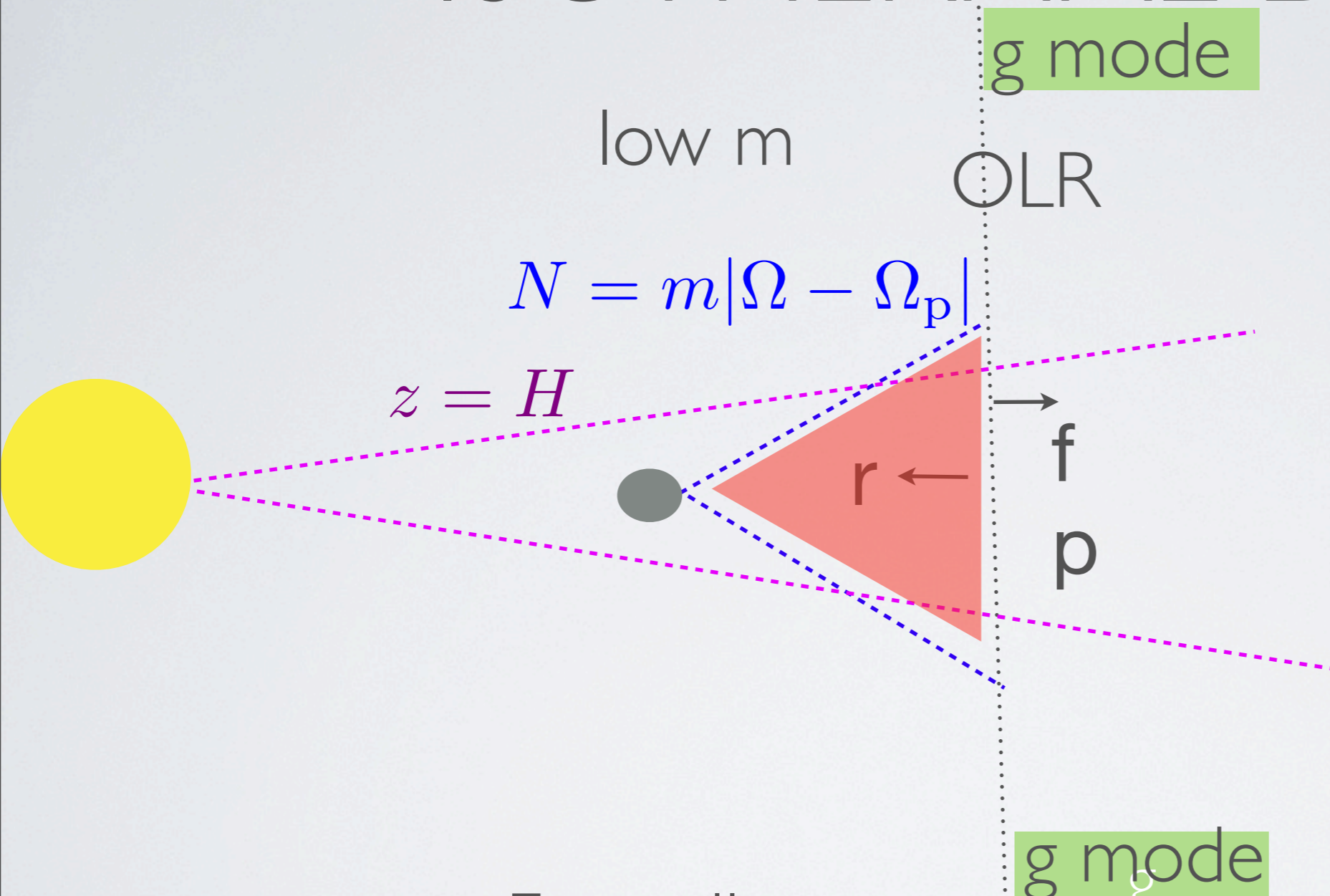
# 3D WAVEGUIDE MODEL

- Consider 3D waves of the form with  $m$  and  $\Omega_p$  given

$$f(r, z) \exp [i(k_r(r)r + m(\phi - \Omega_p t))]$$

- Solve for  $k_r(r)$  and  $f(r, z)$  locally in  $r$  as a function of  $z$ , subject to boundary conditions in  $z$
- Infinitely many modes with different number of nodes in  $z$

# PROPAGATION IN ADIABATIC ISOTHERMAL DISK

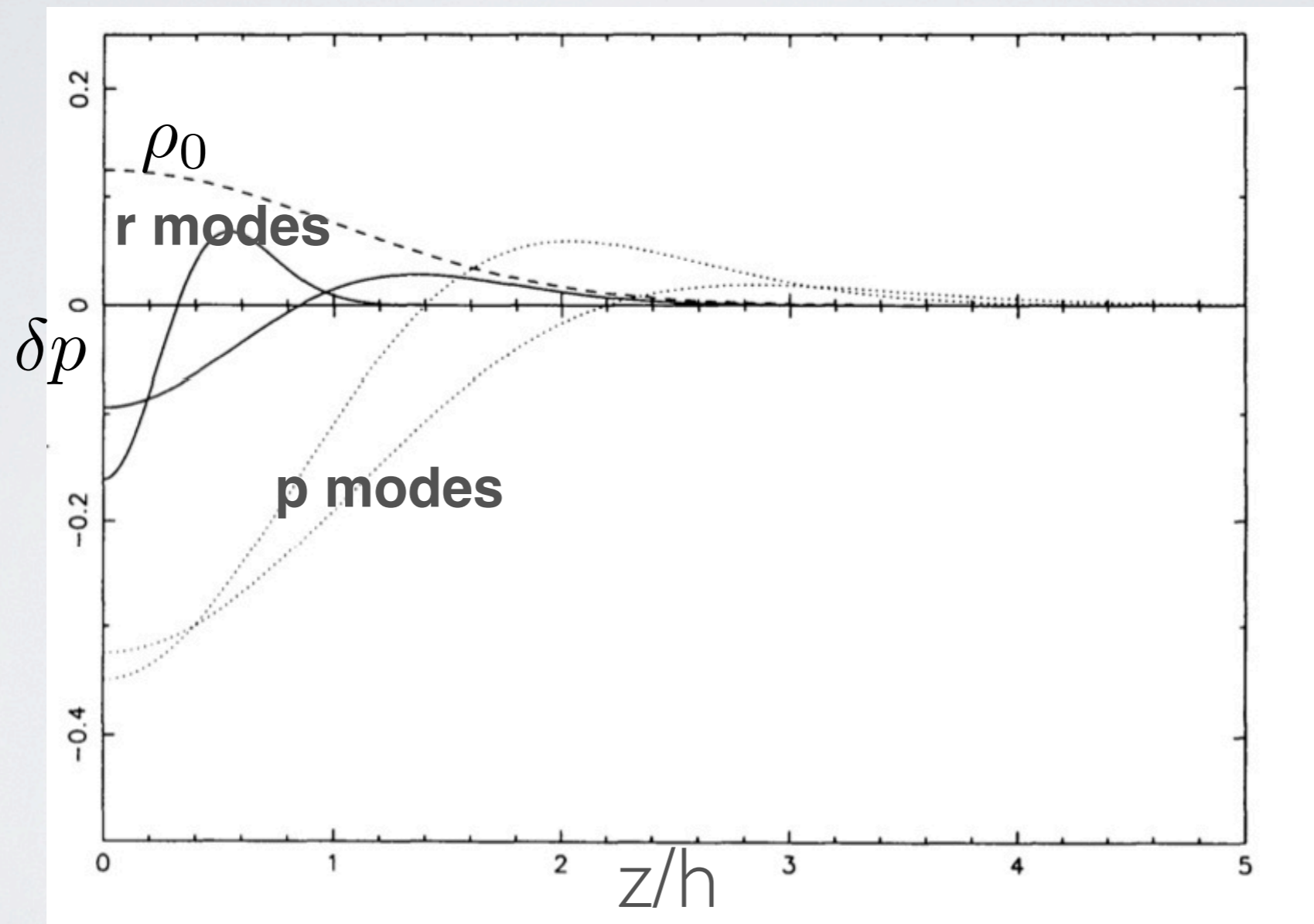


Formally, no g modes.

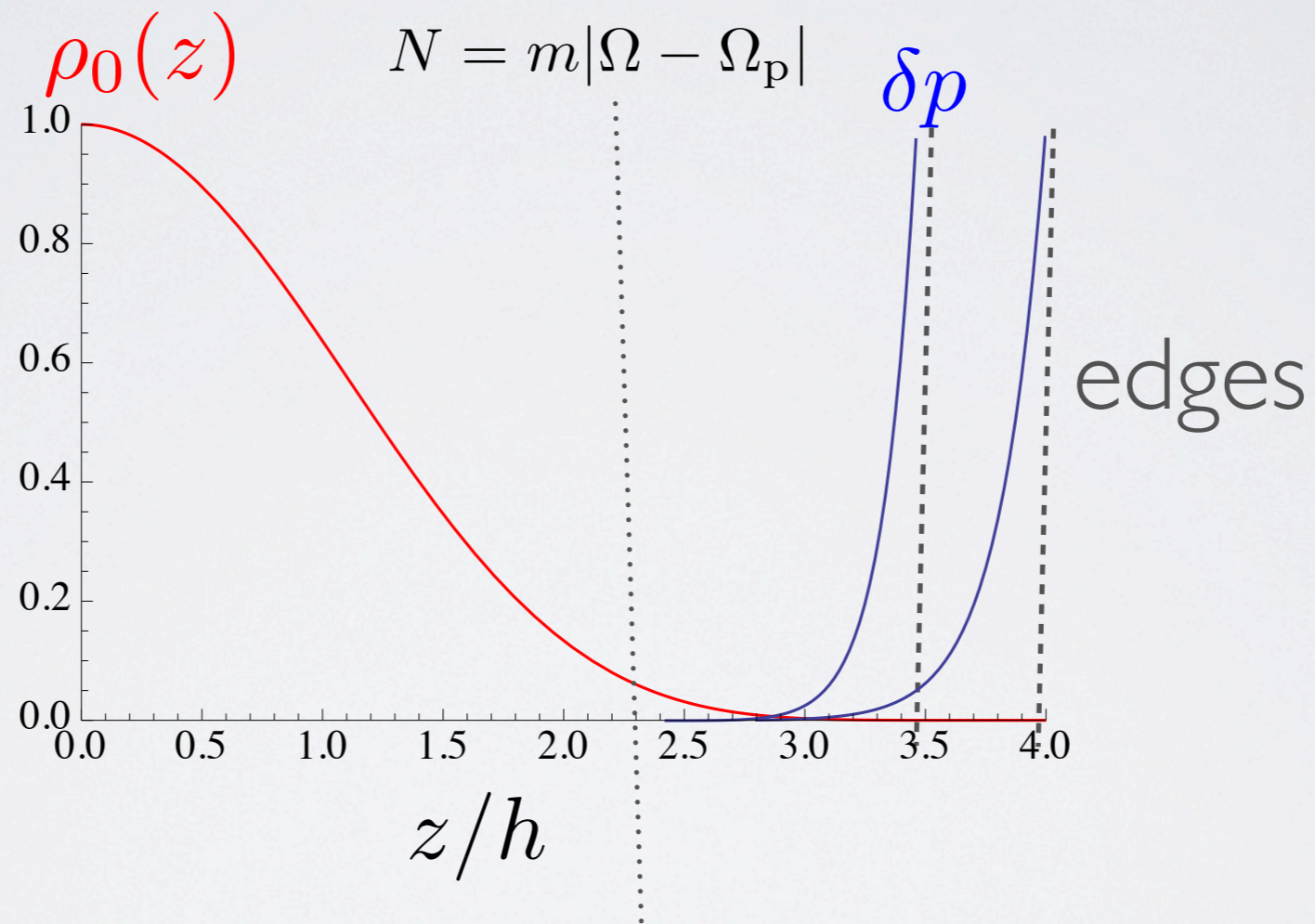
g modes can occur at upper/lower boundaries  
or in nonisothermal disk

# EXAMPLE

$m=0$  waves, low order in  $z$



# GRAVITY MODES



g modes concentrated near z edge, if present

# RESONANCES

- Waves launched where wavelength is long
- So potential can interact with wave
- Occurs at resonances
- Resonance width determined by collective effects: pressure, self-gravity
- Case of planet: high  $m$  LRs set away from planet  $\sim H$
- Torque decreases with  $m > r/H$

# WAVES EXCITED AT 3D LINDBLAD RESONANCES

- Isothermal nonadiabatic disk,  $\gamma = 1$ ,  
wavefronts perpendicular to orbital plane.  
One mode only  
No vertical motion. Same as 2D case
- Isothermal adiabatic,  $\gamma > 1$   
3D motions: p, g, and r modes excited  
2D motions: f mode

# SUMMARY OF 3D WAVES/ RESONANCES

- Torques associated with propagating waves
- Waves launched at resonances: turning points of free waves
- Waveguide model locally sin/cos in  $r$ ,  $\phi$ , and  $t$ .  
Determine structure in  $z$  and radial wavenumber.
- Resonance width controlled by collective effects such as pressure (including corotation)