# Bulk and Thin Film Electrical Contact, RF Heating, and Field Enhancement

**Y. Y. Lau**

Department of Nuclear Engineering and Radiological Sciences

University of Michigan, Ann Arbor

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**Lin-Shu Symposium, Tsinghua University, Beijing**

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# Dedication

# Professor C. C. Lin

# 18.325 Topics in Applied Math

Professor C. C. Lin (MIT, Spring 1977)

I Nature of the course

(a) The applied mathematician's approach (Many aspects)
(Two sheet of notes)

- I Formulation { what scientific object / what issues
- II Solution — Methods — { General concepts / Math. basis (relationship to the logical structure of P.M.)
  (Applied Math. ≠ Methods of A.M.)
- III Interpretation — Refer to I
- IV Implication + Impact

General (Specifics) → Specifics (General) → General → Specifics → General → ...

(b) Comparative study of

- (i) stellar systems → disk shaped galaxies 
- (ii) plasma systems → mirror machine 

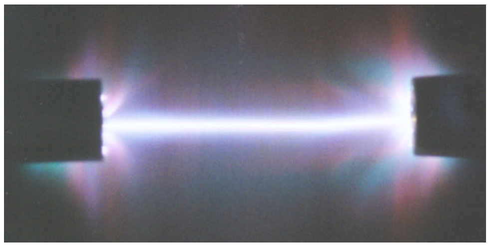
Cooperative phenomena (collisionless) — waves associated with long range forces
— instability modes

Equilibrium — waves — instability
(small amplitude — finite amplitude)

# Introduction

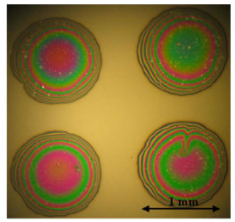
- Electrical contact is important to

**Wire-array Z pinches**



Z-pinch @ UM, Sandia

**Metal-insulator-vacuum junctions**

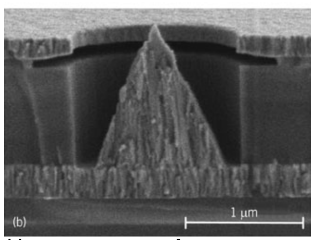



Metal-oxide-junction cathodes @ UM

**High power microwave (HPM) sources**

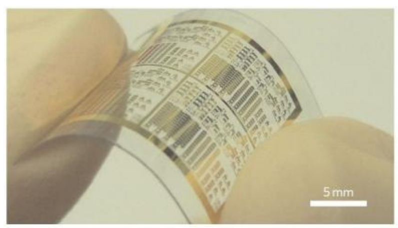
UM/ L-3-Titan relativistic magnetron

**Field emitters**



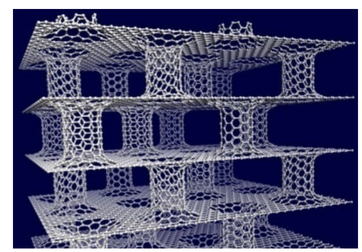

http://accessscience.com

**Thin film devices & integrated circuits**



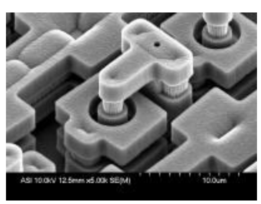

Sun et al, Nature Nanotechnol. 6, 156 (2011)

**Carbon nanotubes based cathodes and interconnects**



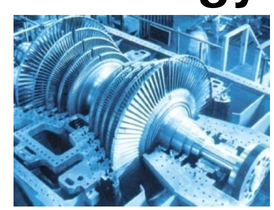
CREDIT: SPIE

**Micro-electromechanical System (MEMS)**



http://www.memx.com/products.htm

**Tribology**



http://machinedesign.com

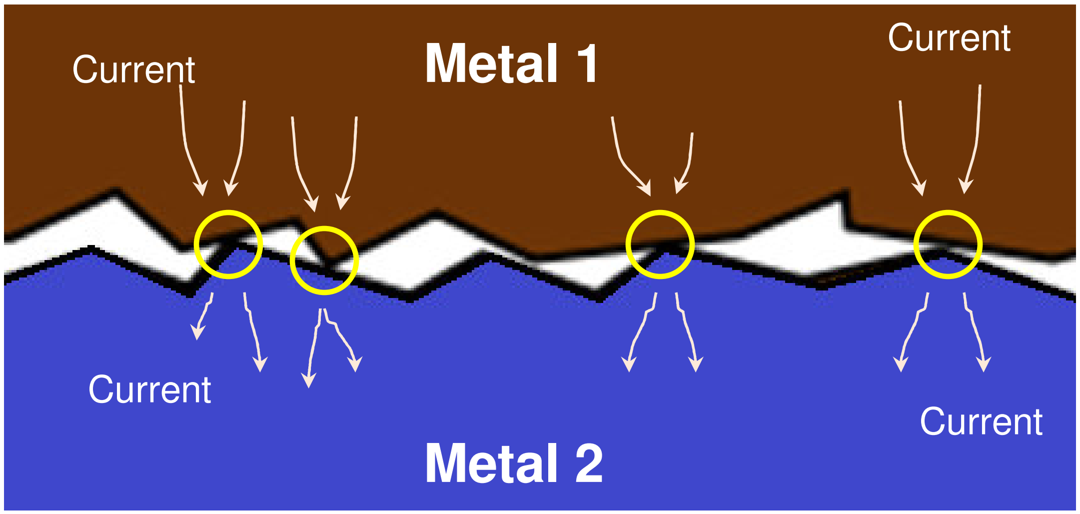
# Introduction

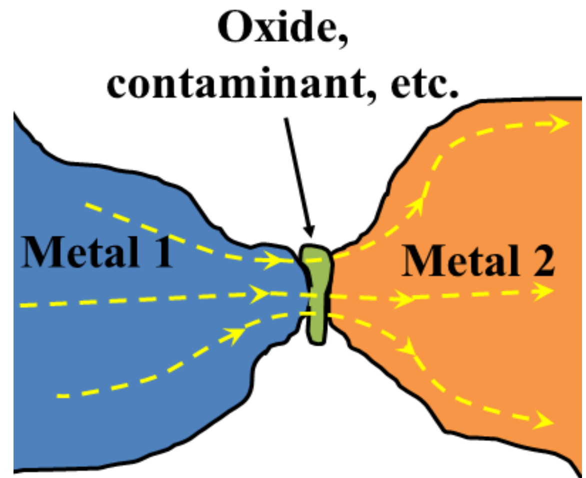
Electrical contact is important to

- Wire-array Z pinches
- Metal-insulator-vacuum junctions
- High power microwave (HPM) sources
- Field emitters
- Thin film devices and integrated circuits
- Carbon nanotubes based cathodes and interconnects
- Tribology
- Contact problems account for 40 percent of all failures in electrical-electronic systems

# Effects of Surface Roughness (I):

## Contact Resistance



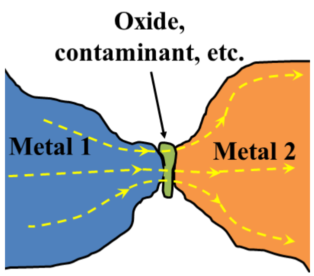



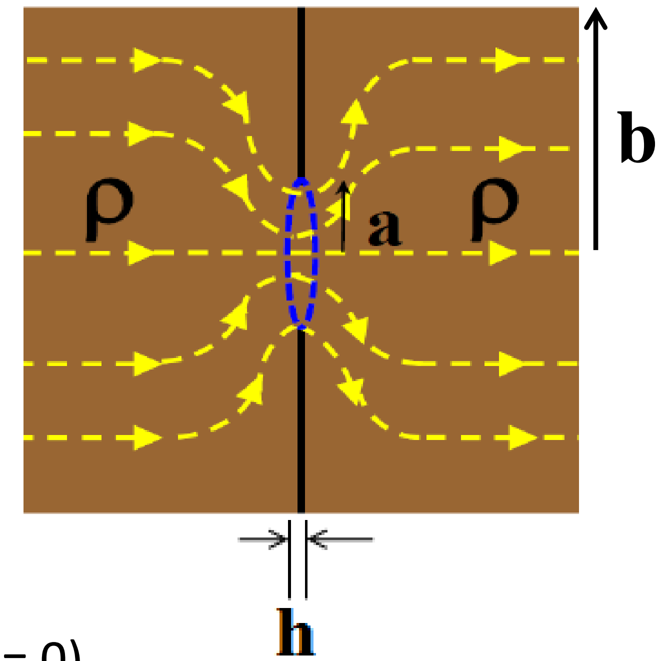

◯ : True points of contact => High contact resistance

- Current flows only through the true points of contact => High contact resistance

- Contact resistance is highly random, affected by surface roughness, pressure, hardness, residing oxides and contaminates, etc.

# Holm's *a*-spot Theory of Electrical Contact




$$R_c = \frac{\rho}{2a}$$

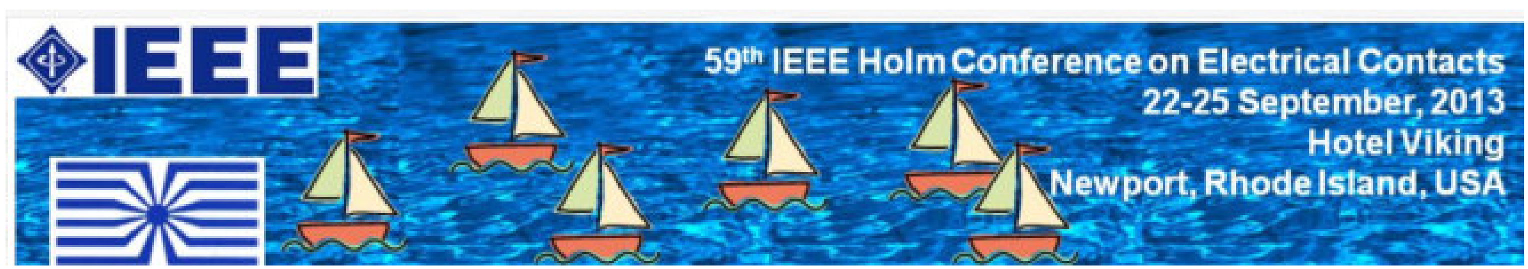
Assumptions on *a*-spot:

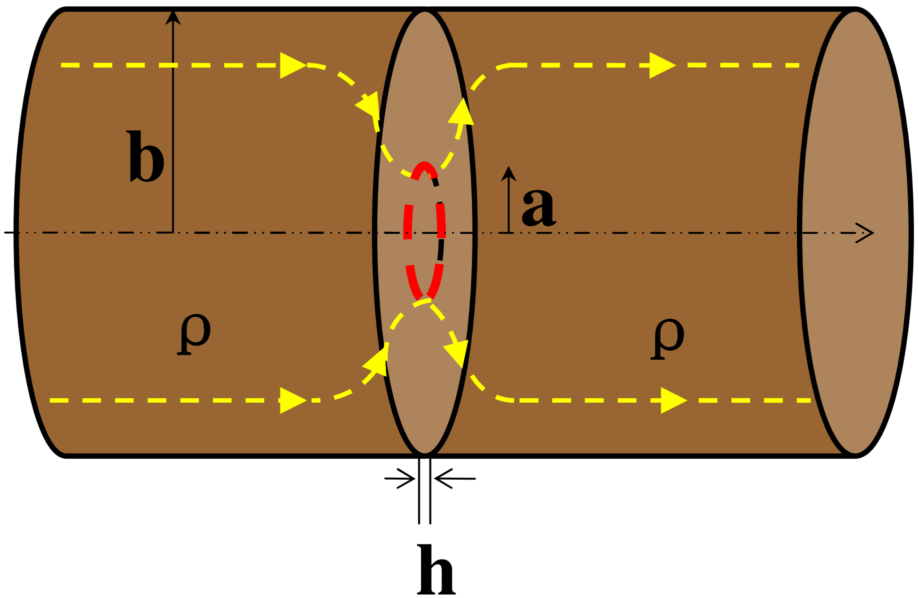
(A) Zero axial length (h = 0)

(B) Infinite transverse dimension ($b \rightarrow \infty$)

(C) Same materials (no contaminants)

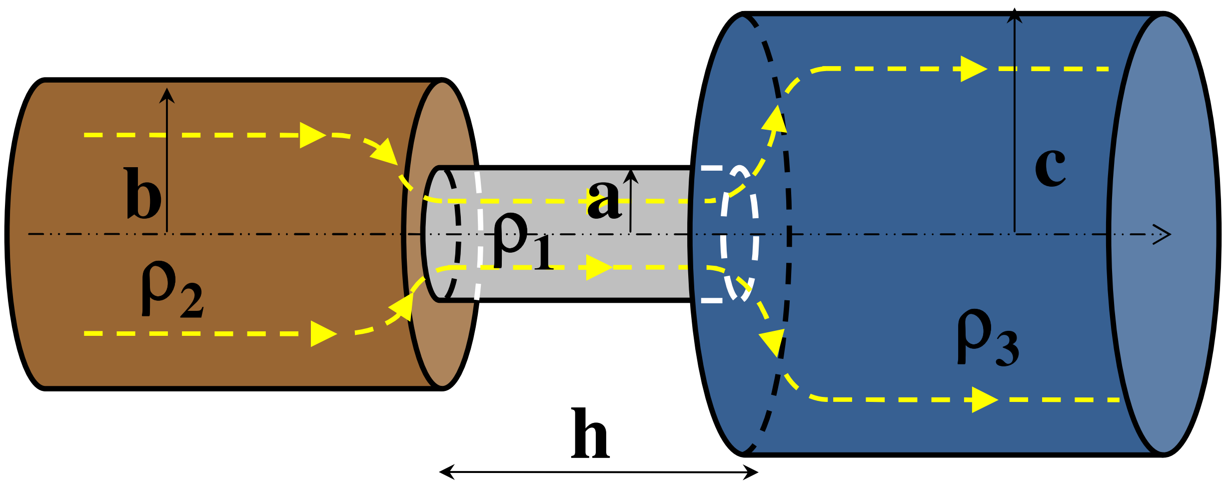
R. Holm, Electrical Contacts (Springer-Verlag, ed.4, 1967)






## Holm's *a*-spot model

$h = 0$




## Our model

$h > 0,$

$a \neq b \neq c,$

$\rho_1 \neq \rho_2 \neq \rho_3$

# Total Resistance of Composite Channel

Cylindrical:

$$R = \underbrace{\frac{\rho_2 L_2}{\pi b^2}}_{\text{Bulk}} + \underbrace{\frac{\rho_2}{4a}\overline{R}_c\left(\frac{b}{a}, \frac{\rho_1}{\rho_2}\right)}_{\text{Interface}} + \underbrace{\frac{\rho_1 \times 2h}{\pi a^2}}_{\text{Bulk}} + \underbrace{\frac{\rho_3}{4a}\overline{R}_c\left(\frac{c}{a}, \frac{\rho_1}{\rho_3}\right)}_{\text{Interface}} + \underbrace{\frac{\rho_3 L_3}{\pi c^2}}_{\text{Bulk}}$$

Cartesian:

$$R = \underbrace{\frac{\rho_2 L_2}{2b \times W}}_{\text{Bulk}} + \underbrace{\frac{\rho_2}{4\pi W}\overline{R}_c\left(\frac{b}{a}, \frac{\rho_1}{\rho_2}\right)}_{\text{Interface}} + \underbrace{\frac{\rho_1 \times 2h}{2a \times W}}_{\text{Bulk}} + \underbrace{\frac{\rho_3}{4\pi W}\overline{R}_c\left(\frac{c}{a}, \frac{\rho_1}{\rho_3}\right)}_{\text{Interface}} + \underbrace{\frac{\rho_3 L_3}{2c \times W}}_{\text{Bulk}}$$

P. Zhang and Y. Y. Lau, J. Appl. Phys. **108**, 044914 (2010)

# Interface Resistance with Dissimilar Materials

A. Cylindrical semi-infinite channel

$$\overline{R}_c\left(\frac{b}{a},\frac{\rho_1}{\rho_2}\right)\cong\overline{R}_{c0}\left(\frac{b}{a}\right)\Big|_{Timsit}+\frac{\Delta}{2}\times\left(\frac{2\rho_1}{\rho_1+\rho_2}\right)\times g\left(\frac{b}{a}\right)$$

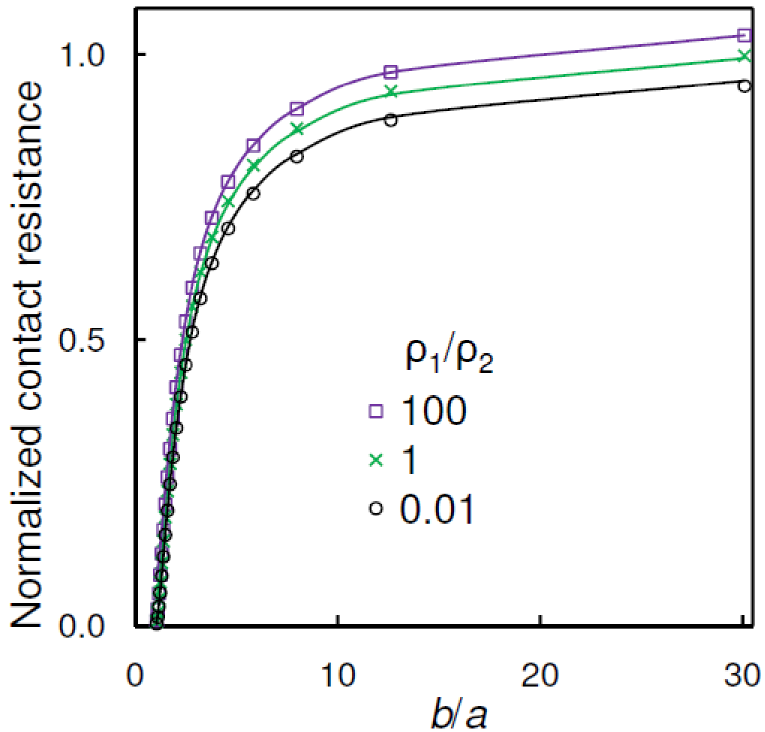
$$\overline{R}_{c0}(b/a)\Big|_{Timsit}=1-1.41581(a/b)+0.06322(a/b)^2$$
$$+0.15261(a/b)^3+0.19998(a/b)^4,$$

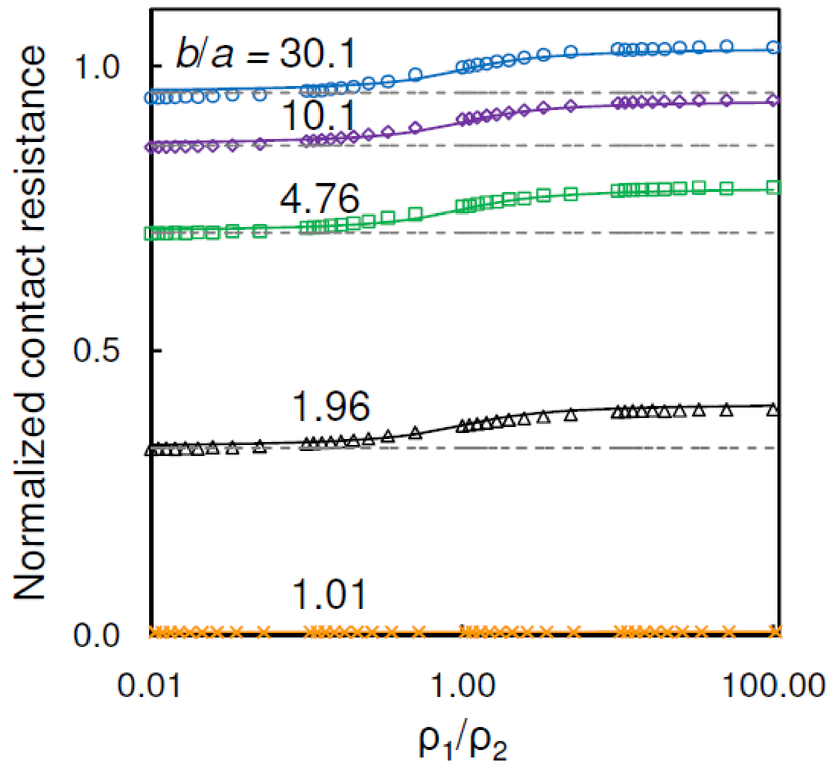
$$g(b/a)=1-0.3243(a/b)^2-0.6124(a/b)^4$$
$$-1.3594(a/b)^6+1.2961(a/b)^8$$

$$\Delta=32/3\pi^2-1=0.08076$$

P. Zhang and Y. Y. Lau, *J. Appl. Phys*. **108**, 044914 (2010)

# Interface Resistance in Cylindrical Semi-Infinite Channel



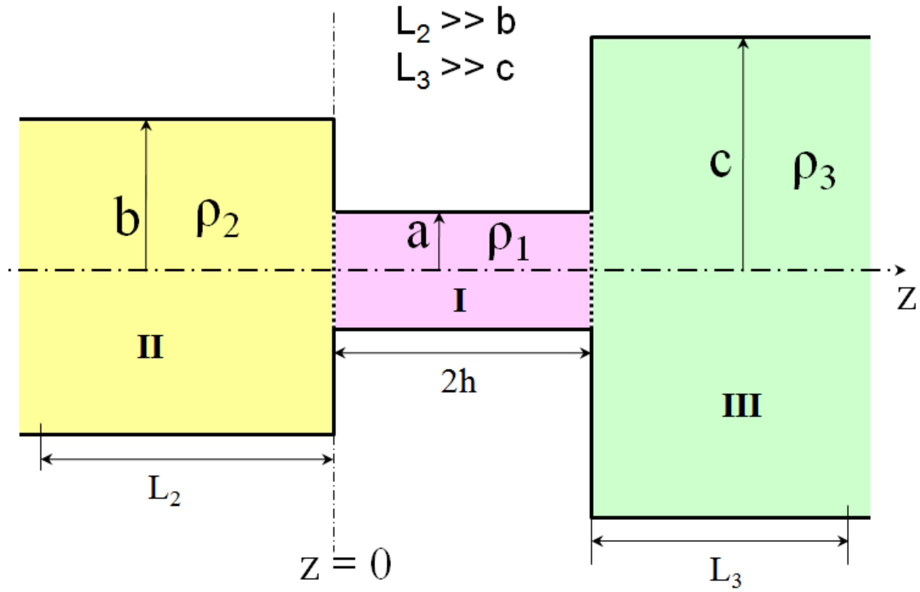




Symbols: Exact theory; Solid lines: Scaling law

# Test of Scaling Laws

**<u>Test A.</u>** $h >> a$

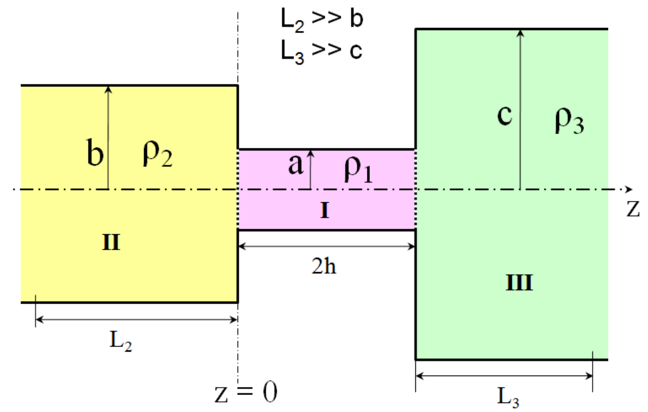
- Electrostatic fringe field at one interface has an exponentially small influence on the other interface
- Both interface resistance the same as in the semi-infinite channel



## **Test B.** $h \rightarrow 0$

- $h = 0$, $\rho_2 = \rho_3$ and $b = c$ ⟹ *a*-spot

  1. Cylindrical $\bar{R}_{c0}(b/a)\big|_{Timsit}$
  2. Cartesian $\bar{R}_{c0}(b/a)\big|_{LTZ}$

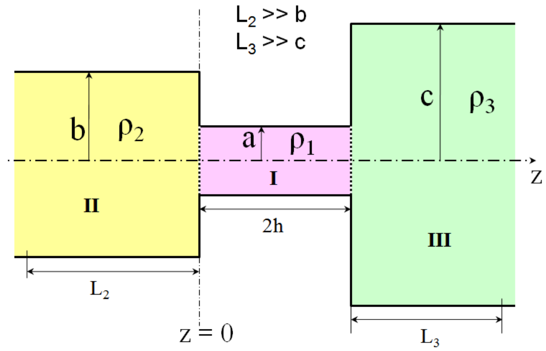



- $\rho_1 \rightarrow 0$, Region I is perfectly conducting, contact resistance at each interface equivalent to ½ of the symmetrical *a*-spot

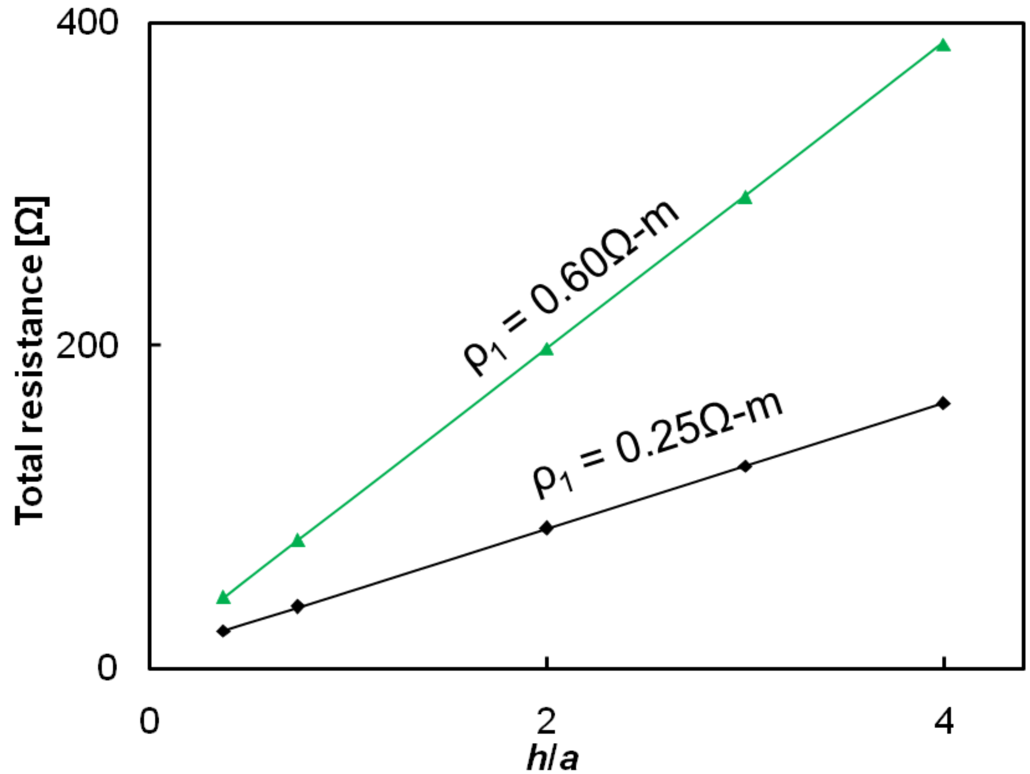
- $b/a \rightarrow \infty$, $c/a \rightarrow \infty$, but $\rho_2 \neq \rho_3$ , the cylindrical scaling laws differs by at most 8% from $(\rho_2 + \rho_3)/4a$

Error in $\bar{R}_c \leq 10\%$ in worst scenario

## **Test C.** Comparison of 3D Maxwell code



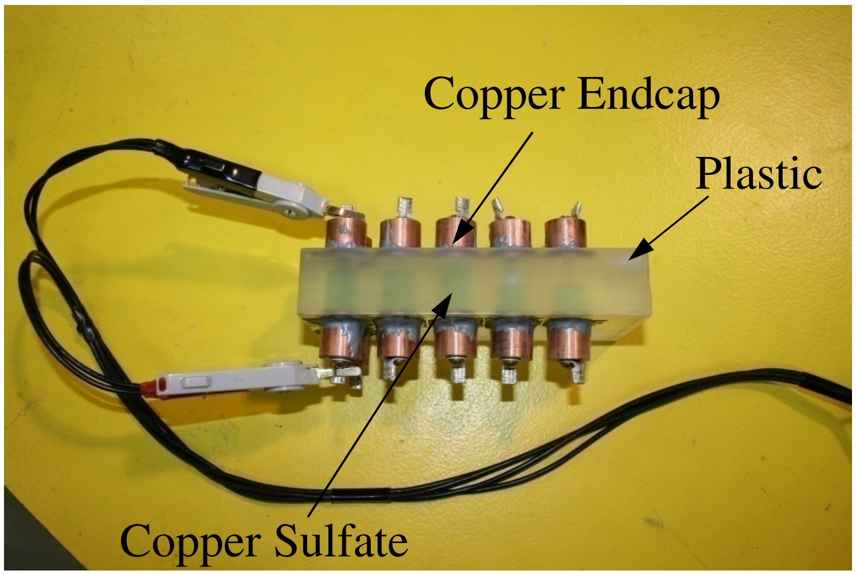

$\rho_2 = 0.038\ \Omega$ m,
$\rho_3 = 0.001\ \Omega$ m,
$a$ = 4mm,
$b$ = 8mm,
$c$ =10 mm,
$L_2 = L_3$,
$2h$ from 1.5 to 16 mm,
$L_2 + 2h + L_3$ = 80 mm,
Excitation voltage = 10V



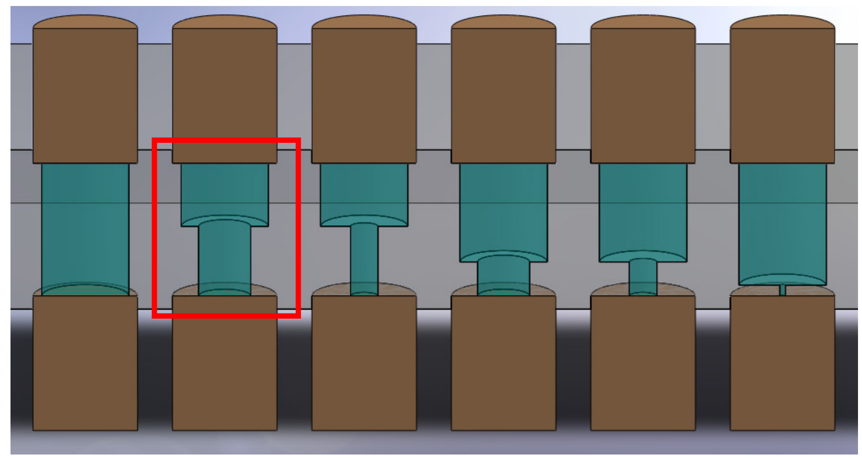

Symbols: Maxwell 3D
Solid lines: Scaling law

# Validation: Experiment
## ($\rho_1 = \rho_2 = \rho_3$, $b = c$)

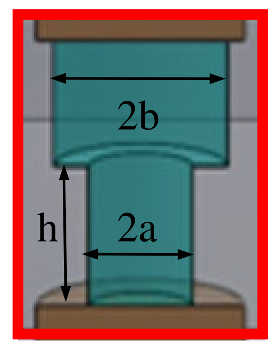
Experimental Configuration




Cross Sectional View

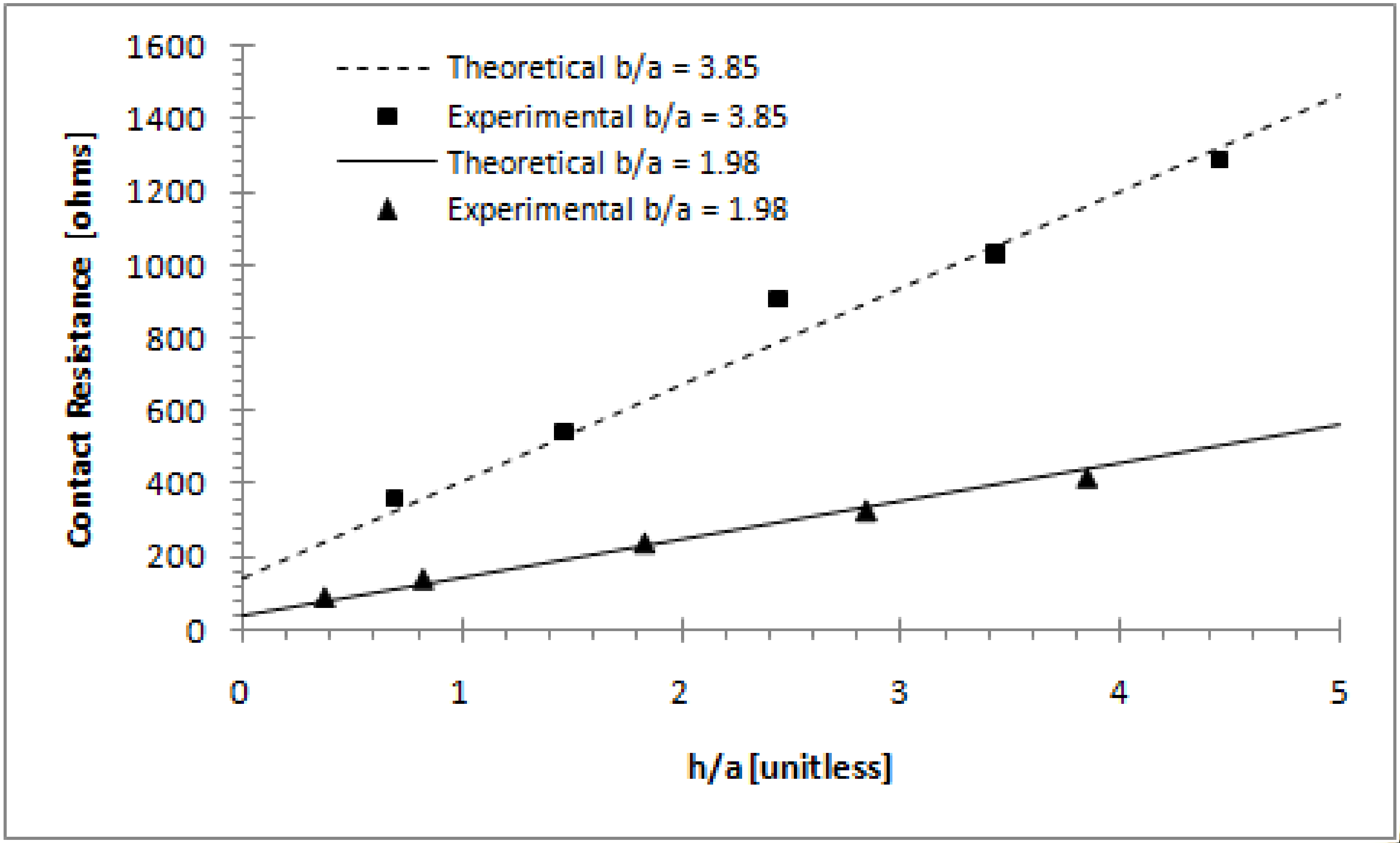


Case: A B C D E




Theoretical contact resistance geometry was mimicked by machining holes of varying diameter in a piece of plastic and filling it with copper sulfate.

# Experimental Validation
# ($\rho_1 = \rho_2 = \rho_3$, $b = c$)




*M. R. Gomez et al., Appl. Phys. Lett. **95**, 072103 (2009)

# Summary of Bulk Contact

- We vastly generalized Holm's classical *a*-spot theory (1967) of contact resistance to higher dimensions, with dissimilar materials.

- Simple scaling laws for contact resistance, with dissimilar materials and aspect ratios are constructed. They have been validated in various limits, in simulations, and in experiments.

- If the contaminant at contact is highly resistive, its bulk resistance dominates over the interface resistance; the interface resistance depends mainly on resistivity of main current channels and is insensitive to that of the contact region.

- Great uncertainty in numerical code or experiments, if there is large contrast between $\rho_1$ , $\rho_2$ , and $\rho_3$, or between $h$, $a$, $b$, $c$, $L_2$, and $L_3$.



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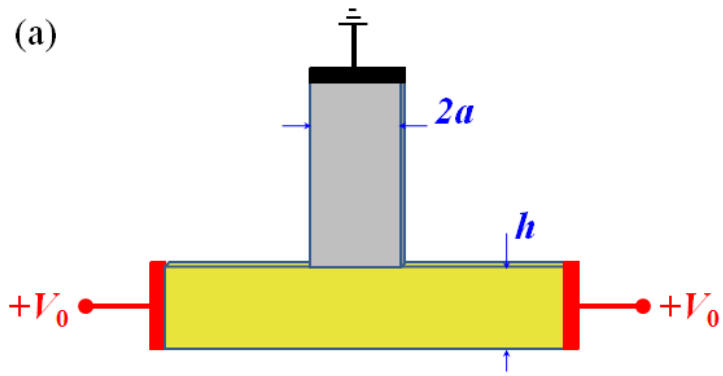
# Contact Resistance

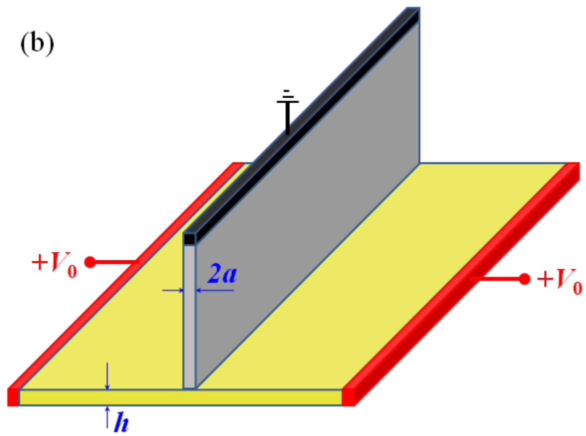
## Thin-Film Geometry

# Thin Film Contact Model

## Cartesian

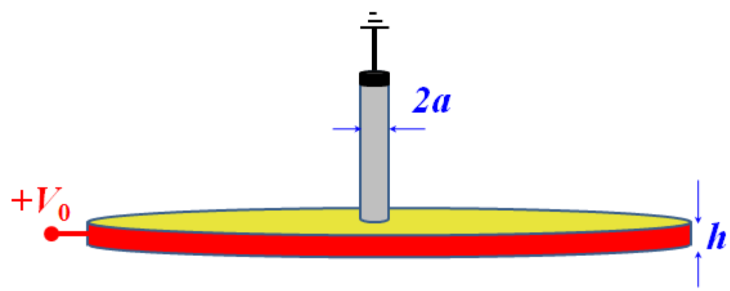



Thin film resistor




Heat sink

## Cylindrical

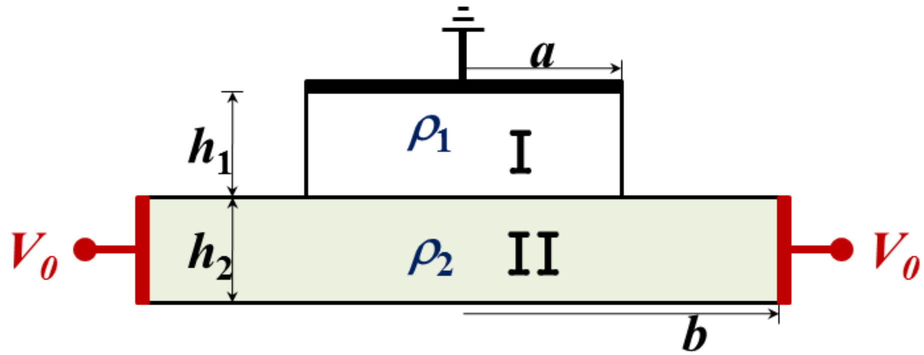



Field Emitter

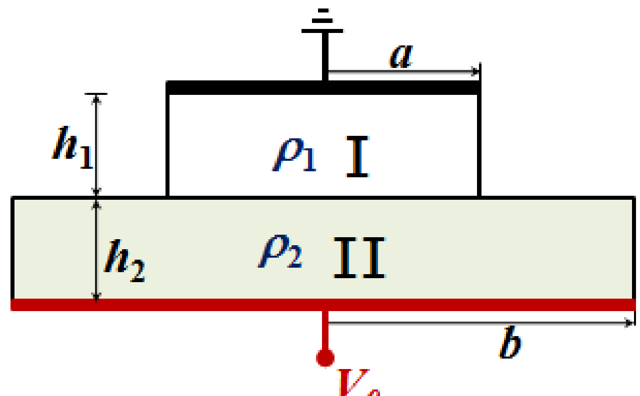
# Thin Film Contact Model

Cartesian and Cylindrical

Horizontal Type [1]




Vertical Type [2]

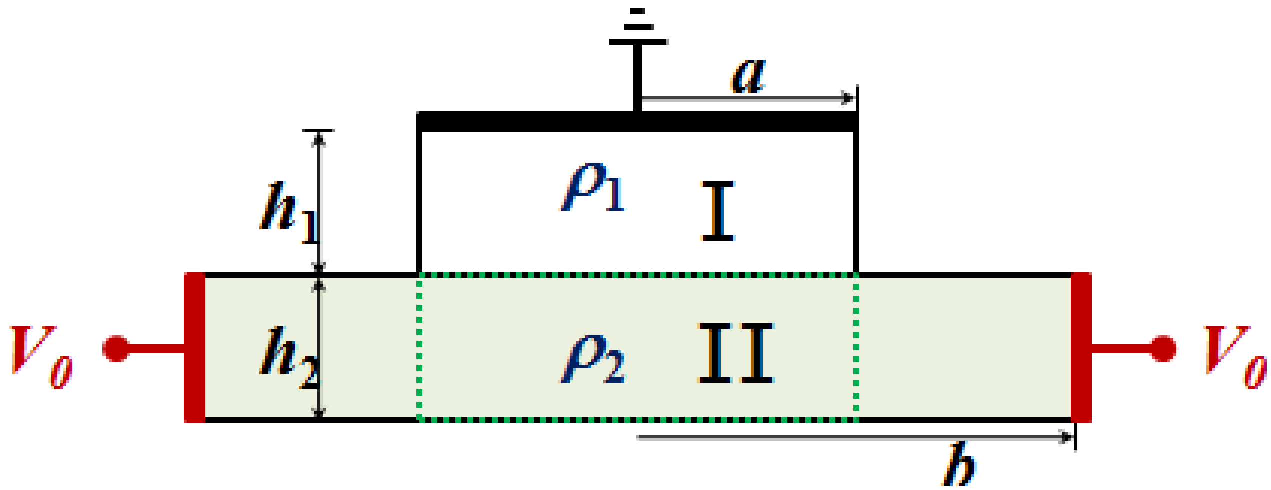



Results: severe current crowding in Horizontal type;
Much less severe current crowding in Vertical type.


[1] Peng Zhang, D. Hung, Y. Y. Lau, J. Phys. D: Appl. Phys., 46, 065502 (2013); Corrigendum, ibid, 46, 209501 (2013).

[2] Peng Zhang and Y. Y. Lau, IEEE J. Electron Devices Soc., in the press (2013).

# Thin Film Contact Model: Horizontal Type



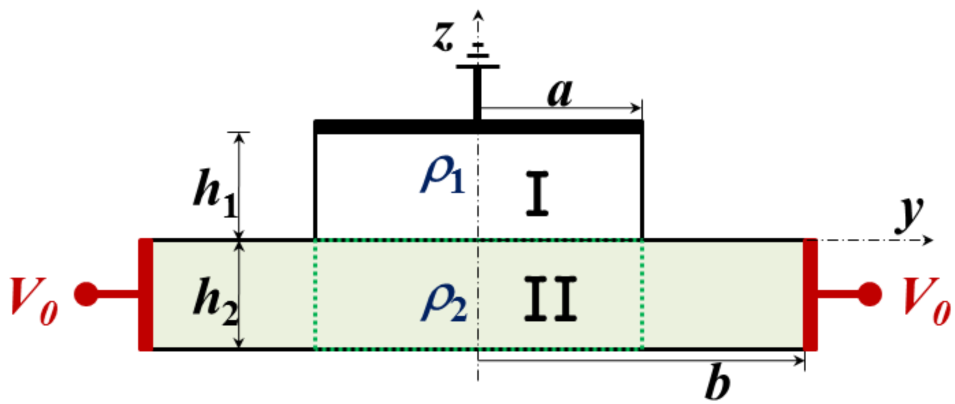

Cartesian

$$R = \underbrace{\frac{\rho_1 h_1}{2a \times W}}_{R_I} + \underbrace{\frac{\rho_2}{4\pi W}\overline{R}_c\left(\frac{a}{b},\frac{a}{h_2},\frac{h_1}{a},\frac{\rho_1}{\rho_2}\right)}_{\text{Contact resistance } R_c} + \underbrace{\frac{\rho_2 (b-a)}{2h_2 \times W}}_{R_{II}}$$

Cylindrical

$$R = \underbrace{\frac{\rho_1 h_1}{\pi a^2}}_{R_I} + \underbrace{\frac{\rho_2}{4a}\overline{R}_c\left(\frac{a}{b},\frac{a}{h_2},\frac{h_1}{a},\frac{\rho_1}{\rho_2}\right)}_{\text{Contact resistance } R_c} + \underbrace{\frac{\rho_2}{2\pi h_2}\ln\left(\frac{b}{a}\right)}_{R_{II}}$$

# Cartesian Thin Film Contact




Boundary conditions

$$\Phi_+ = \Phi_-, \qquad z = 0, |y| \in (0, a)$$

$$\frac{1}{\rho_1}\frac{\partial \Phi_+}{\partial z} = \frac{1}{\rho_2}\frac{\partial \Phi_-}{\partial z}, \quad z = 0, \quad |y| \in (0, a)$$

$$\frac{\partial \Phi_-}{\partial z} = 0, \qquad z = 0, \quad |y| \in (a, b)$$

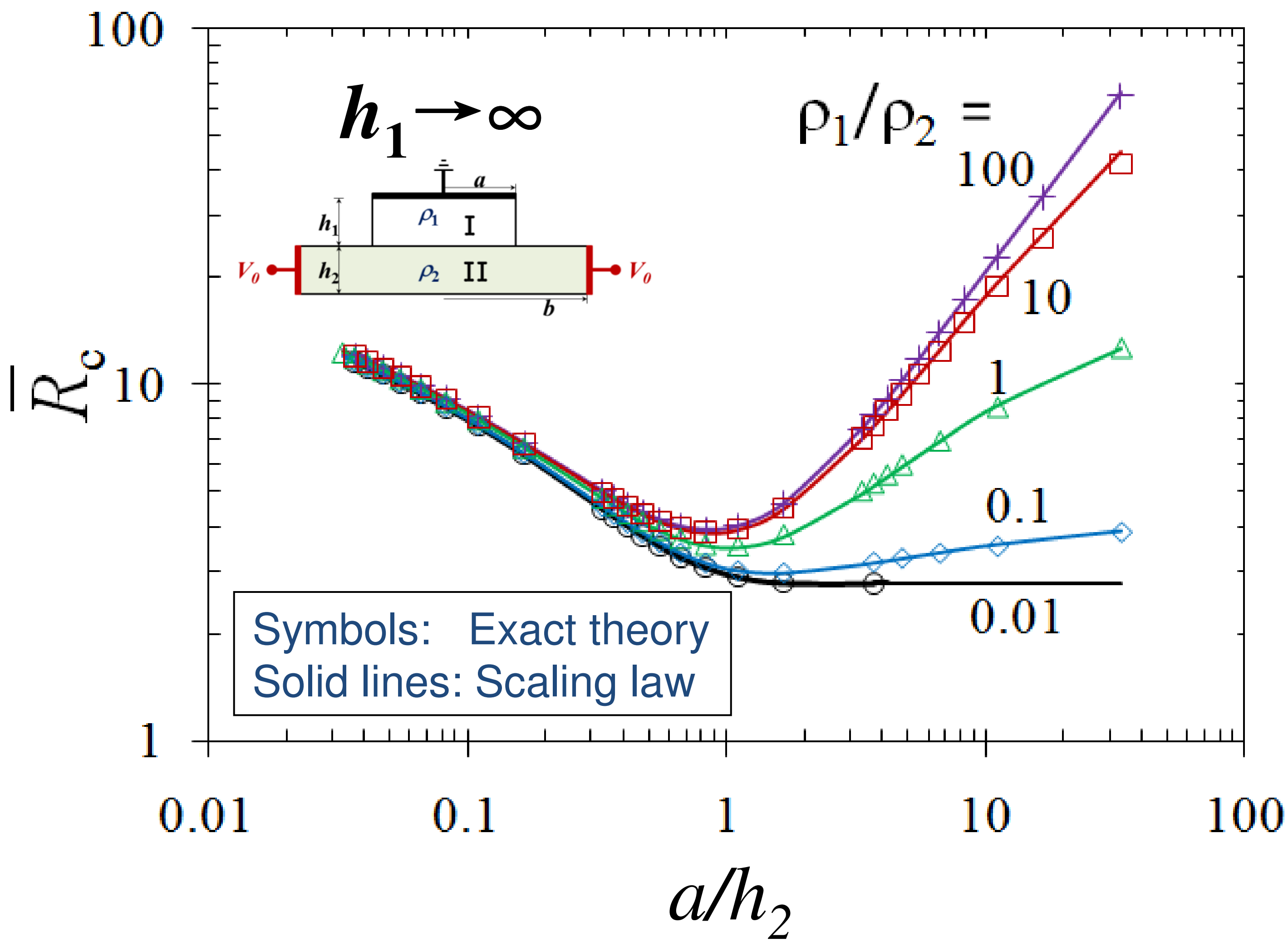
Laplace's equation

$$\Phi_+(y,z) = A_0(z-h_1) + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi y}{a}\right) \times \sinh\left(n\pi \frac{z-h_1}{a}\right), \qquad 0 < z < h_1, |y| \in (0,a),$$

$$\Phi_-(y,z) = V_0 + \sum_{n=1}^{\infty} B_n \cos\left(\frac{(n-1/2)\pi y}{b}\right) \times \frac{\cosh\left((n-\frac{1}{2})\pi \frac{z+h_2}{b}\right)}{\sinh\left((n-\frac{1}{2})\pi \frac{h_2}{b}\right)}, \qquad -h_2 < z < 0, |y| \in (0,b).$$

→ Potential profile and contact resistance

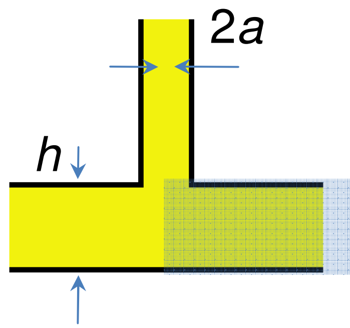
# Cartesian Thin Film Contact




Zhang, Lau, Gilgenbach, *Appl. Phys. Lett*. **97**, 204103 (2010);
*J. Appl. Phys*. **109**, 124910 (2011).

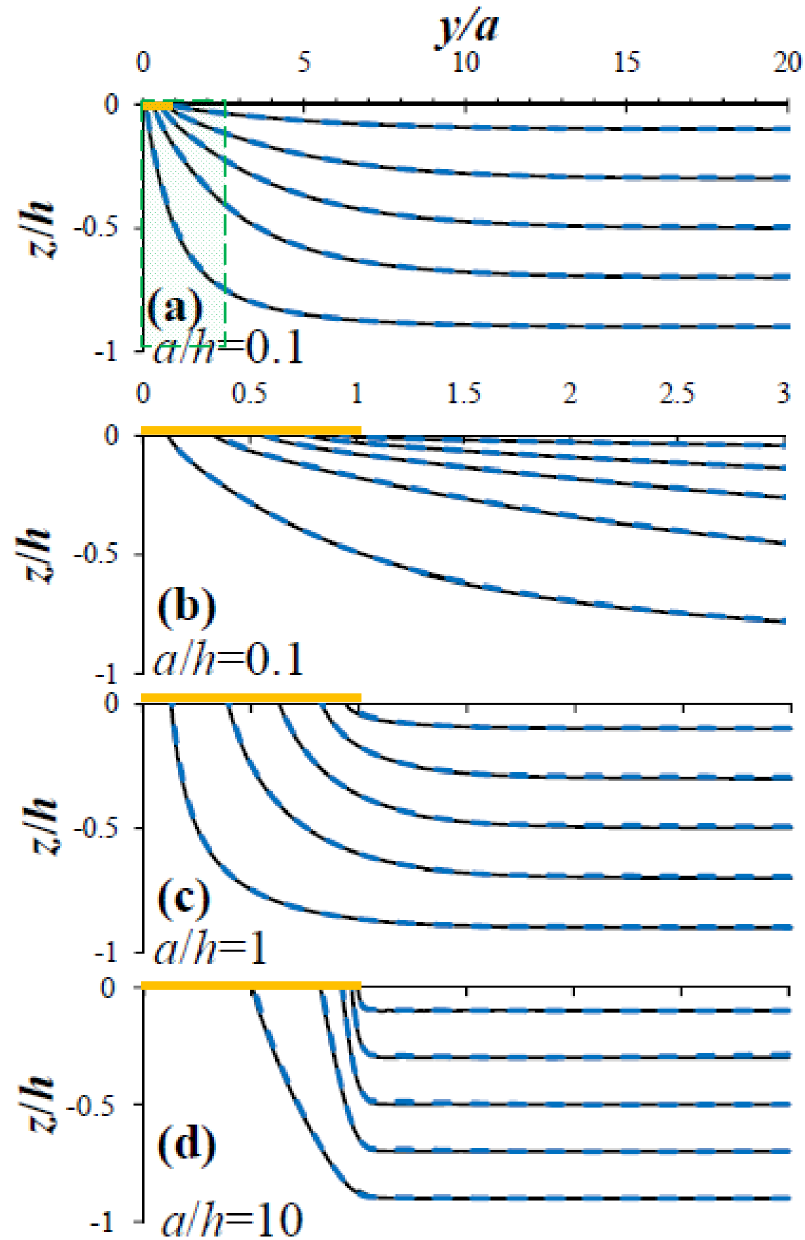
# Cartesian Thin Film Contact – Field Lines

$$\rho_1/\rho_2 = 1$$

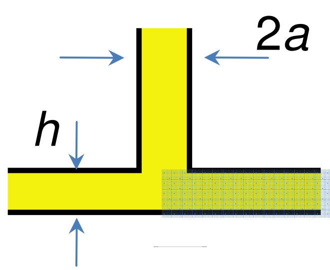


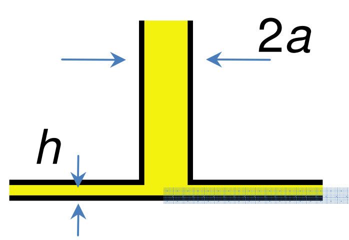

Dashed lines:
Conformal Mapping

Solid lines:
Series Expansion

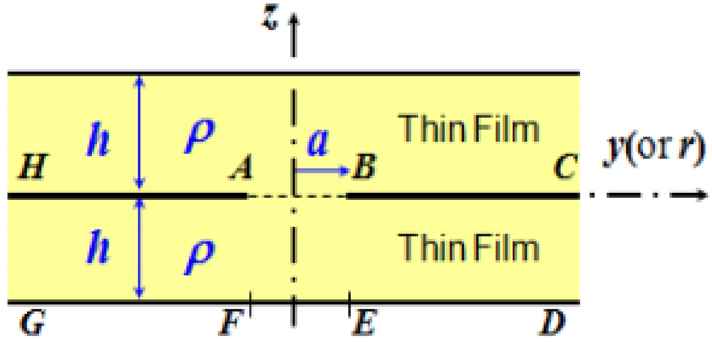



Crowding of field lines gives rise to contact resistance



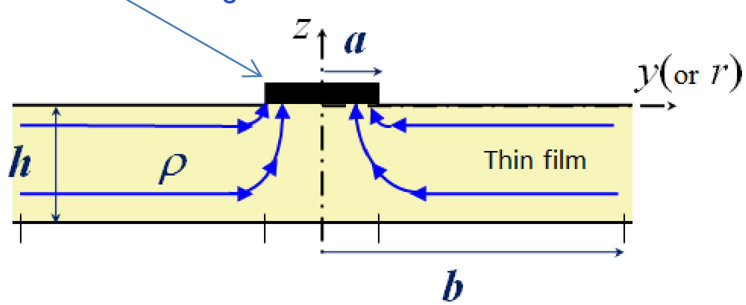



# Application to Thin-film Contact – MEMS




## Same as Metal-Metal Contact, by Symmetry ($h_1 = 0$)

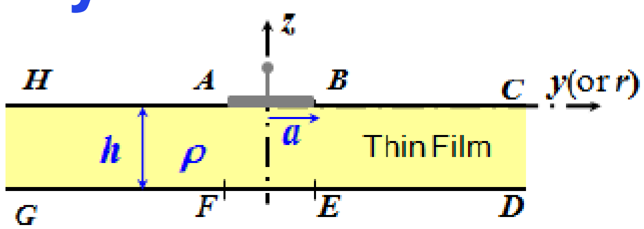
Contact resistivity $r_c$=0

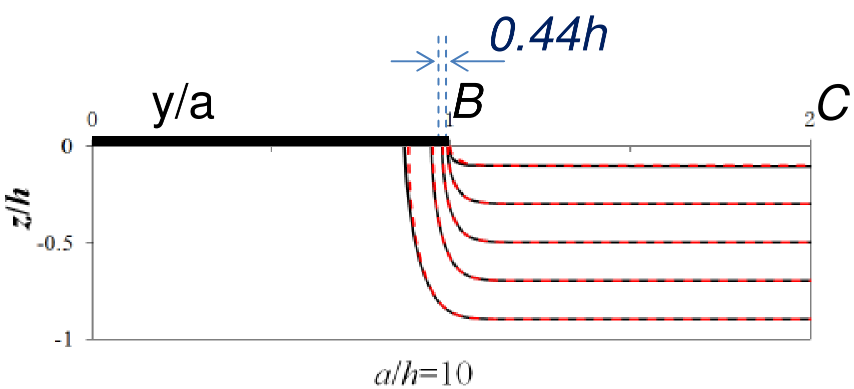



Peng Zhang, Y. Y. Lau, and R. S. Timsit, "*On the spreading resistance of thin-film contacts*", IEEE Trans. Electron Devices **59**, 1936 (2012).

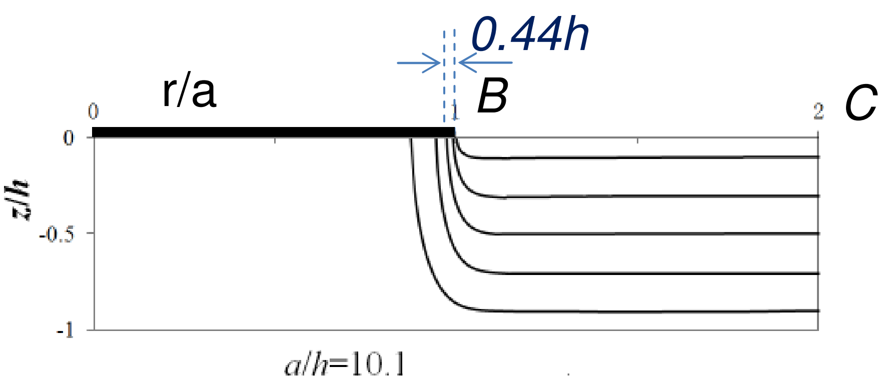
# Identical Field Line Patterns for Both Cartesian and Cylindrical Thin Film Contacts

As h→0,



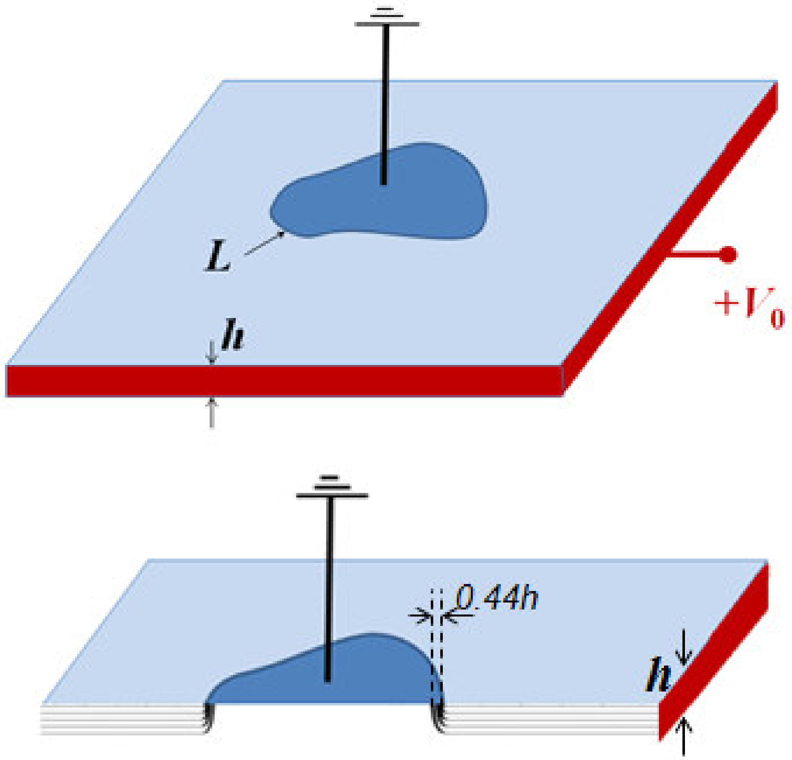

Cartesian




Cylindrical

Note: Enhanced heating at Edge "B" (& "A").

# Contact Resistance in an Electrode of Arbitrary Shape*




$$R_s = \left(\frac{\rho}{L}\right)\frac{2\ell n2}{\pi}, \quad h \to 0$$

*L*: circumference of electrode

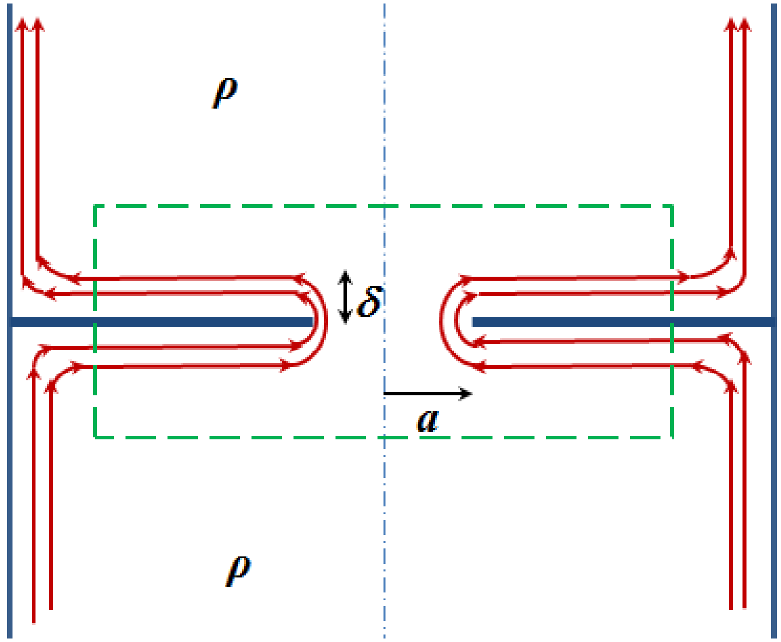
Current transfer length

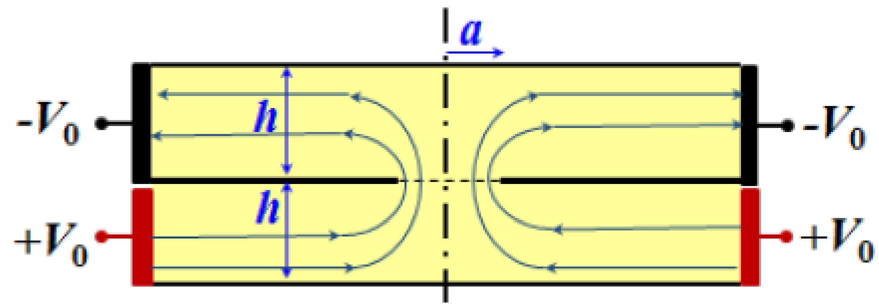
$L_T \sim 0.44h$

*Note:* Intense local heating on electrode rim. This formula reduces to those for a-spot in circular and Cartesian geometry.


Peng Zhang, Y. Y. Lau, and R. S. Timsit, "*On the spreading resistance of thin-film contacts*", IEEE Trans. Electron Devices **59**, 1936 (2012).

# Relation of high frequency AC bulk contact and DC thin - film contact




## AC Case (Bulk Contact)

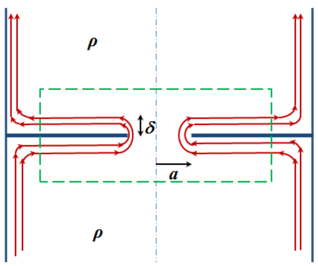
Lavers and Timsit, *IEEE Trans. Compon. Packag. Technol.*, **25**, 446–452, 2002.

## DC Case (Thin Film)

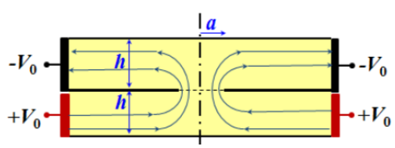
P. Zhang, Y. Y. Lau, and R. S. Timsit, *IEEE Trans. Electron Devices* **59**, 1936 (2012).

Rule: Identify skin depth ( $\delta$ ) with thin-film thickness (*h*).

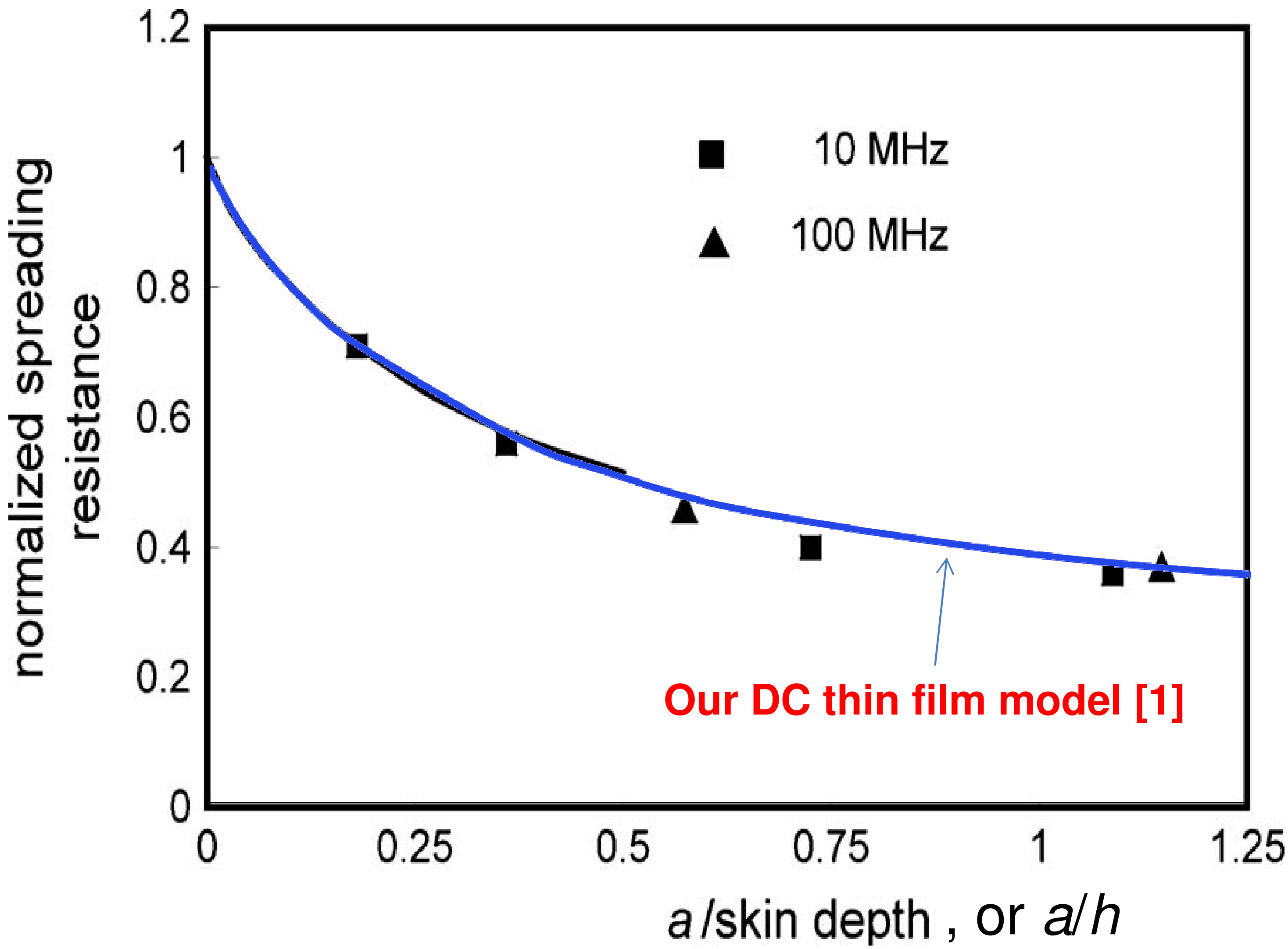
# AC bulk constriction resistance for frequencies ranging from dc to 1 GHz*




AC case




Our DC model




*Lavers and Timsit, *IEEE Trans. Compon. Packag. Technol.*, 25, 446–452, 2002.

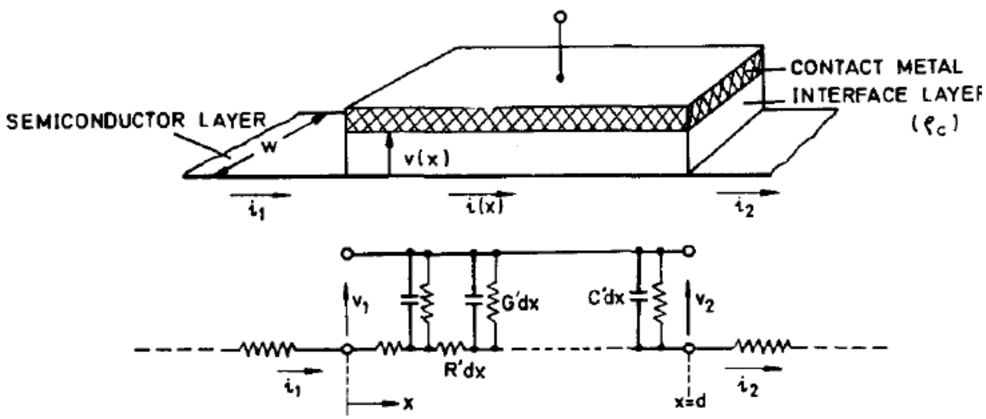
[1]Peng Zhang, Y. Y. Lau, and R. S. Timsit, IEEE Trans. Electron Devices **59**, 1936 (2012)

DC thin film resistance formula is applicable to AC bulk contact resistance from low to high frequency!!

# General 3-terminal Thin Film Devices

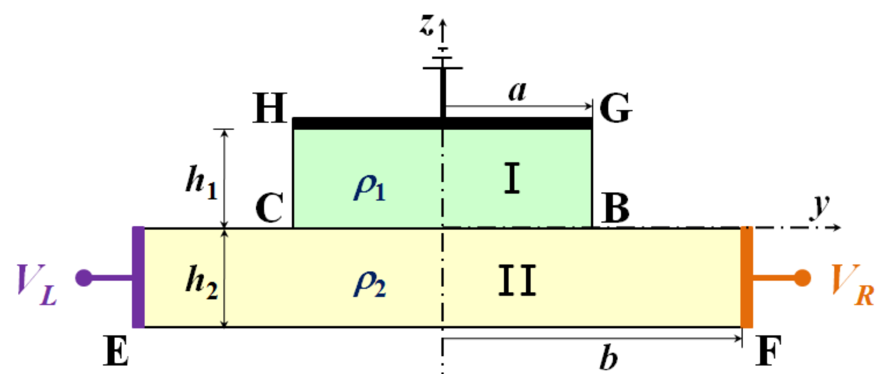
## (Arbitrary voltages on all 3 terminals)

(a)



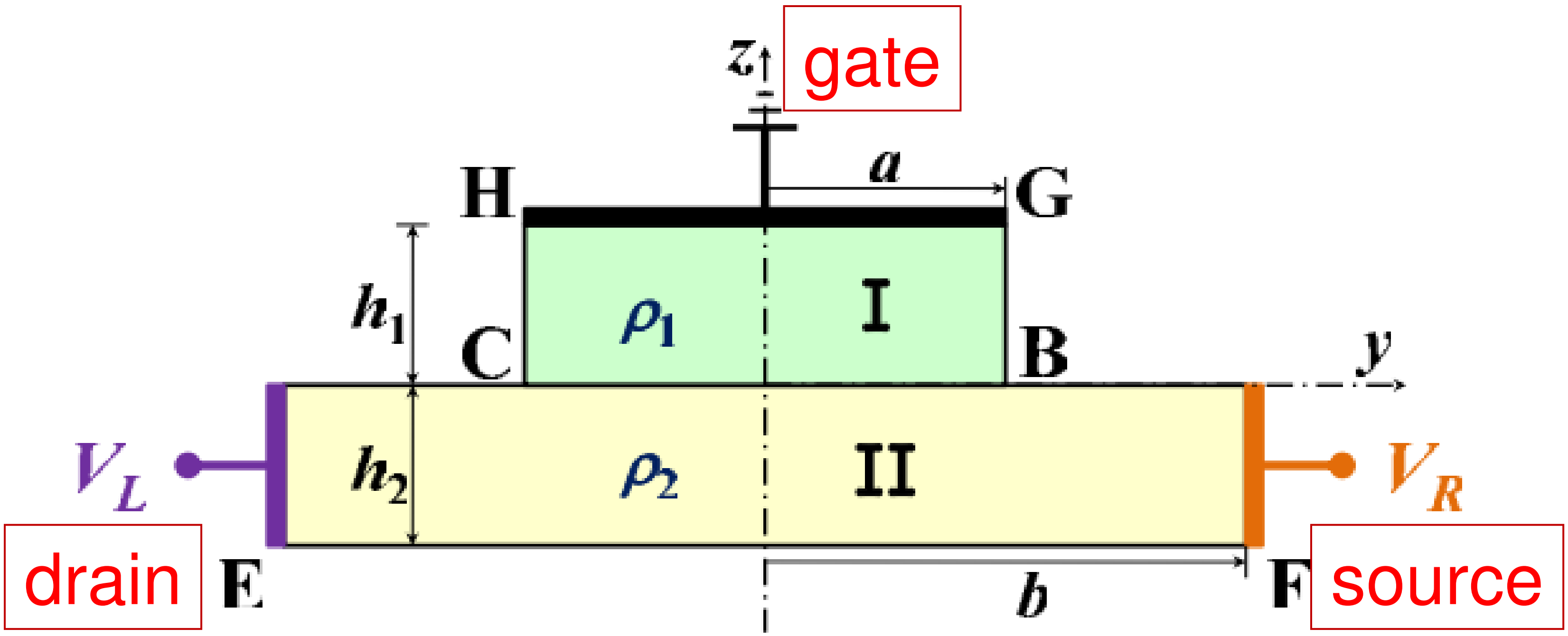

Transmission Line Model

(b)




Generalization

# General 3-termial Thin Film Contact [1]



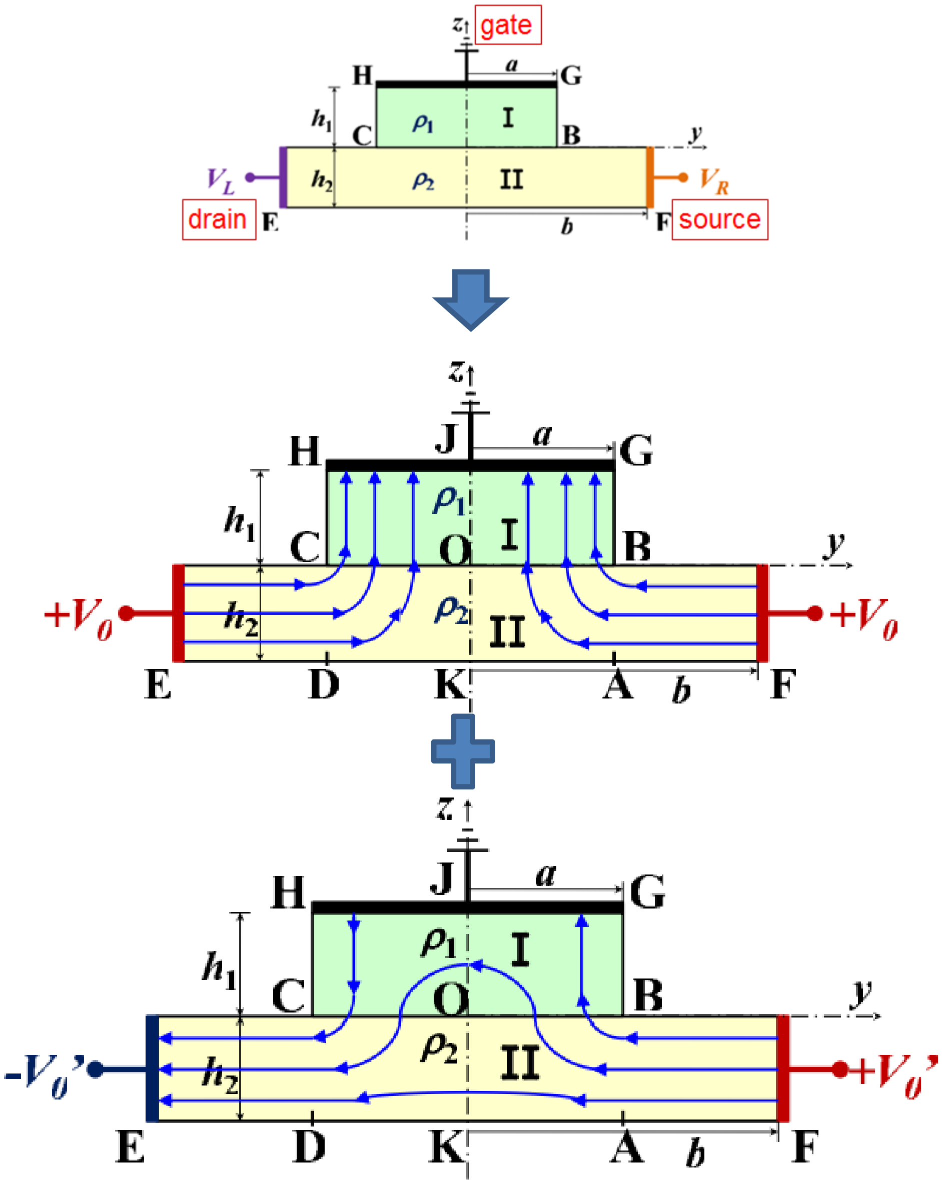

Cartesian

$V_L \neq V_R$

Top terminal (HG) grounded.
Arbitrary $h_1$, $h_2$, $\rho_1$, $\rho_2$, $a$, $b$, $V_R$, $V_L$.

[1] Peng Zhang, D. Hung, Y. Y. Lau, J. Phys. D: Appl. Phys., 46, 065502 (2013), Corrigendum, ibid, 46, 209501 (2013).

[Decomposition into Even and Odd Solutions]()




## Even solution

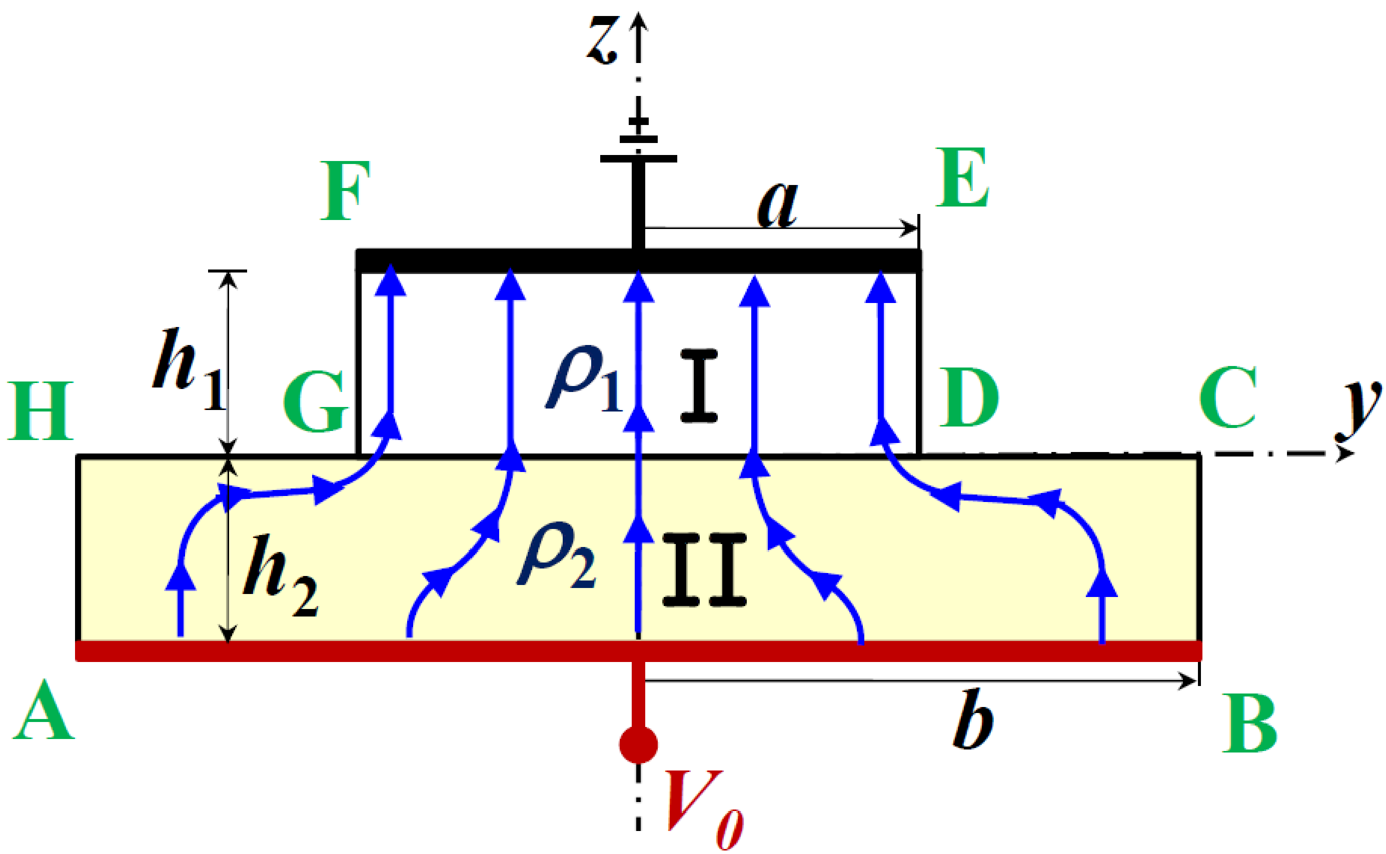
$$V_0 = (V_L + V_R)/2$$

## Odd solution

$$V_0^{'} = (V_R - V_L)/2$$

**Even solution accounts for all current transport to the gate (HG).**
**Odd solution gives only current transport to the drain (E).**
**=> Most intense heating on edge (B) of the "source" side.**

# Vertical Type of Thin Film Contact*




* Peng Zhang and Y. Y. Lau, IEEE J. Electron Devices Soc., in the press (2013).

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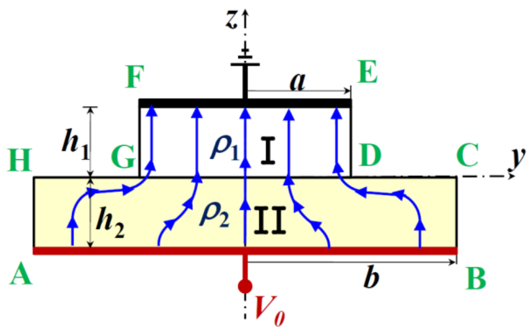
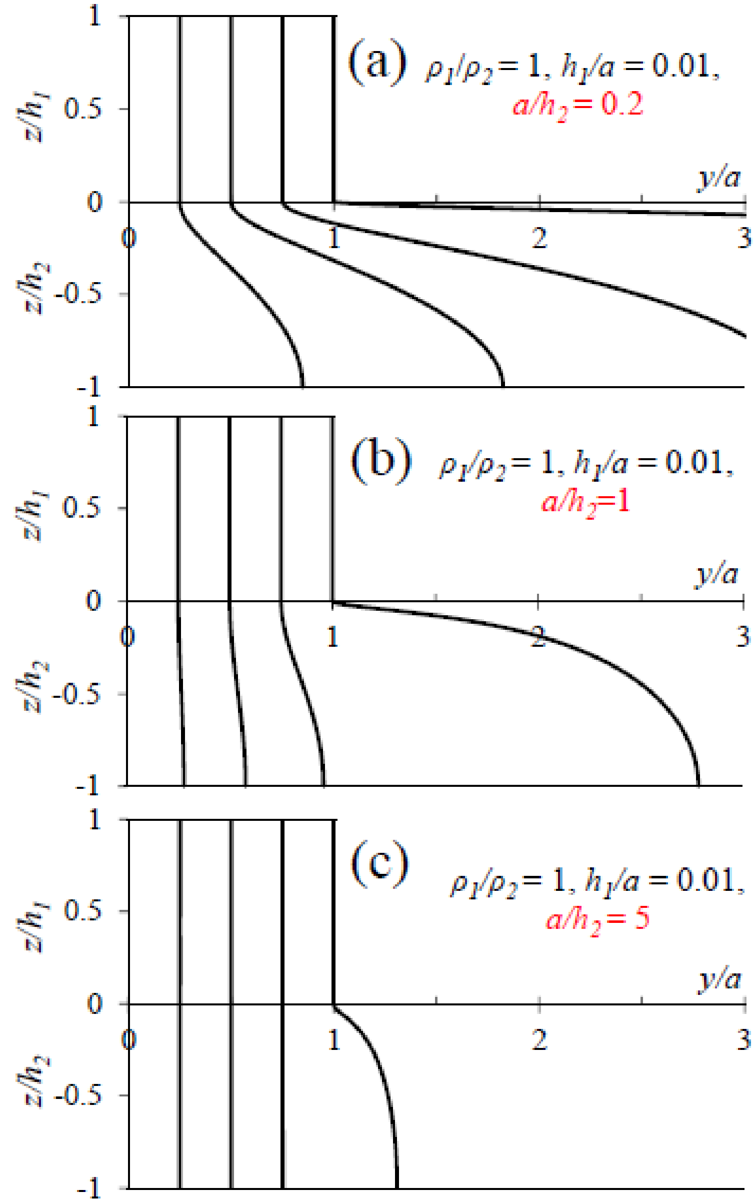
# General Thin Film Contact: Vertical Type

Cartesian




If $h_2$ decreases:




Almost uniform current flow in Region I, implying very little enhanced edge heating

# Summary of Thin Film Contact

- Accurate scaling laws are synthesized over a large range of resistivity ratio and aspect ratio, for both horizontal and vertical contacts, including 3-terminal devices.
- Horizontal contacts suffer intense edge heating on the "source side" of thin-film. There is little enhanced edge heating in vertical contacts.
- Thin film contact model is extended to an *a*-spot of arbitrary geometry.
- The DC thin film contact model turns out to be applicable to the bulk contact resistance, from DC to high frequency (!)
- A new transfer length, due to fringing field, is identified and compared to the transmission line model.
- Issues on experimental measurements are being explored.

# Effects of Surface Roughness (II):

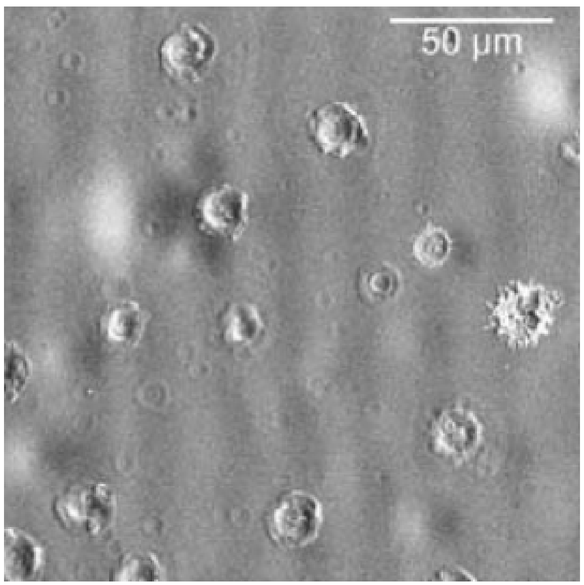
## Enhanced RF Losses

and

## Field Enhancements

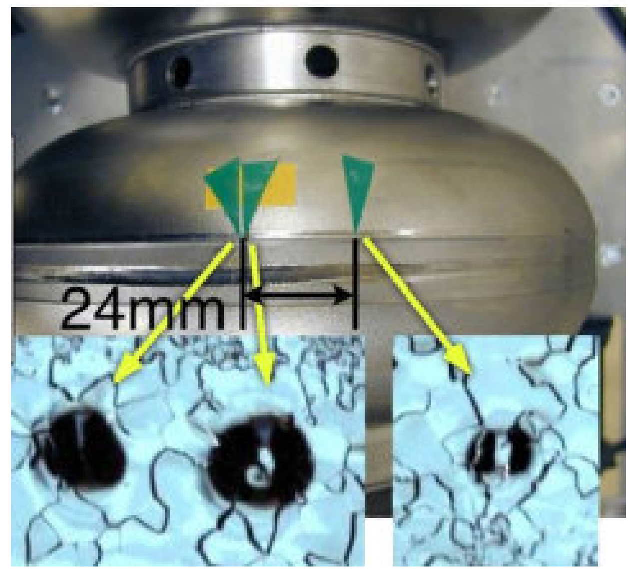
# Introduction

- Surface roughness increases RF absorption, causes E-field enhancement (triggers breakdown) and B-field enhancement (quenching of superconducting cavities)




Splashes on the metallic surface

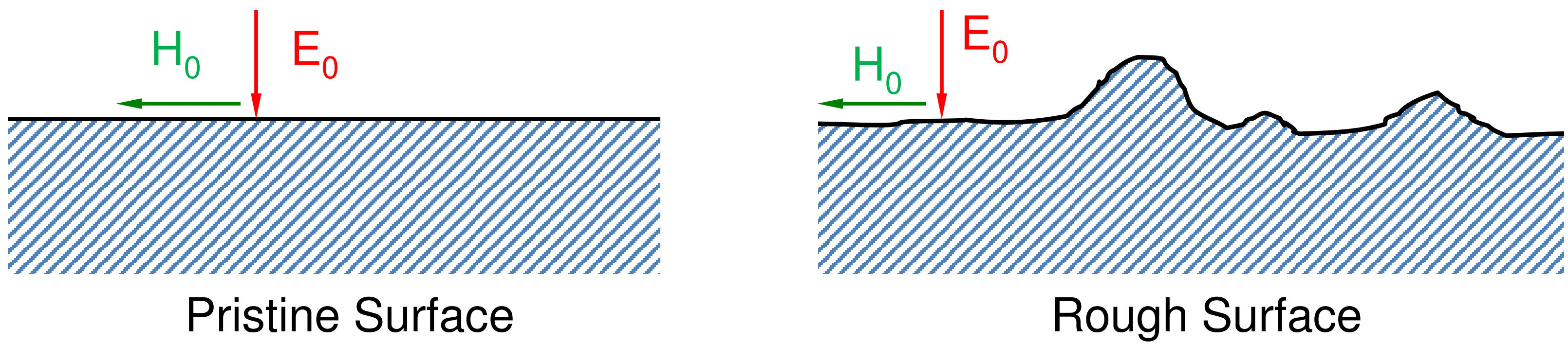
S. E. Harvey et al.,
Report No. SLAC-PUB-10197, 2003




Hot spots found in superconductor cavities

Yoshihisa Iwashita,
Phys. Rev. ST Accel. Beams 11, 093501 (2008)

# Rough Surface



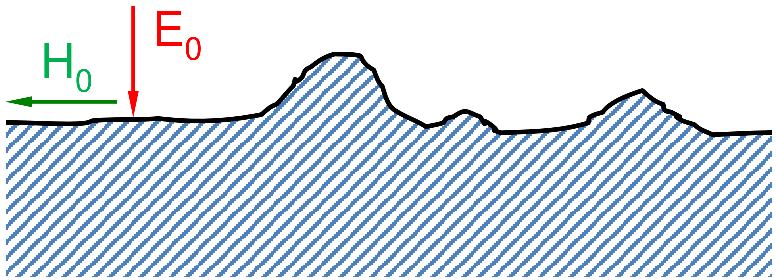

(1) Cause enhanced heating

(2) Cause local field enhancement

$E_{rf}$ ⟹ Field Emission ⟹ Breakdown

$H_{rf}$ ⟹ Quenching of Superconductor Cavity

# Enhanced Heating on a Rough Surface



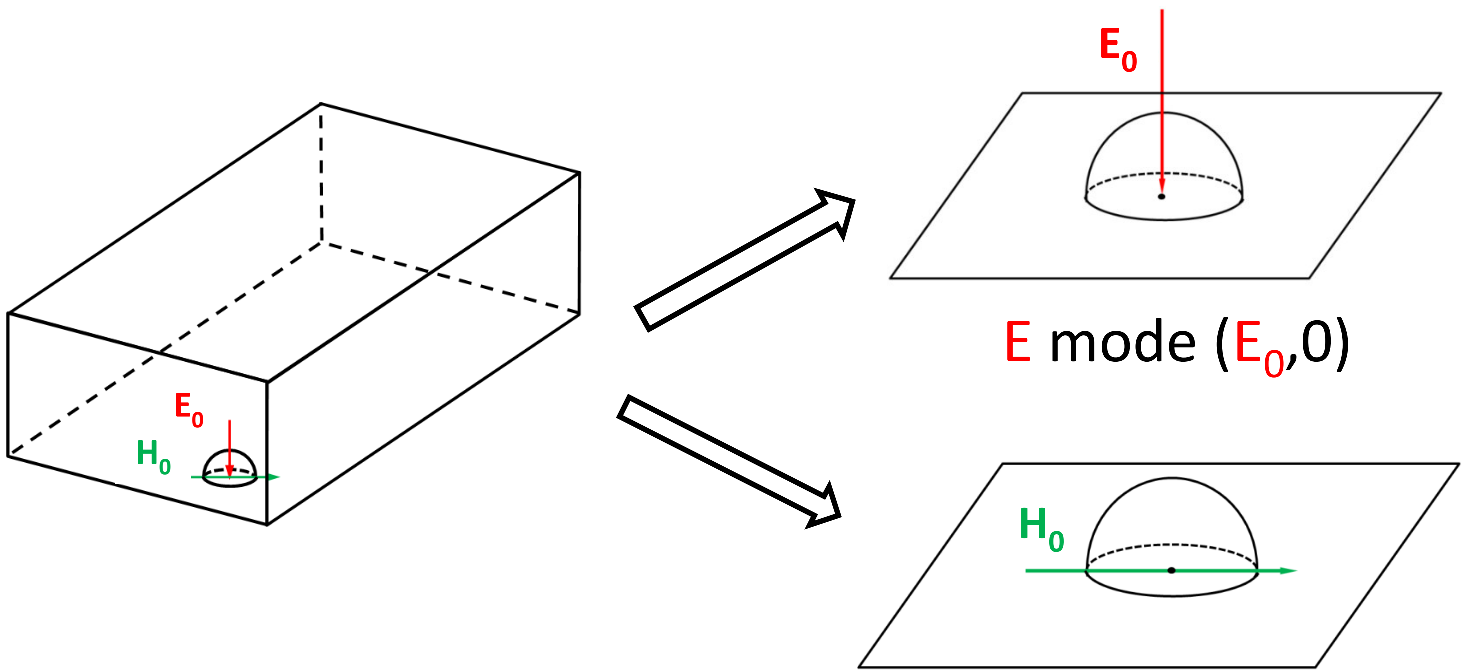

Rough Surface

Questions: Enhanced heating by E or by H?
Parametric dependence ($\lambda$, $\sigma$, a, etc.)?

Model: Consider a small hemispherical protrusion of radius $a \ll \lambda$, with arbitrary $\varepsilon,\mu,\sigma$, and arbitrary $E_0$ and $H_0$ (in the absence of protrusion).

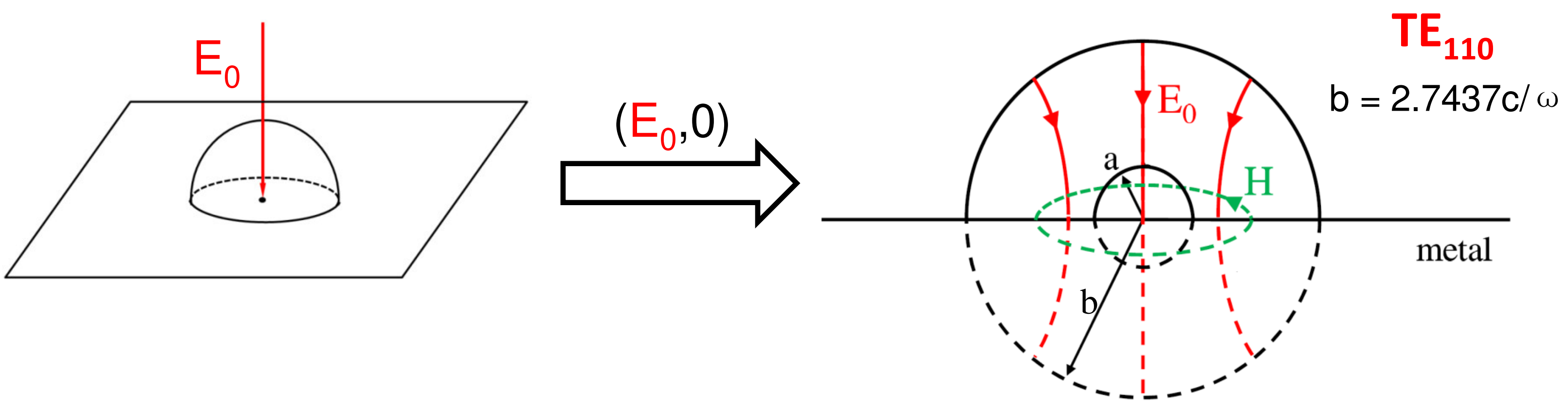
# Hemispherical protrusion on the surface ($a \ll \lambda_{exterior}$). Note: $E_0$ and $H_0$ are arbitrary.

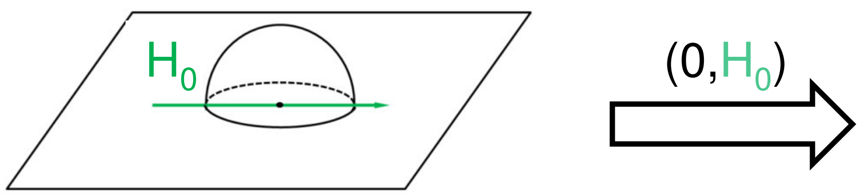
$$(E_0, H_0) = \underbrace{(E_0, 0)}_{\text{E mode}} + \underbrace{(0, H_0)}_{\text{M mode}}$$

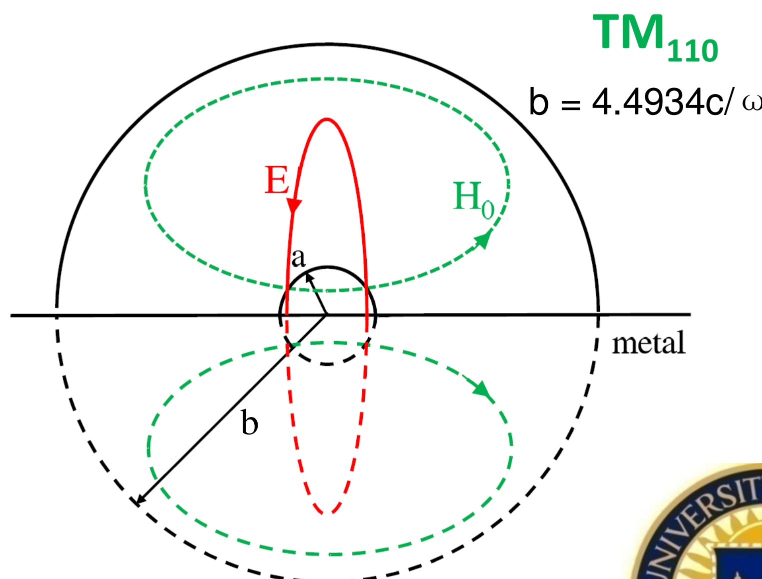



M mode $(0, H_0)$

# Hemispherical protrusion on the surface




The irregular geometry “hemispherical protrusion on a surface” transformed into an equivalent, but highly symmetrical problem “spherical particulate in a spherical cavity”

# Hemispherical protrusion on the surface

(A) Perturbation on Eigenvalue gives power dissipated by particulate

$$2\omega_i = \frac{P_d}{U}$$

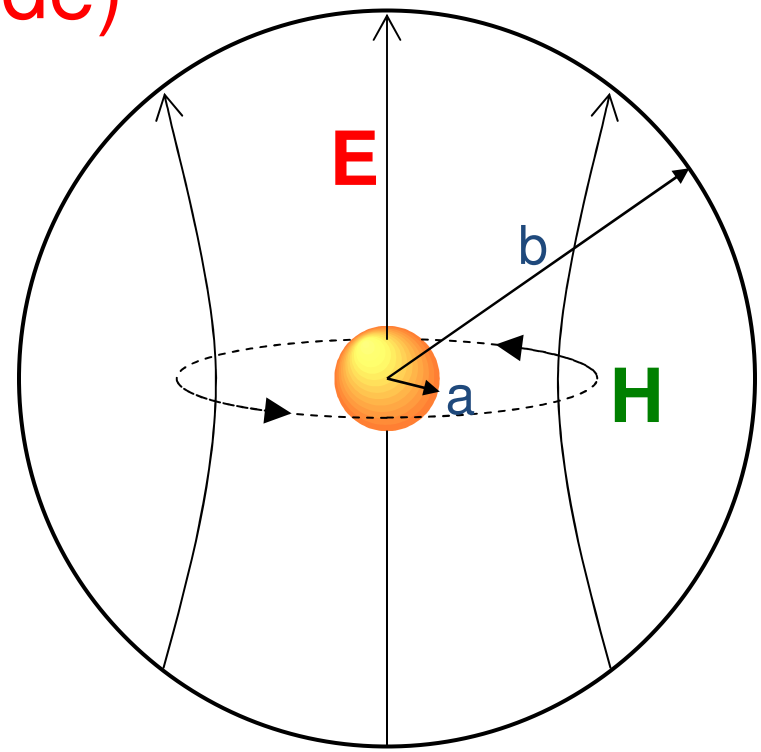
(B) Perturbation on Eigenfunction gives RF field enhancements

Note: (a) ($\varepsilon_r$, $\sigma$, $\mu$) of protrusion may be arbitrary.

(b) Perturbed $TE_{110}$ & $TM_{110}$ mode calculated exactly, consistent with full set of Maxwell equations.

P. Zhang, Y. Y. Lau, R. M. Gilgenbach, J. Appl. Phys. 105, 114908 (2009).

# RF **electric** field heating
## (**TE** mode)




- Dispersion relation for the **$TE_{110}$** mode:

$$\frac{J'(\xi_1)}{J(\xi_1)} = \frac{\sqrt{\mu_2/\varepsilon_2}}{\sqrt{\mu_1/\varepsilon_1}} \frac{M'(\xi_2)}{M(\xi_2)}$$

- For a << b ~ $\lambda$ (the only assumption):

$$\frac{\gamma_E}{\omega_E} = \frac{Y'(\eta_E)}{\eta_E J''(\eta_E)} \frac{J(\xi_{2E})}{Y(\xi_{2E})} \left[ \frac{Y'(\xi_{2E})}{\xi_{2E} Y(\xi_{2E})} - \frac{J'(\xi_{2E})}{\xi_{2E} J(\xi_{2E})} \right] \mathbf{Im}\left(\frac{1}{Z_E}\right)$$

$\gamma$ => Q => $\alpha_E$



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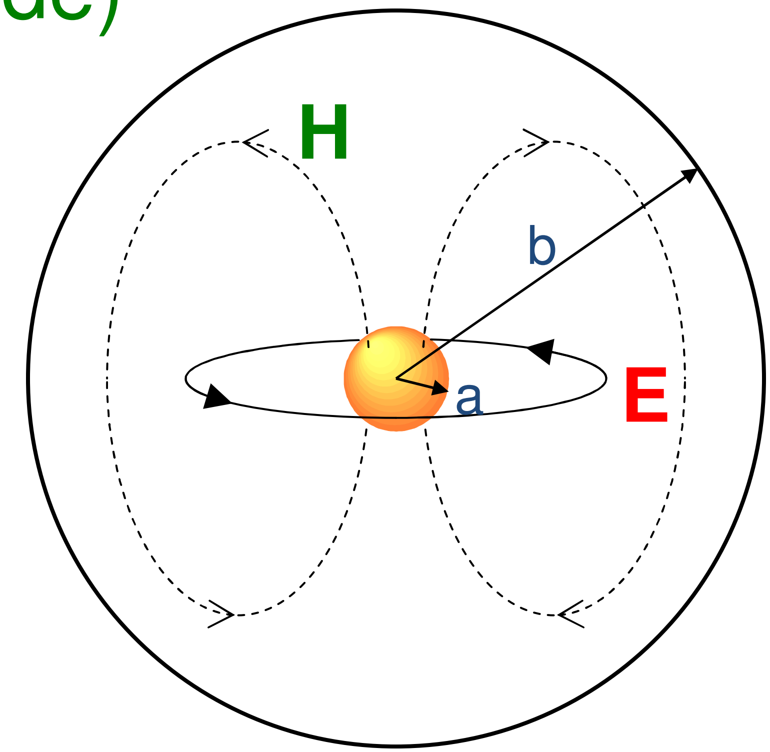
# RF **magnetic** field heating
## (**TM** mode)




- Dispersion relation for the **TM$_{110}$** mode:

$$\frac{J'(\xi_1)}{J(\xi_1)} = \frac{\sqrt{\mu_1/\varepsilon_1}}{\sqrt{\mu_2/\varepsilon_2}}\frac{N'(\xi_2)}{N(\xi_2)}$$

- For a << b ~ $\lambda$ (the only assumption):

$$\frac{\gamma_M}{\omega_M} = \frac{y(\eta_M)}{\eta_M\, j'(\eta_M)}\frac{J'(\xi_{2M})}{Y'(\xi_{2M})}\left[\frac{Y(\xi_{2M})}{\xi_{2M}Y'(\xi_{2M})} - \frac{J(\xi_{2M})}{\xi_{2M}J'(\xi_{2M})}\right]\mathbf{Im}\left(\frac{1}{Z_M}\right)$$

$$\gamma \Rightarrow Q \Rightarrow \alpha_H$$



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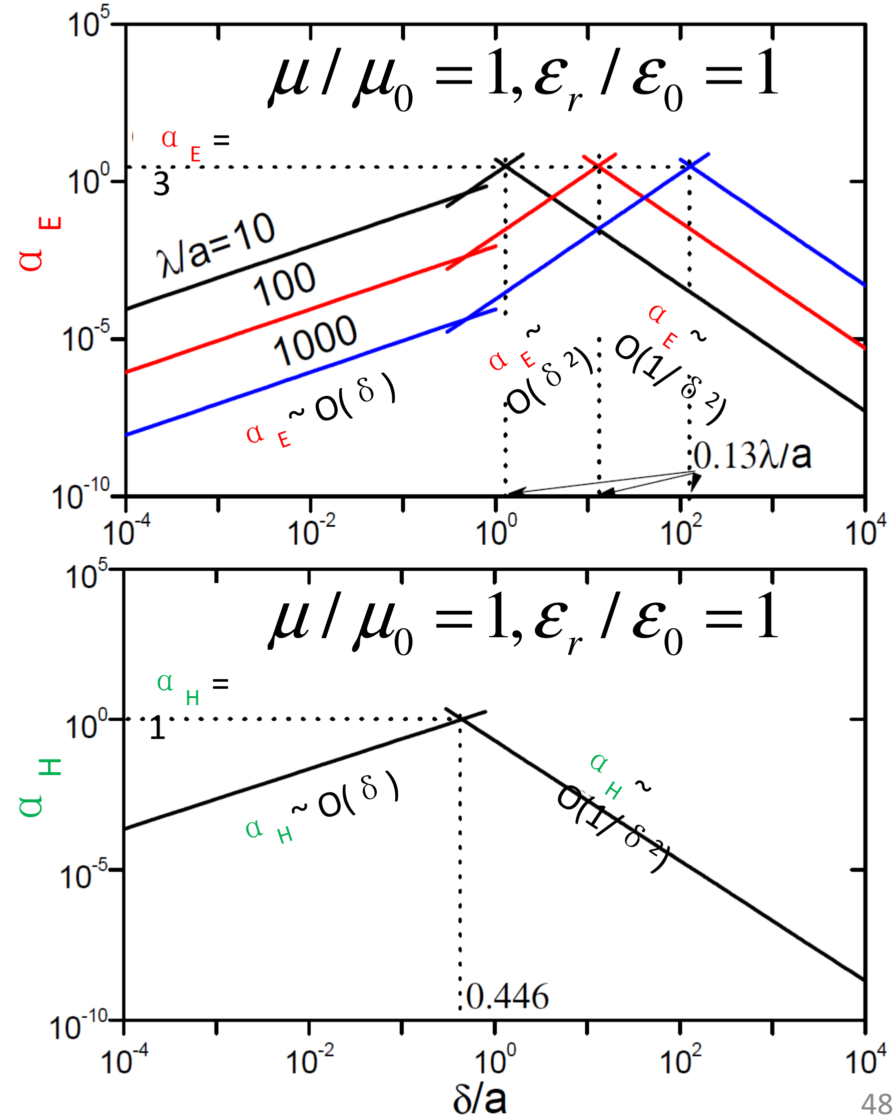
# RF Power absorption due to small hemispherical protrusion on the surface




$$P_E = \alpha_E \omega \left( \frac{1}{2} \varepsilon_0 E_0^2 \right) V$$

$\alpha_E$: Electrical polarizability

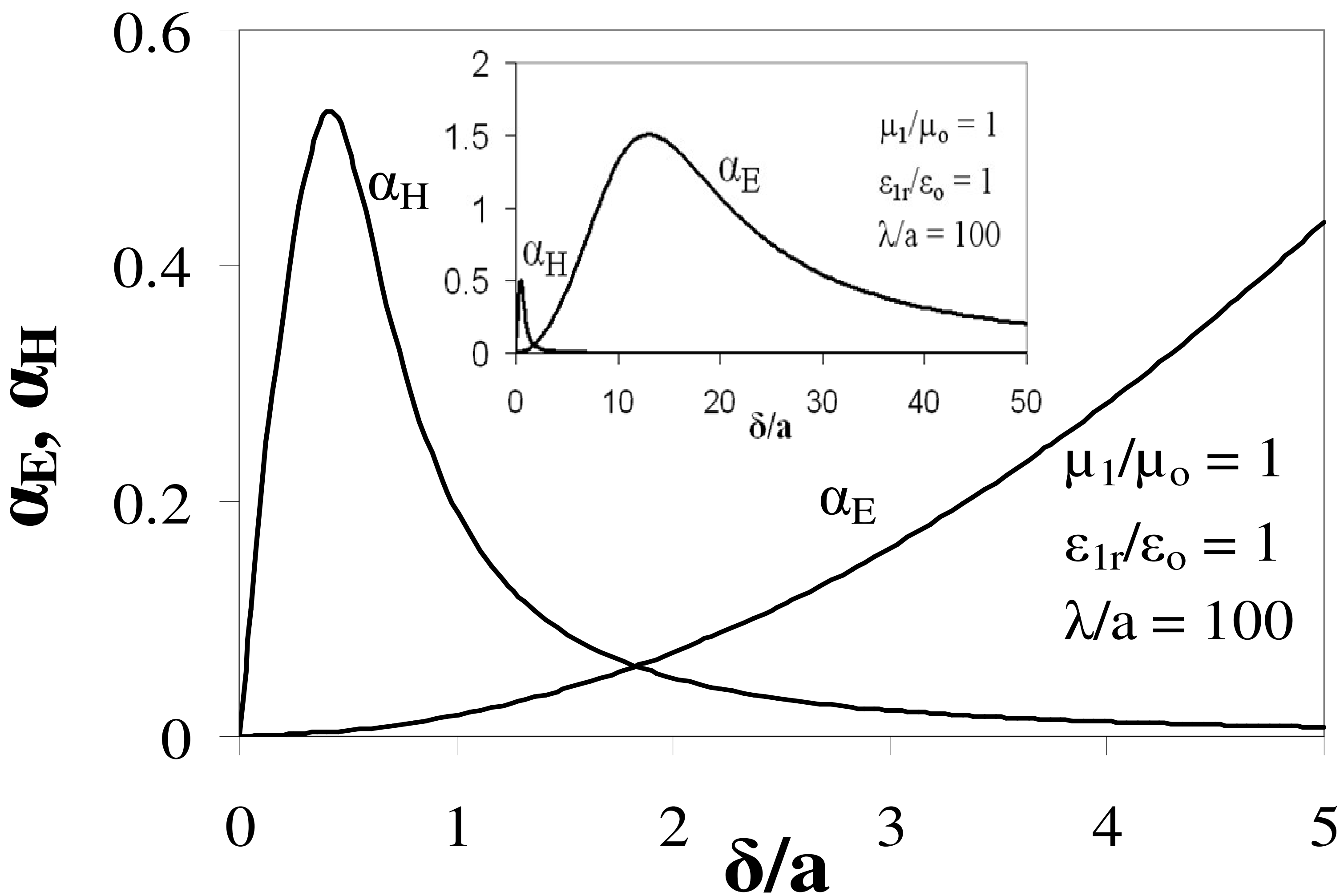
$$V = \frac{2\pi}{3} a^3 = \text{Volume of protrusion}$$

$$P_M = \alpha_H \omega \left( \frac{1}{2} \mu_0 H_0^2 \right) V$$

$\alpha_H$: Magnetic polarizability

$$\delta = \sqrt{2/\omega\mu\sigma} = \text{skin depth}$$

# Comparison of RF electric and magnetic field heating




Heating by **H** greatly exceeds heating by **E** whenever $\delta << a$.
**H** generates an inductive E, which heats conductor ohmically.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A (non-magnetic) metallic protrusion dissipates a lot more magnetic RF energy than the electric RF energy if $\delta \ll a$.

Macroscopically, collections of such lossy particulates may effectively contribute to an Imaginary Part of $\mu$, even though these particulates are NON-magnetic.


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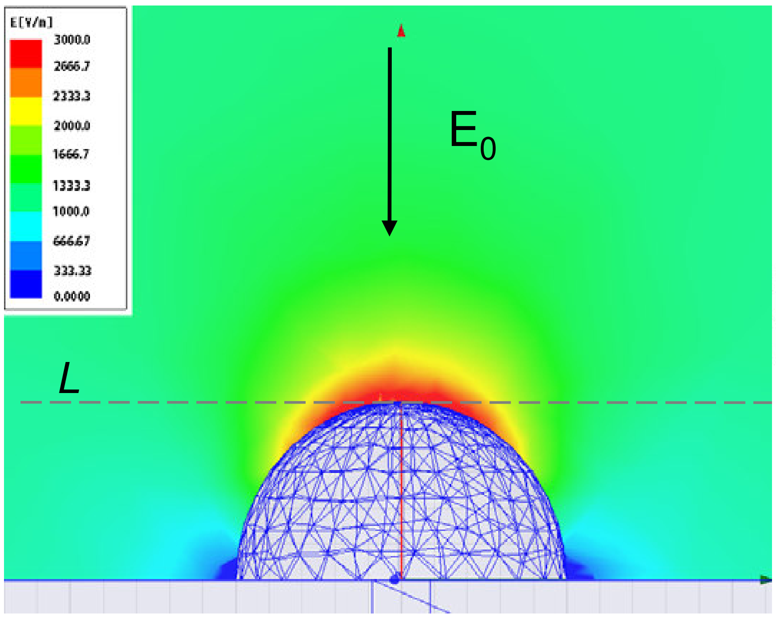
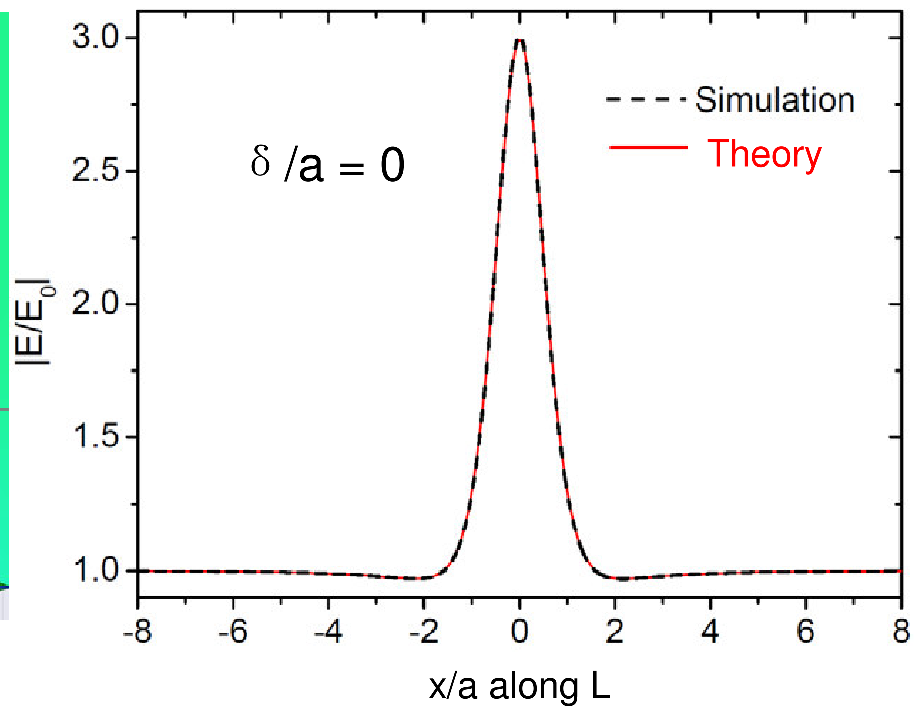
# Electric field enhancement due to hemispherical protrusion on the surface

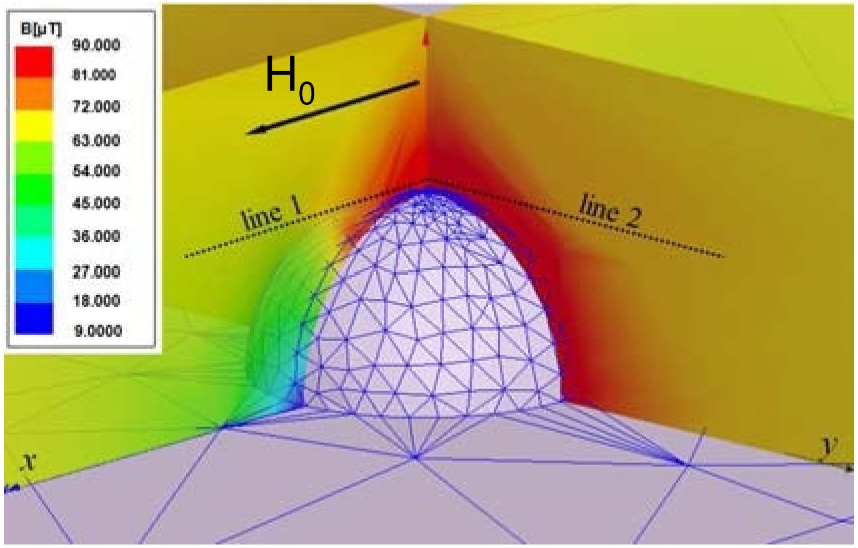


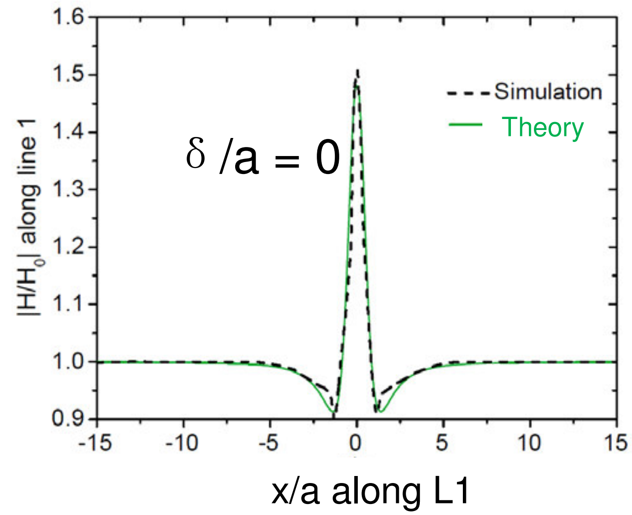




SIMULATION = MAXWELL 3D

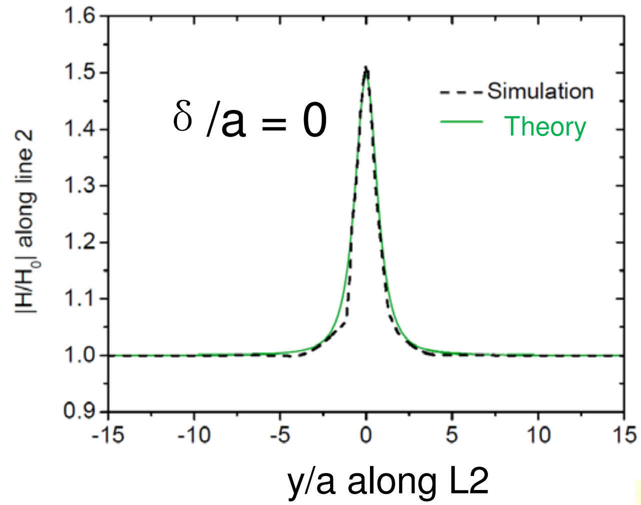
THEORY = Perturbation of Eigenfunction by Particulate

# Magnetic field enhancement due to hemispherical protrusion on the surface




Line 1




Line 2

SIMULATION = MAXWELL 3D

THEORY = Perturbation of Eigenfunction by Particulate

# Conclusions

- The RF absorption by a small hemispherical protrusion is accurately calculated for arbitrary values of ($\varepsilon_r$, $\sigma$ , $\mu$ ).
- A (non-magnetic) metallic protrusion dissipates a lot more magnetic RF energy than the electric RF energy if $\delta << a$.
- RF electric and magnetic field enhancements have been calculated from the perturbed eigenfuctions.
- Since $\alpha_E$ and $\alpha_H$ are computed for all $\omega$, $\sigma$, $\varepsilon$, $\mu$, the enhanced surface resistance may readily be assessed, once the distribution and composition of the surface roughness is postulated.
- Essentially calculated the scattered radiation of an arbitrary incident wave by a protrusion.
- Unsolved: close-by bumps, a dimple, origin of the *HFSS scaling*, and *Microwave Handbook scaling*.

Ref: Zhang, Lau, Gilgenbach, J. Appl. Phys. 105, 114908 (2009)

# Potential Future Works

- Frequency response and RF heating at contacts (a very difficult problem to analyze – huge contrasts in length scales such as resistive skin depths, sharp corners, film thickness; $a \sim \lambda$ regime)
- For bulk contacts, perform statistics on asperity geometries and resistivities using the newly derived analytic scaling laws (application specific)
- Evaluate ohmic heating at thin-film and bulk contacts based on the newly calculated current flow profile (watch out for temperature dependence of electrical resistivity, thermal conductivity, dimensions, and heat loss mechanisms)
- Comparison with measurements of contact resistance (very challenging if there is a huge contrast in dimensions or in resistivity in the contacting members, and if geometry is irregular in experiments)

# 18.325 Topics in Applied Math

Professor C. C. Lin (MIT, Spring 1977)

I Nature of the course

(a) The applied mathematician's approach (Many aspects)
(Two sheets of notes)

I Formulation { what scientific object; what issues

II Solution — Methods — { General concepts; Math. basis
(Applied Math. ≠ Methods of A.M.) (relationship to the logical structure of P.M.)

III Interpretation — Refer to I

IV Implication + Impact

General (specifics) → Specifics (general) → General → Specifics → General → ...

(b) Comparative study of

(i) stellar systems → disk shaped galaxies 

(ii) plasma systems → mirror machine 

Cooperative phenomena (collisionless) — waves associated with long range forces
— instability modes

Equilibrium — waves — instability
(small amplitude — finite amplitude)

# Acknowledgement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Jacob C. Zier

**Faculty:**
Ronald M. Gilgenbach