Reflections on Fifty Years of SDWT

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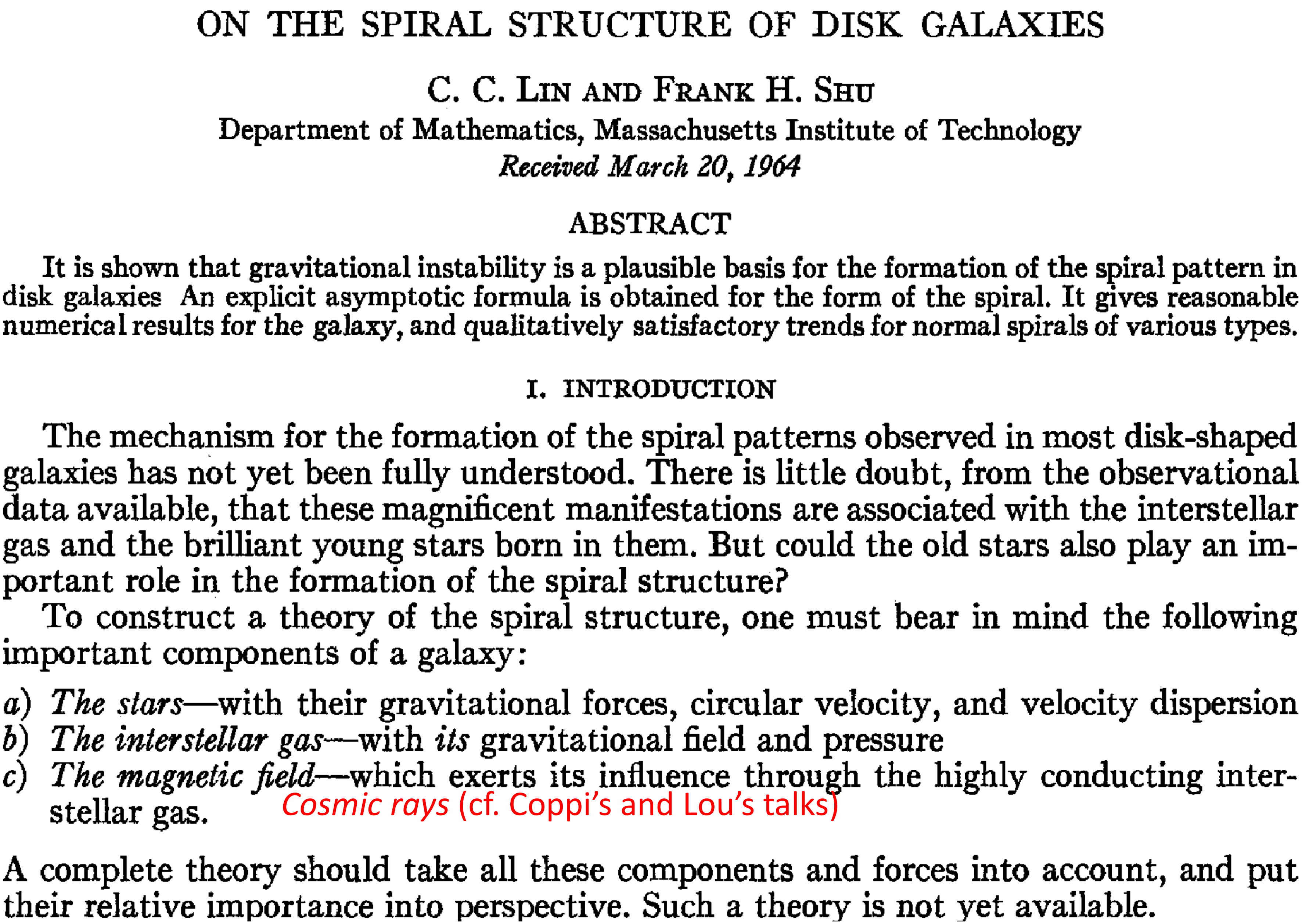
Outline

- Linear and Nonlinear SDWs in Disk Galaxies
- Observational Consequences & Tests
  - Spiral Patterns and Explanation of Hubble Sequence
  - Compression of ISM and Magnetic Field
  - Gas Kinematics
  - Migration of OB Stars and Color Gradients Across Spiral Arms
- Resonantly Forced SDWs and Applications
  - Saturn’s Rings
  - Disk Truncation
  - Planet Migration in Protoplanetary Disks
  - Bar Forcing in Central Regions of Active Galaxies
- Growing Normal Modes
  - SDWs with Negative/Positive Densities of Energy & Angular Momentum
  - Over-reflection at CR by WASER & Swing
  - Feedback from Central Regions
  - Saturation of Growing Stellar Density Waves & the QSSS Hypothesis
  - Observational evidence
- Drawing Connections

Linear & Nonlinear SDWs



ON THE SPIRAL STRUCTURE OF DISK GALAXIES

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Received March 20, 1964

ABSTRACT

It is shown that gravitational instability is a plausible basis for the formation of the spiral pattern in disk galaxies An explicit asymptotic formula is obtained for the form of the spiral. It gives reasonable numerical results for the galaxy, and qualitatively satisfactory trends for normal spirals of various types.

I. INTRODUCTION

The mechanism for the formation of the spiral patterns observed in most disk-shaped galaxies has not yet been fully understood. There is little doubt, from the observational data available, that these magnificent manifestations are associated with the interstellar gas and the brilliant young stars born in them. But could the old stars also play an important role in the formation of the spiral structure?

To construct a theory of the spiral structure, one must bear in mind the following important components of a galaxy:

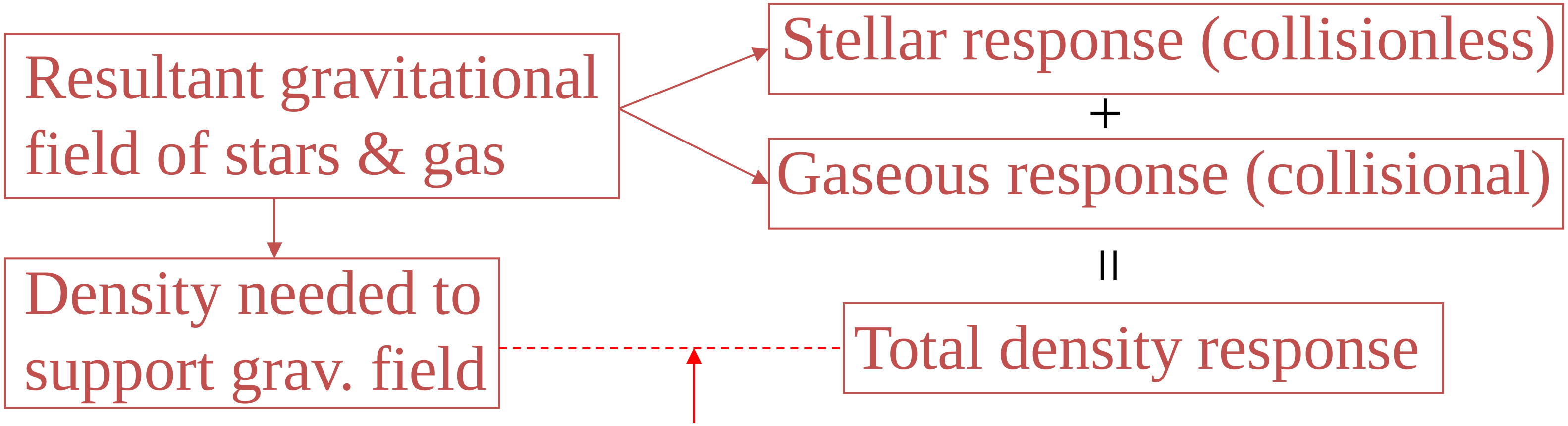
a) *The stars*—with their gravitational forces, circular velocity, and velocity dispersion
b) *The interstellar gas*—with *its* gravitational field and pressure
c) *The magnetic field*—which exerts its influence through the highly conducting interstellar gas.

Cosmic rays (cf. Coppi's and Lou's talks)

A complete theory should take all these components and forces into account, and put their relative importance into perspective. Such a theory is not yet available.

QSSS Hypothesis, Asymptotics (controversial, esp. regarding existence of Long SDWs) Hong

Outline of Density Wave Theory
(Lin & Shu 1966)



Resultant gravitational field of stars & gas

Stellar response (collisionless)

+

Gaseous response (collisional)

||

Total density response

Density needed to support grav. field

This equation determines properties of spiral density waves

- Linear asymptotic dispersion relation (leading/trailing; short/long)

$$|k| = \frac{\kappa^2(1-\nu^2)}{2\pi G\left[\Sigma_{\mathrm{g}} + \Sigma_* \mathcal{F}_\nu(x)\right]}, \quad \nu = \frac{\omega - m\Omega}{\kappa}, \quad x = \frac{k^2\left\langle c_\varpi^2 \right\rangle}{\kappa^2},$$

$$\mathcal{F}_\nu(x) = \frac{1-\nu^2}{x}\left[1 - \frac{\nu\pi}{\sin\nu\pi} \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-x(1+\cos s)} \cos(\nu s)\ ds\right]$$

Hubble Sequence; $-1 \le \nu \le +1$ PR, $\nu = \pm 1, \pm 2, \ldots$ I-OLRs, $\nu = 0$ CR.

ON THE SPIRAL STRUCTURE OF DISK GALAXIES
III. COMPARISON WITH OBSERVATIONS*

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ceived June 21, 1968; revised August 16, 1968

ABSTRACT

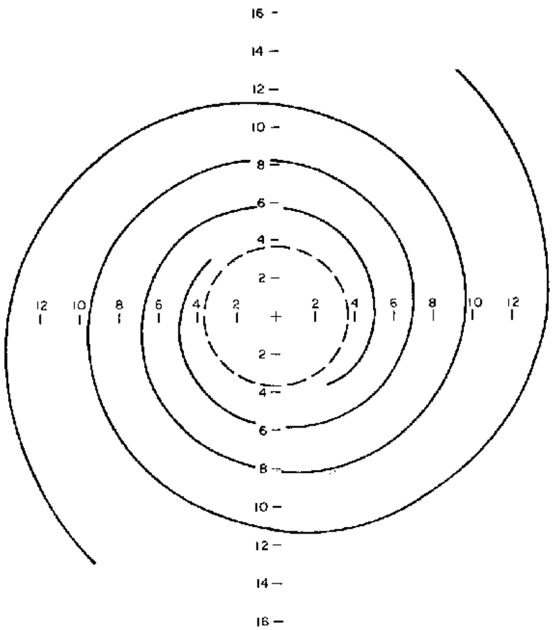


FIG. 3.—Spiral patterns at $\Omega_p = 11$ km sec^{-1} kpc^{-1}

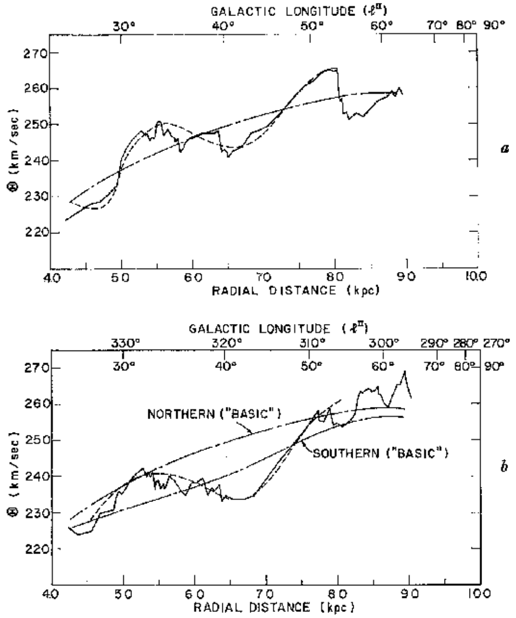


FIG. 5.—*a*, Northern Hemisphere. *b*, Southern Hemisphere. Rotation curves. Observation (Kerr 1964)

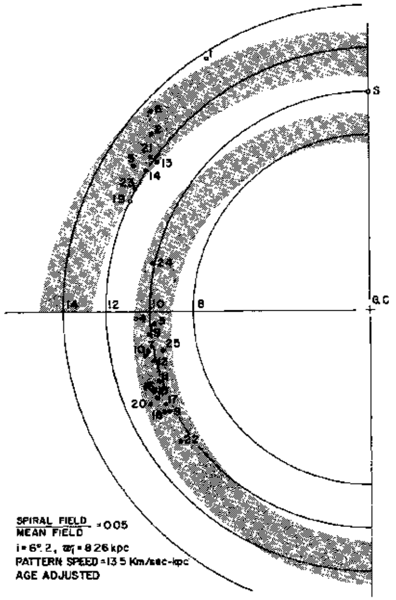


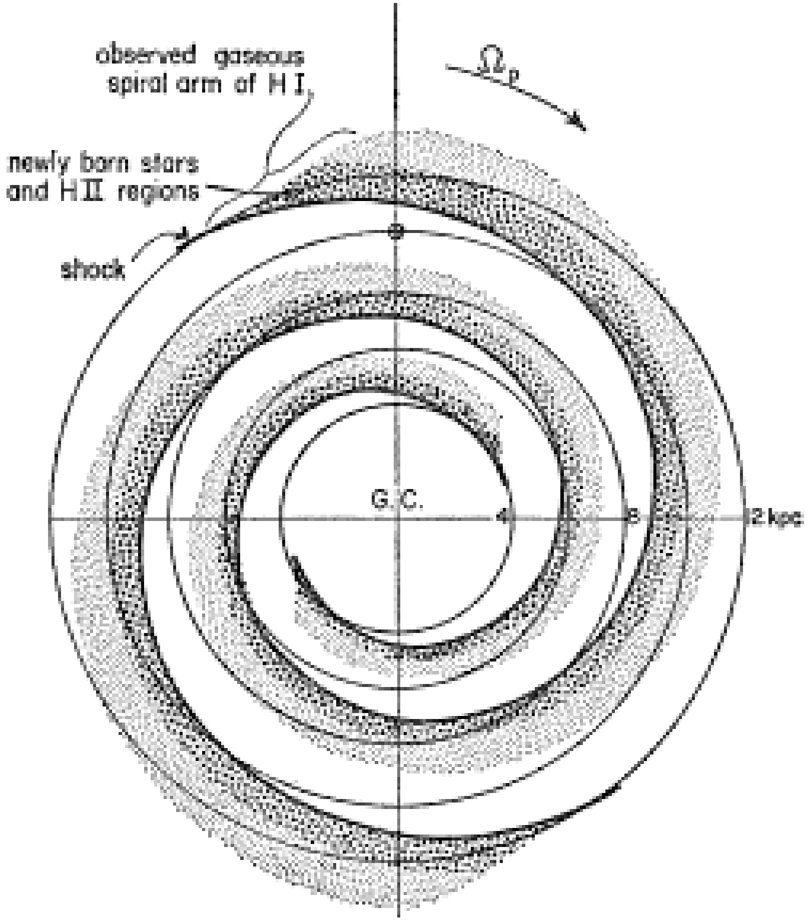
FIG. 7.—Places of formation of stars as determined with the inclusion of a spiral gravitational field travelling at a pattern speed of 13.5 km sec^{-1} kpc^{-1} and having an intensity equal to 5 per cent of that of the symmetrical gravitational field.

Trailing short-wave
$Q_* = 1$, Ω_p adjustable
Ω_p underestimated
- DM halo (Adams; cf. Shatsky)
- thickness correction
  (Griv) Thick/Thin disks
  (Effect of form/evol Elmegreens)

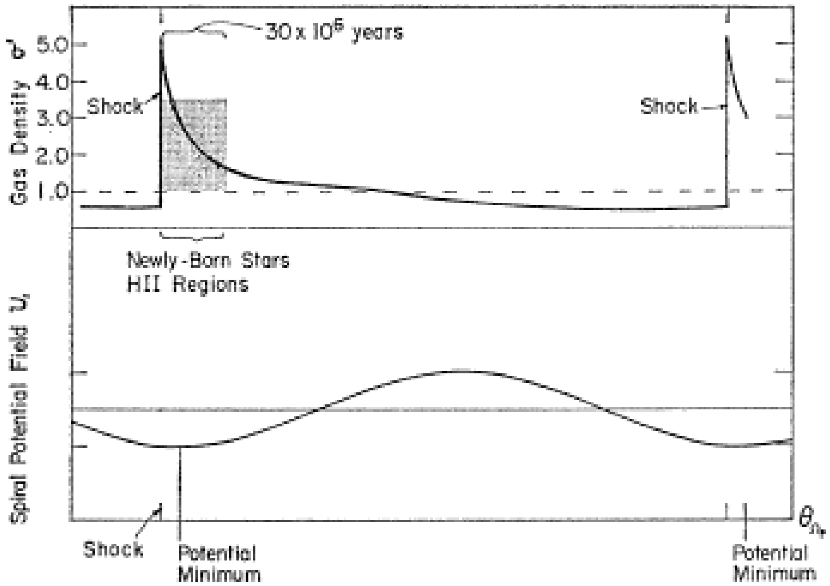
Wiggles on 21 cm
rotation curve, *F* = 5%

Birthplace of OB stars
with given Ω_p, *F* = 5%
Nice update: Grosbol

Nonlinear TASS: Roberts (1969)



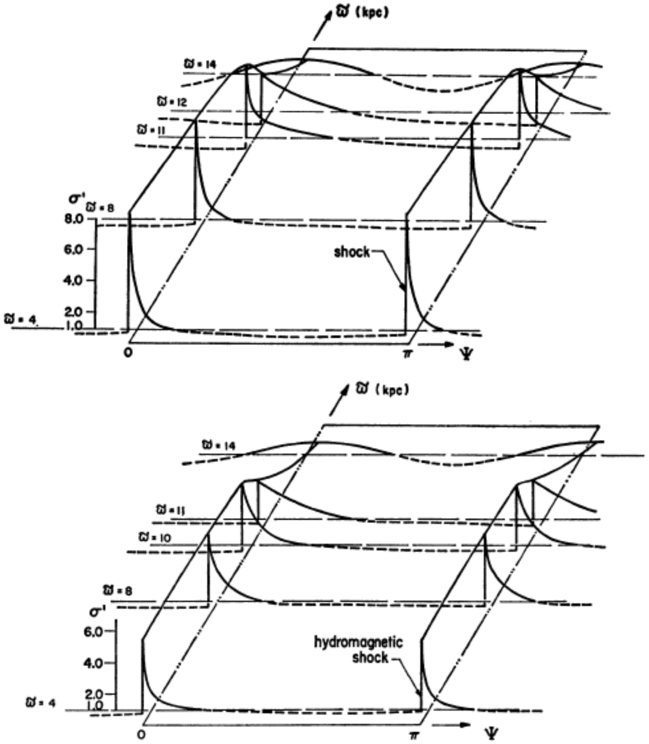
Roberts (1969)



Out-of-phase gaseous response damps stellar spiral density-wave (Kalnajs 1972). Presence of shockwave guarantees non-closure of streamlines (accretion inside CR) & helps to saturate growth of stellar density wave (Roberts & Shu 1972).

Observational Tests

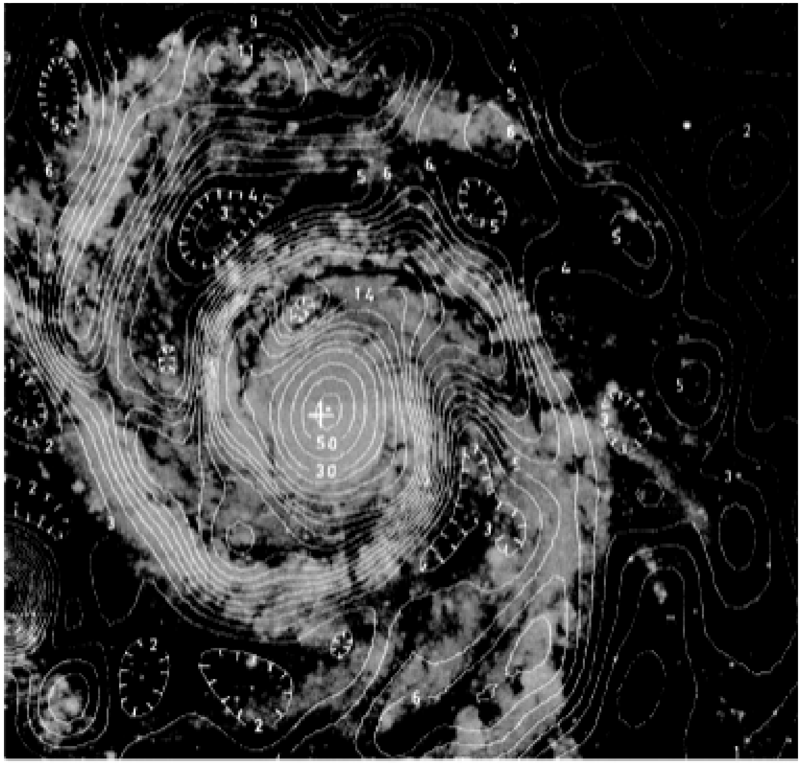
Shock Compression of Magnetic Field of ISM



$B_0 = 0$

$B_0 = 2\ \mu G$

Roberts & Yuan (1970)



Mathewson, van der Kruit, Brouw (1972)

But synchrotron emission $\propto B^{2.7}$ if 1-D compression. Inflation by CR e's?

Kinematic Signatures in H I and CO

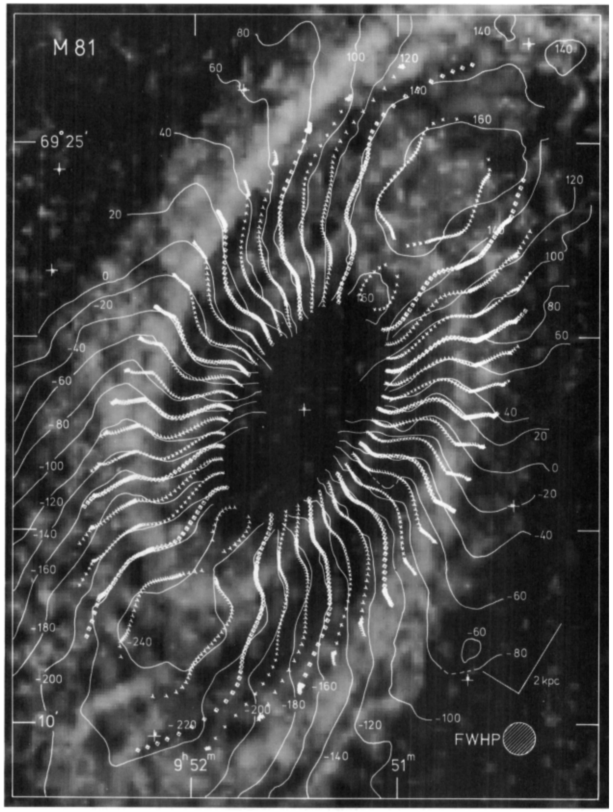
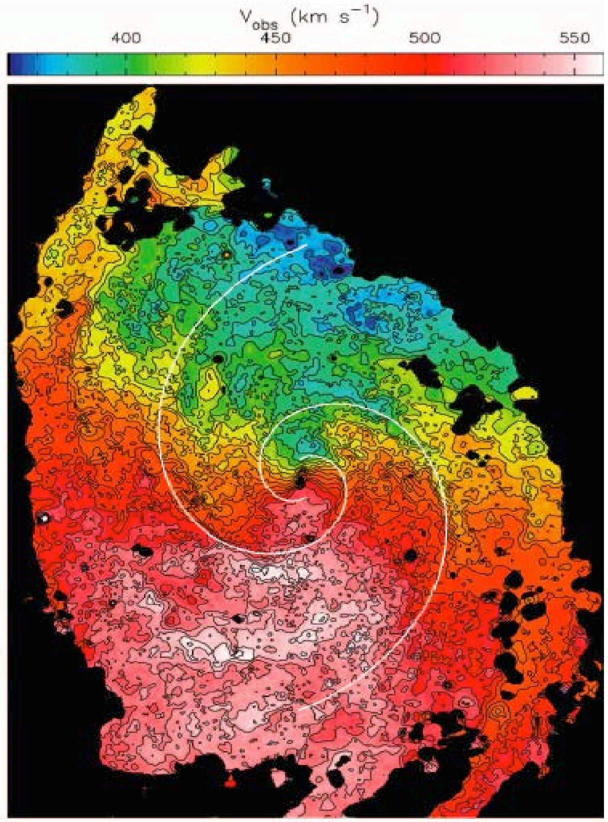


Fig. 5. The radial-velocity field of the final model (symbols) together with the observed velocity field (full and dashed lines) at an angular resolution of 50", superimposed on a radiograph of the density distribution of the atomic hydrogen at 25" resolution. See also the caption of Fig. 4

Visser (1980); Wang (this symp)



Shetty et al. (2007)

Fortuitous match of theoretical uncertainty of ISM physics & angular res of observations (Allen). Truth in sci arrived at by successive approx.

Relationship Between Atomic and Molecular Gas?

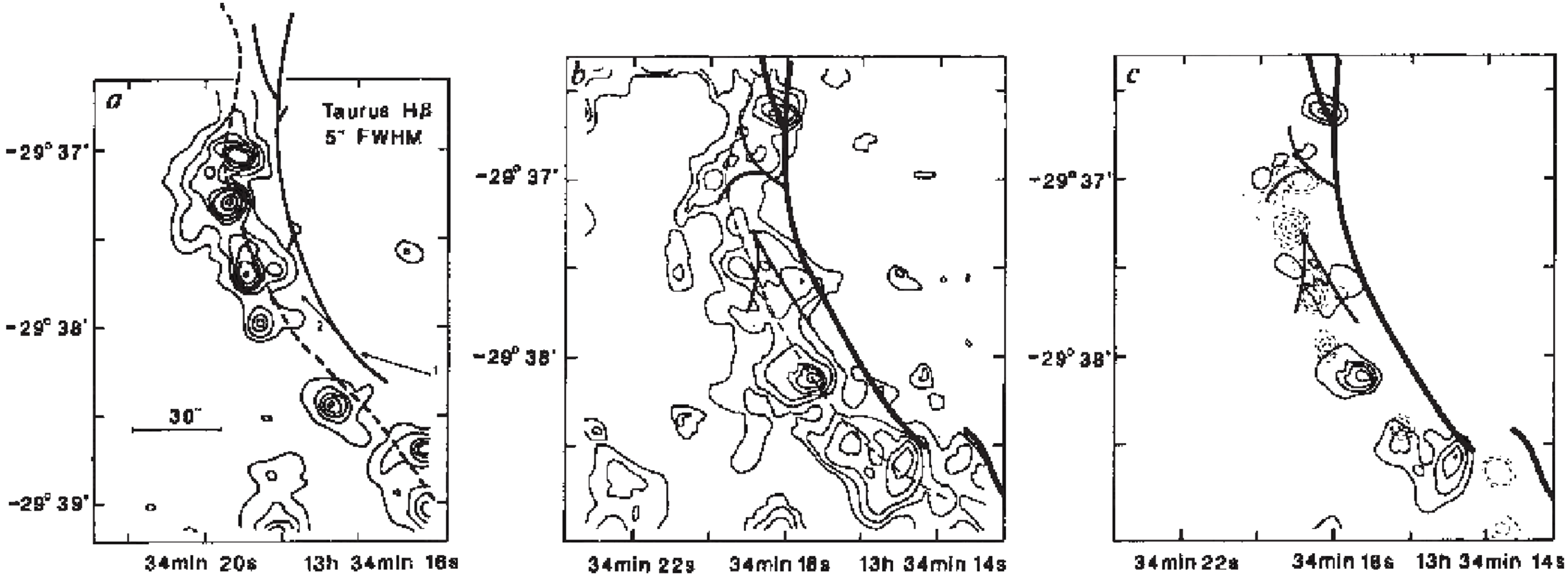
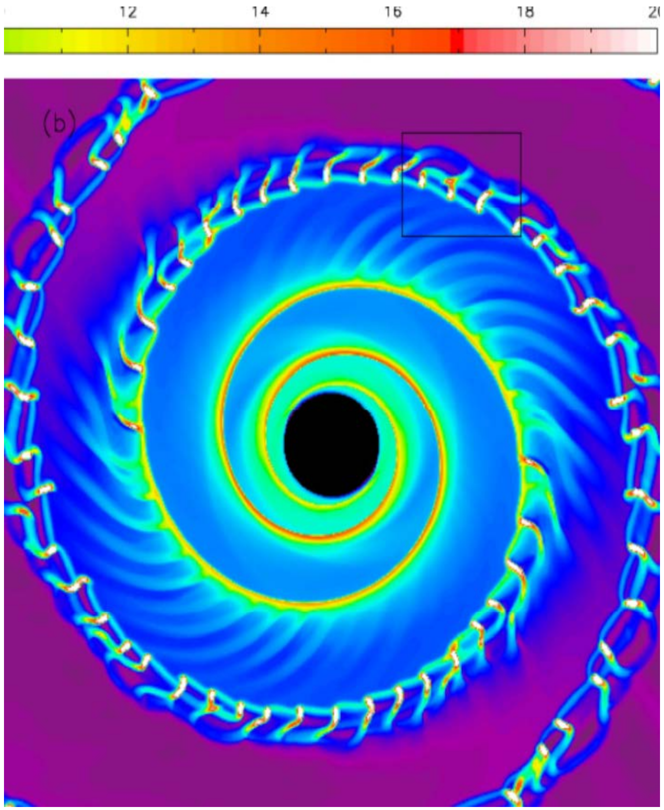


Fig. 2 *a*, The distribution of dust (thick solid line), H β emission (contours, 5 arc s resolution), and the ridge line of the H I from the contours in Fig. 1 (dashed line) are shown here for a section of the inner eastern spiral arm in M83. The horizontal bar at the lower left indicates 30 arc s, corresponding to 530 pc or 1,280 pc along the major axis of M83 for assumed distances of 3.7 and 8.9 Mpc, respectively. The arrows marked 1 and 2 indicate the pre- and post-shock gas flow. *b*, Contours of surface density of atomic hydrogen at 10 arc s resolution, along with the H I ridge line (dashed) from the 25 arc s map of Fig. 1. *c*, Peaks of H I (solid contours), H β (dashed contours) and dust lanes in the eastern arm of M83.

In eastern arm of M83, H I peaks with Hα, not at dust lanes where H_2 (and CO), peaks. (Allen, Atherton, Tilanus 1986). Atomic hydrogen is dissociation product of, not precursor to, molecular hydrogen.

Parasitic Feathering Instability



Elmegreen (1980) Lubow, Cowie, Balbus (1986), Balbus (1988), Kim & Ostriker (2006) Shetty & Ostriker (2006), Lee & Shu (2012)

NASA and The Hubble Heritage Team (STScI/AURA)
Hubble Space Telescope WFPC2 • STScI-PRC01-07

Expected Color Gradients from Triggering of OB Star Formation

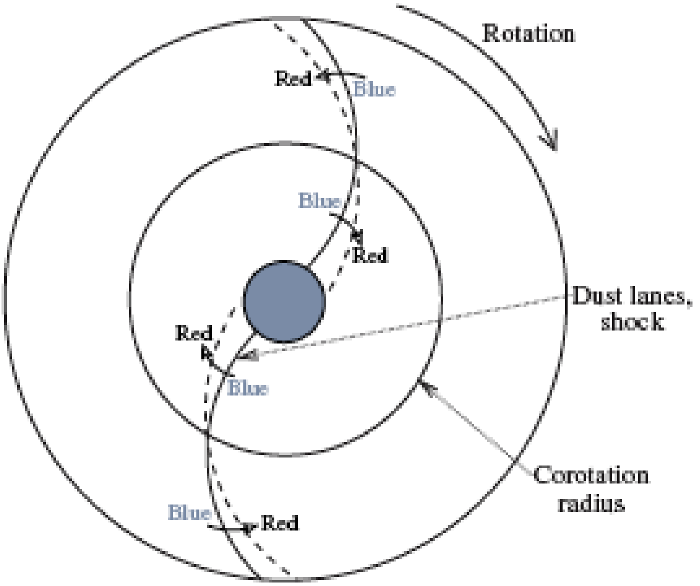
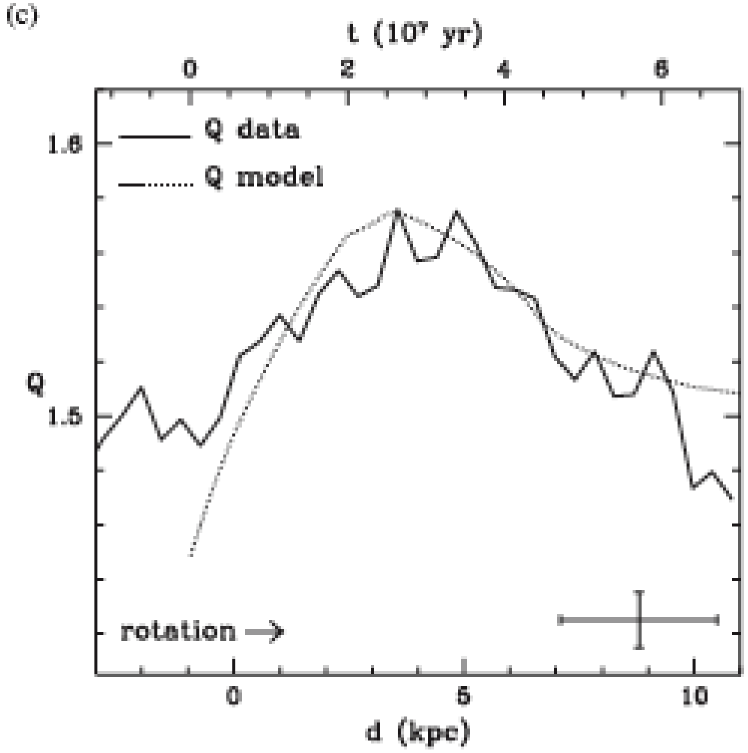


Figure 1. Stellar age gradients across the spiral arms are indicated by arrows that go from blue to red. The azimuthal age gradients are produced by stars born in the spiral shock, where the shocked interstellar medium forms a dust lane, that later drift away as they age. The direction of the gradients changes at the corotation radius, R_{CR}. Inside this radius, the disk material overtakes the spiral wave, and beyond it the spiral wave catches up with the material (see also Figure 1 in Puerari & Dottori (1997)).



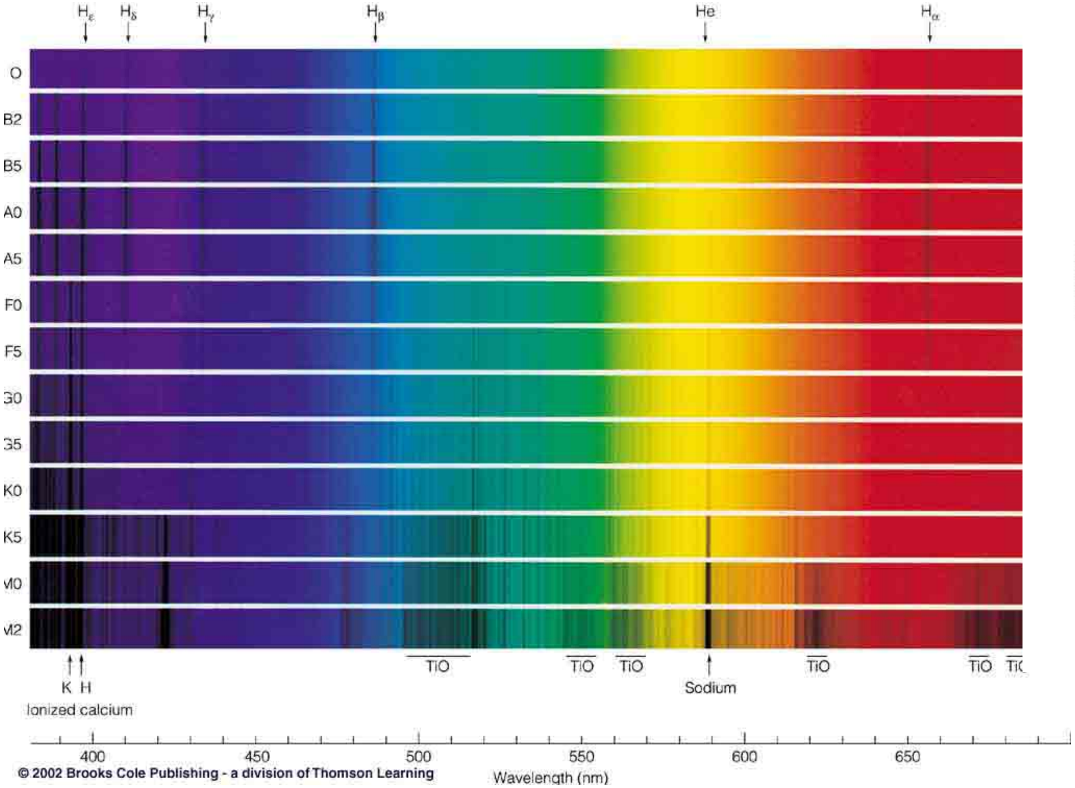
10 out of 13 SA and SAB galaxies show predicted color gradients

Martinez-Garcia, Gonzalez-Lopezlira, Bruzual-A (2009)

Key to succsess: (1) Avoid large H II regions. (2) Dust-independent color index.

If shock waves cause SF at 3-6 km/s or 100 km/s, why not at 30 km/s?

O Stars & Recombination Cascade in Ionization Bounded H II Regions (Stromgren)



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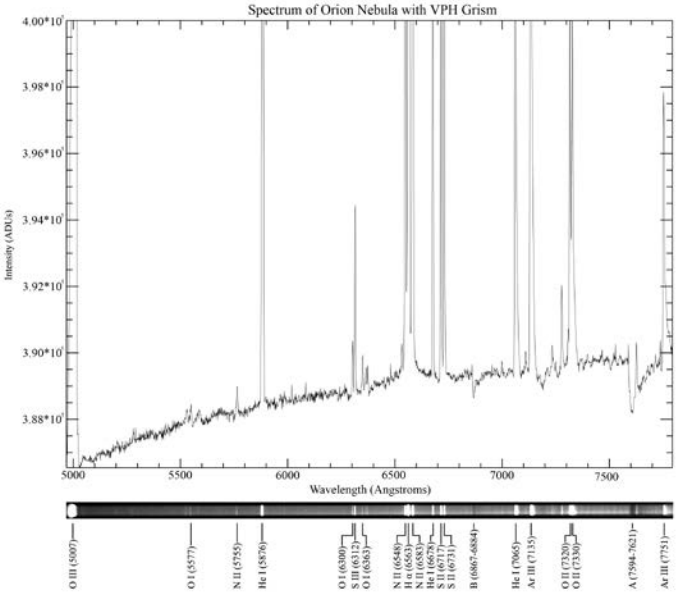
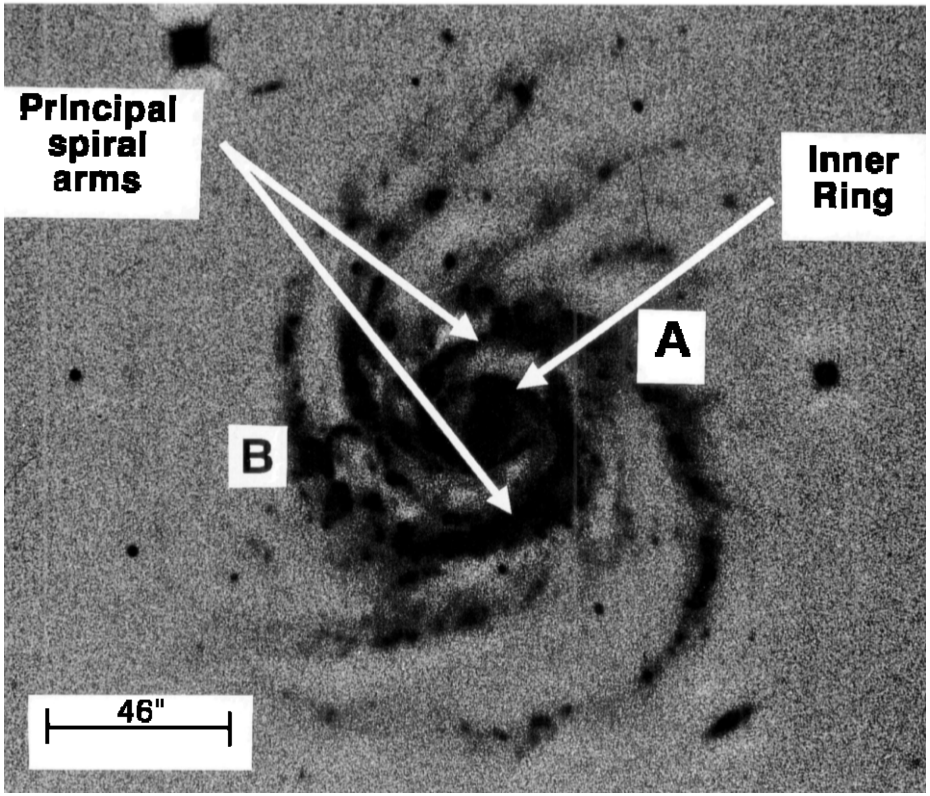
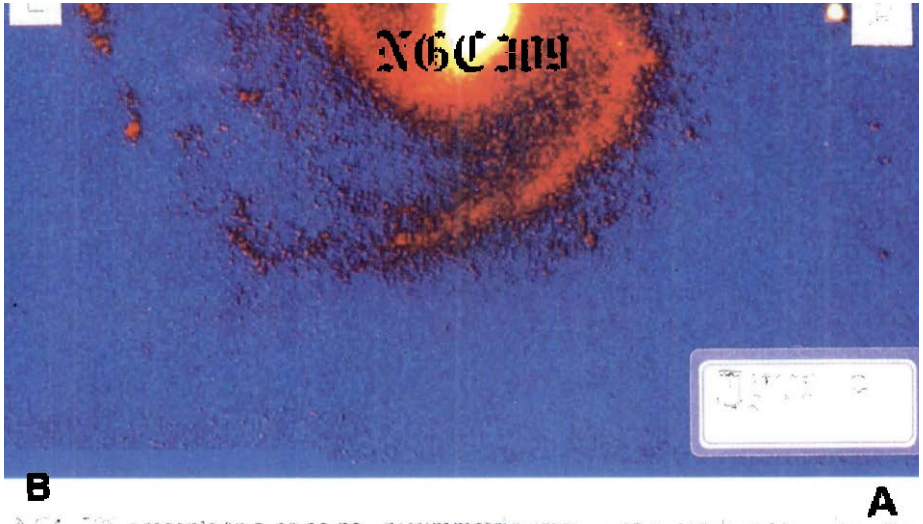


Background Disk Stars
Block et al. (1994)



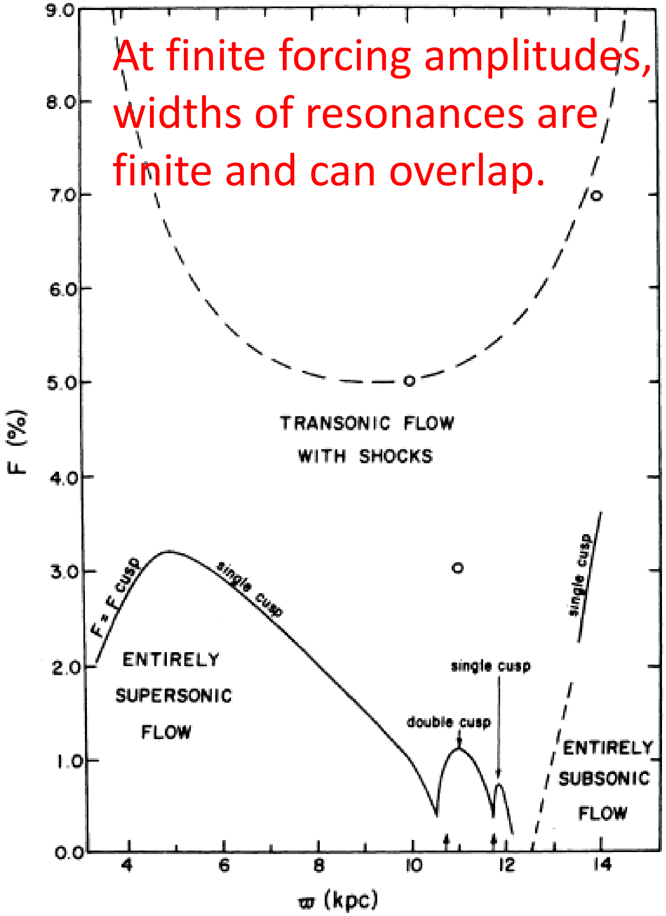


NGC 309 in blue light
Young stars born from ISM

NGC 309 ininfrared light
Old stars in stellar disk

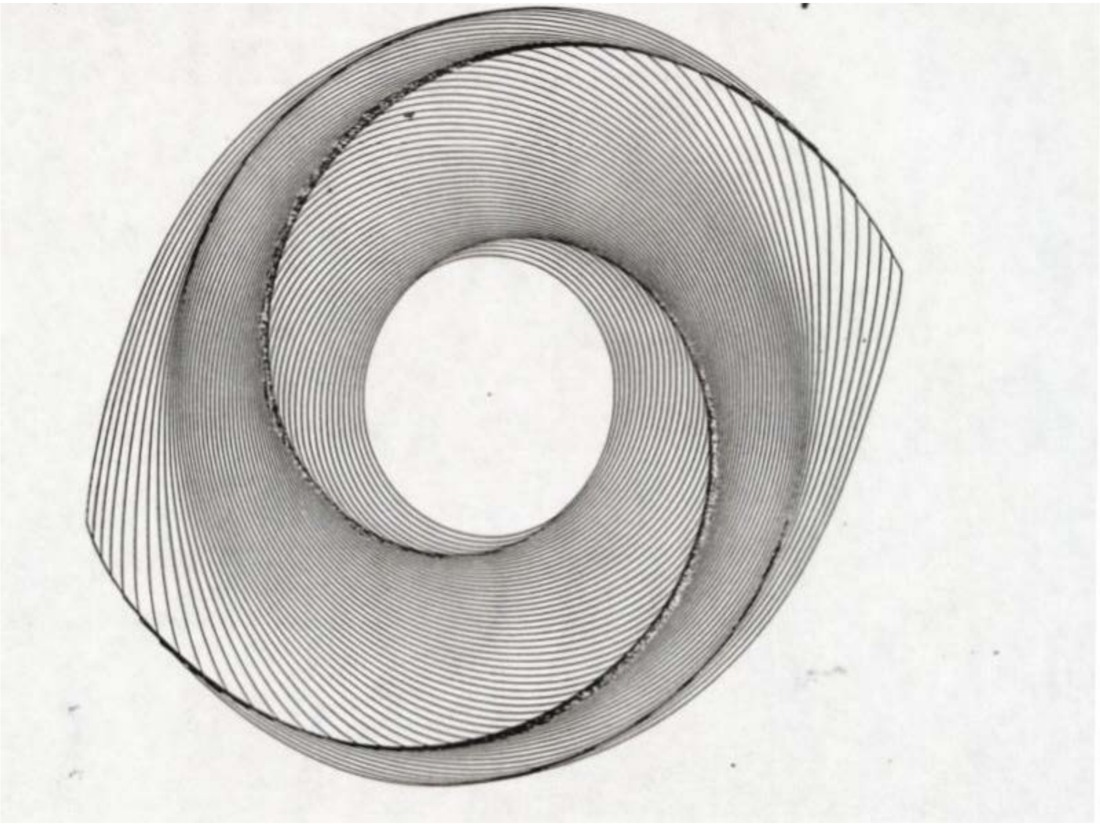
Nonlinearity of gaseous response leads to extra arms/substructure

Subharmonic Resonances & Chaos



At finite forcing amplitudes, widths of resonances are finite and can overlap.

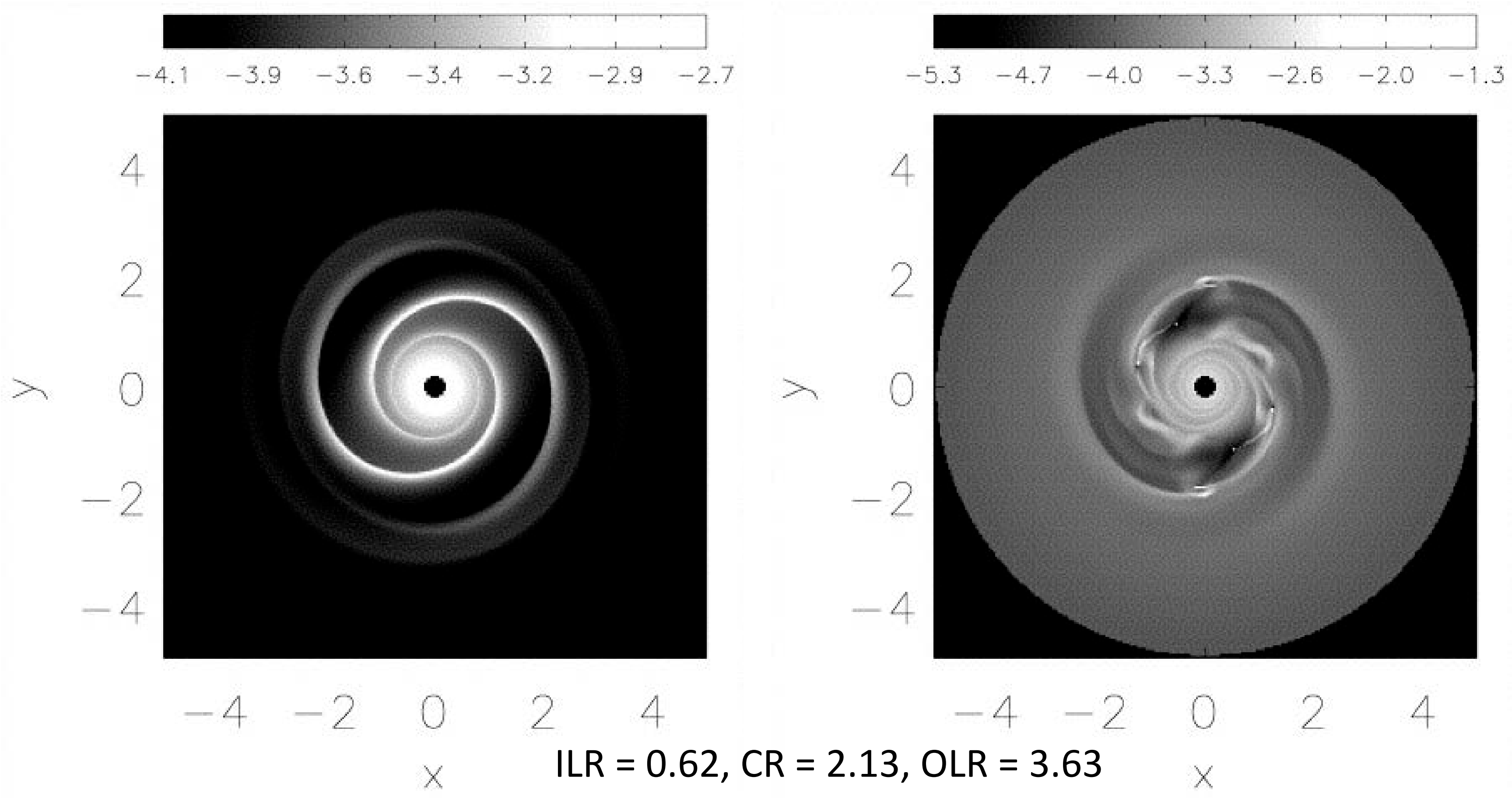
Shu, Milione, Roberts (1972)



Visser (1980)

- Overapping resonances only mechanism known for chaos in celestial mechanics
- Free energy of shear is classical cause of turbulence for flow in pipes, over airfoils, etc (capturable by analysis Wada & Koda 2004, but not by Lee & Shu 2012; 3D effects?).

Chakrabarti, Laughlin, Shu (2002)
$Q_g = 2.48$, f=0.1, F=5%



ILR = 0.62, CR = 2.13, OLR = 3.63

576 Myr

2870 Myr

Resonantly Forced SDWs

Stellar Density Wake of Point Mass at CR Julian & Toomre (1966)

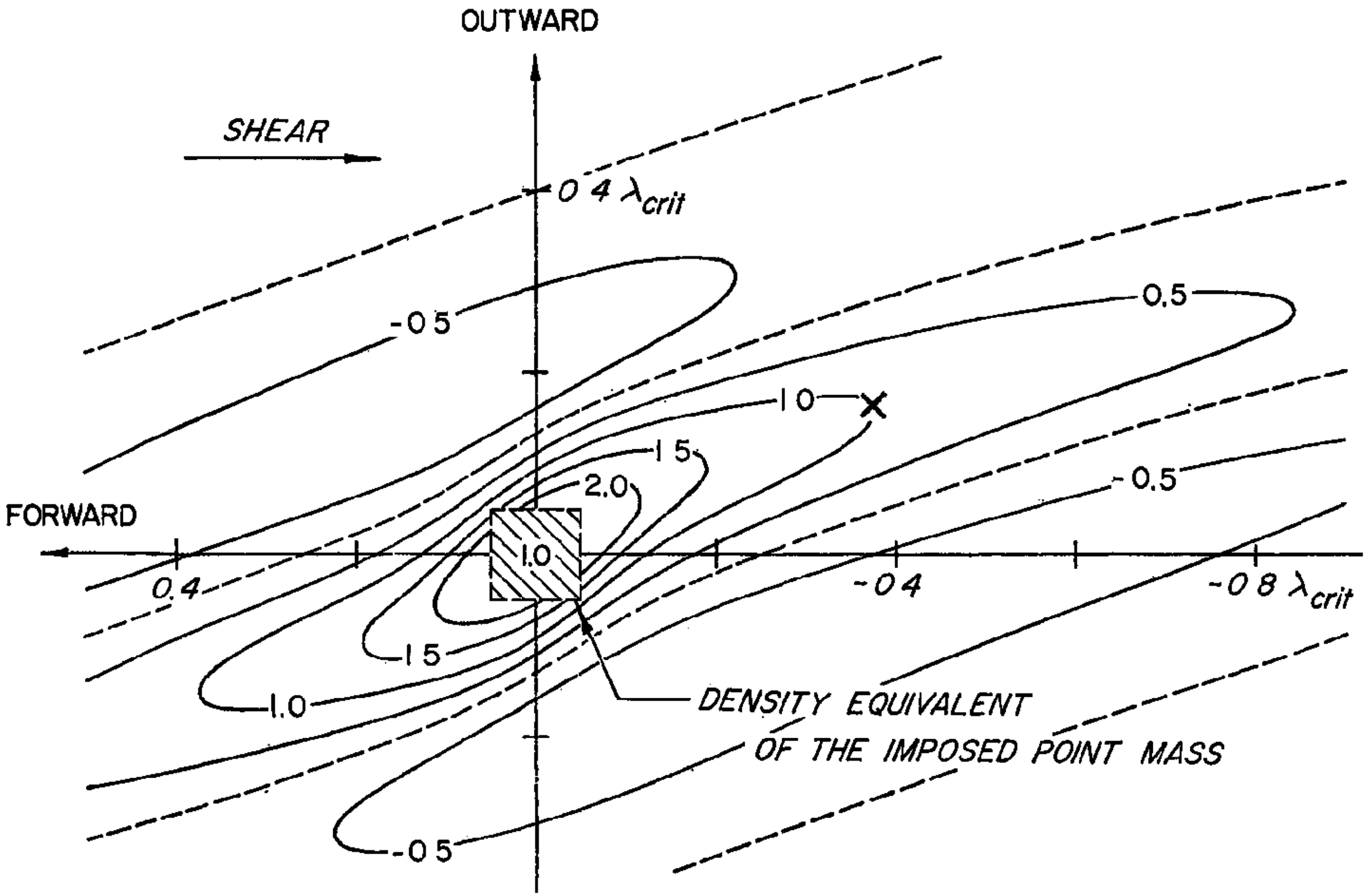
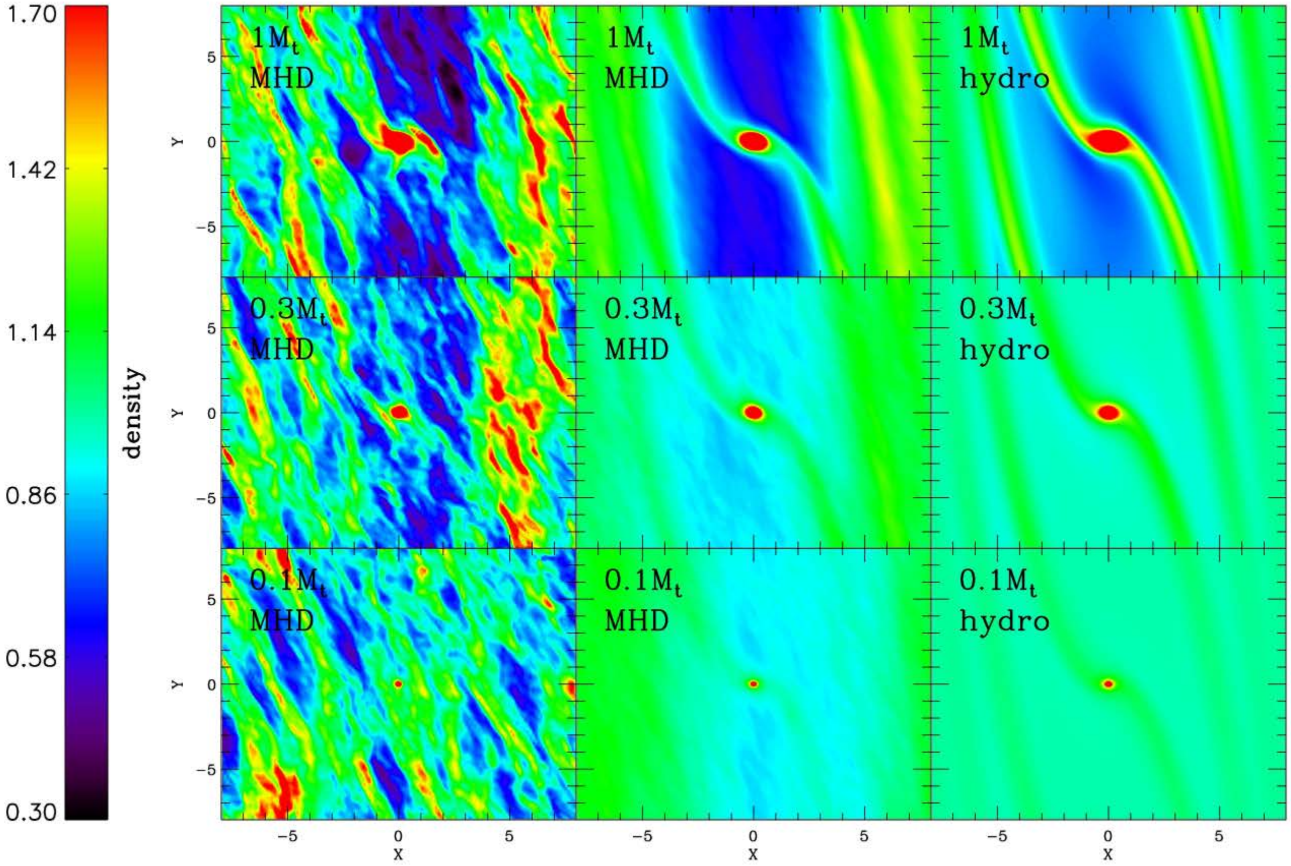


FIG 7 —Steady response densities $\mu'(x,y,t \to \infty)$ for the case $Q = 1.4$, $\Gamma = 1$ (i.e., $V(r) = \text{const}$).

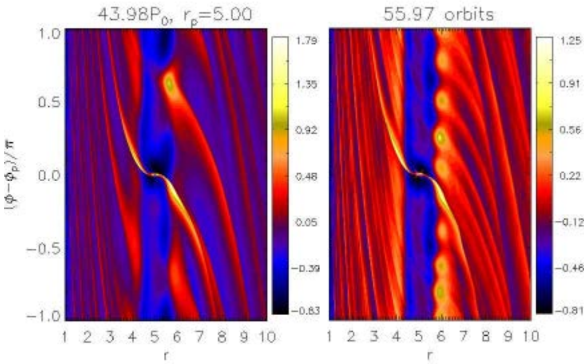
Not responsible for feathers (co-rotation with point mass gives winding dilemma. But seen in Saturn's rings! (Dones)

Gap Opening in MRI & Alpha Disks

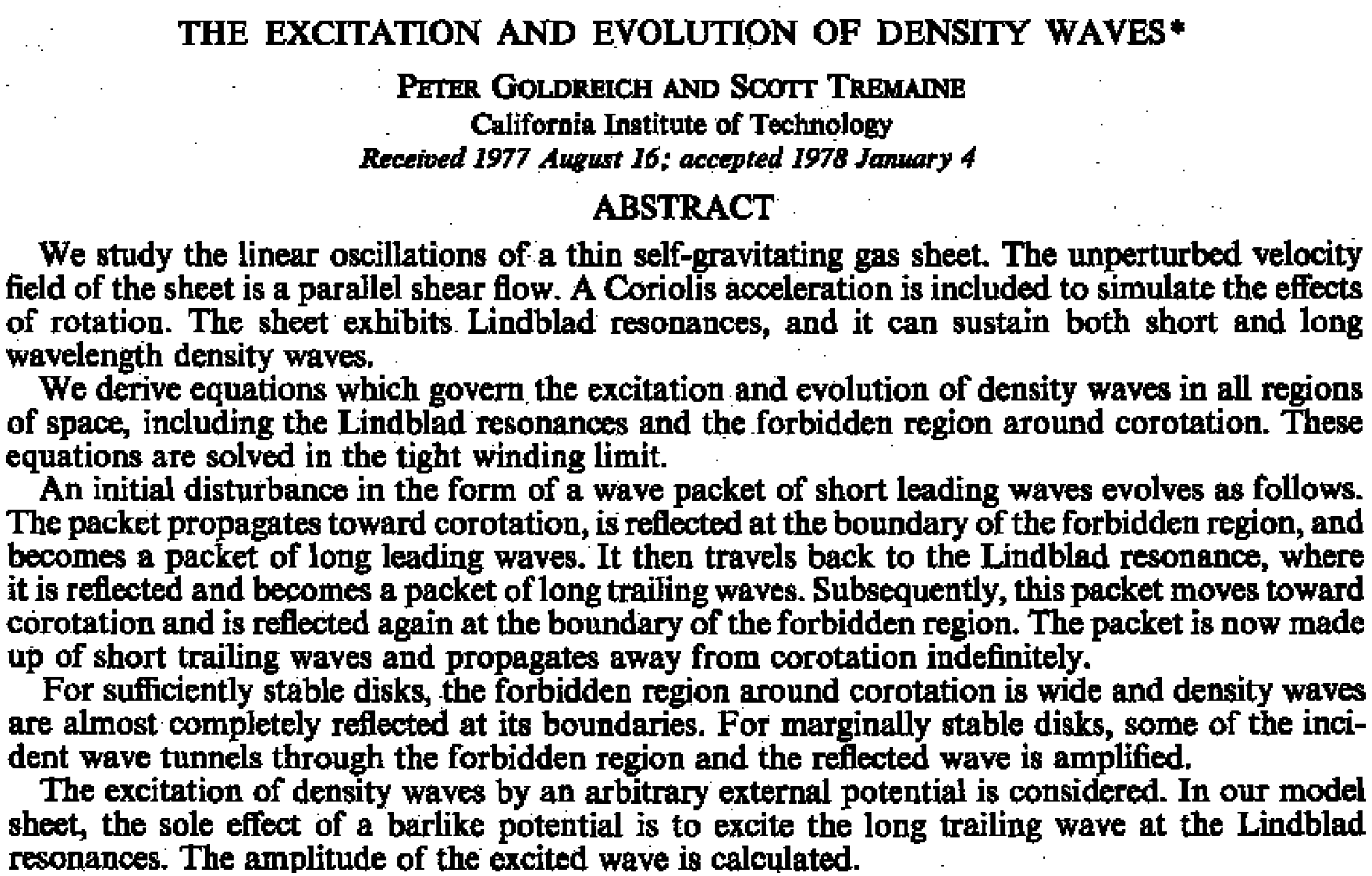


Zhu (this Symp)

Vortex Street Induced Near CR in Inviscid Self-Gravitating Sheets



M. Lin & Papaloizou(2011)



THE EXCITATION AND EVOLUTION OF DENSITY WAVES*

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Received 1977 August 16; accepted 1978 January 4

ABSTRACT

We study the linear oscillations of a thin self-gravitating gas sheet. The unperturbed velocity field of the sheet is a parallel shear flow. A Coriolis acceleration is included to simulate the effects of rotation. The sheet exhibits Lindblad resonances, and it can sustain both short and long wavelength density waves.

We derive equations which govern the excitation and evolution of density waves in all regions of space, including the Lindblad resonances and the forbidden region around corotation. These equations are solved in the tight winding limit.

An initial disturbance in the form of a wave packet of short leading waves evolves as follows. The packet propagates toward corotation, is reflected at the boundary of the forbidden region, and becomes a packet of long leading waves. It then travels back to the Lindblad resonance, where it is reflected and becomes a packet of long trailing waves. Subsequently, this packet moves toward corotation and is reflected again at the boundary of the forbidden region. The packet is now made up of short trailing waves and propagates away from corotation indefinitely.

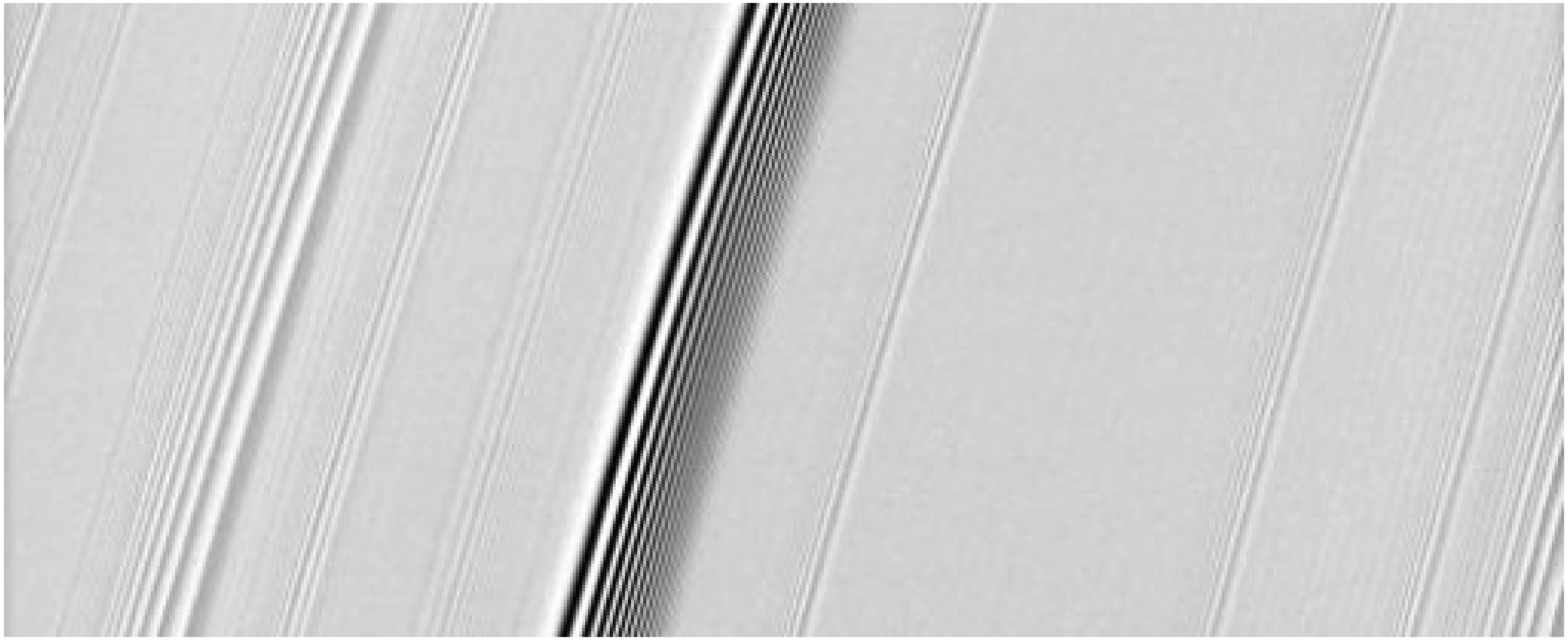
For sufficiently stable disks, the forbidden region around corotation is wide and density waves are almost completely reflected at its boundaries. For marginally stable disks, some of the incident wave tunnels through the forbidden region and the reflected wave is amplified.

The excitation of density waves by an arbitrary external potential is considered. In our model sheet, the sole effect of a barlike potential is to excite the long trailing wave at the Lindblad resonances. The amplitude of the excited wave is calculated.

SDW coupling to mass points and/or bars at LRs depends on existence of *long* waves.
Physical relationship to Tagger's branch cut from $k = im$ (cf. papers on Saturn's rings).

Long SDWs and BWs in Saturn's Rings

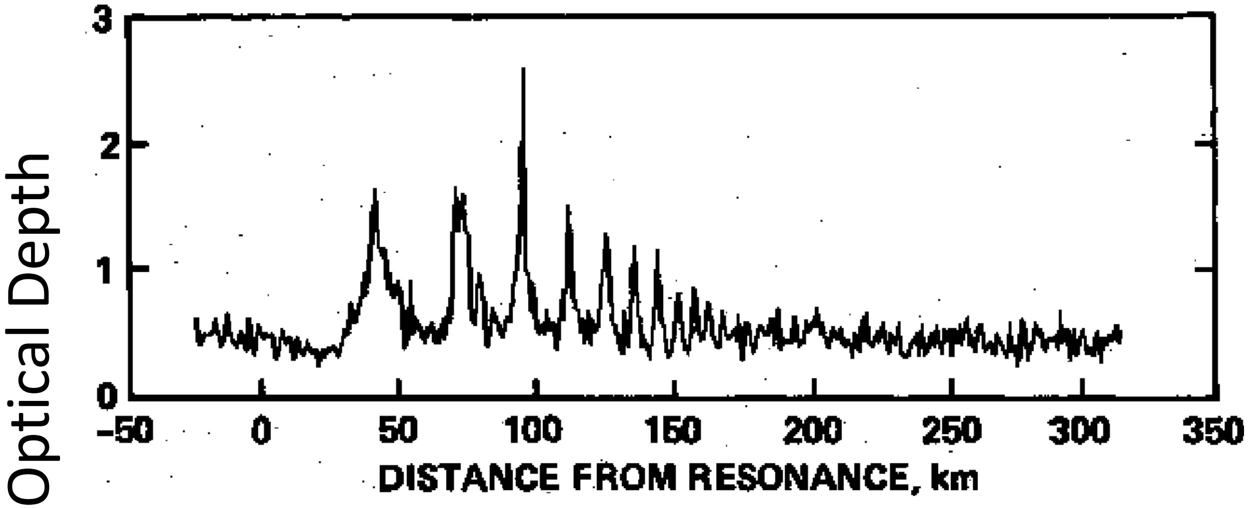
First pair discovered & explained: Shu, Lissauer, Cuzzi (1983)

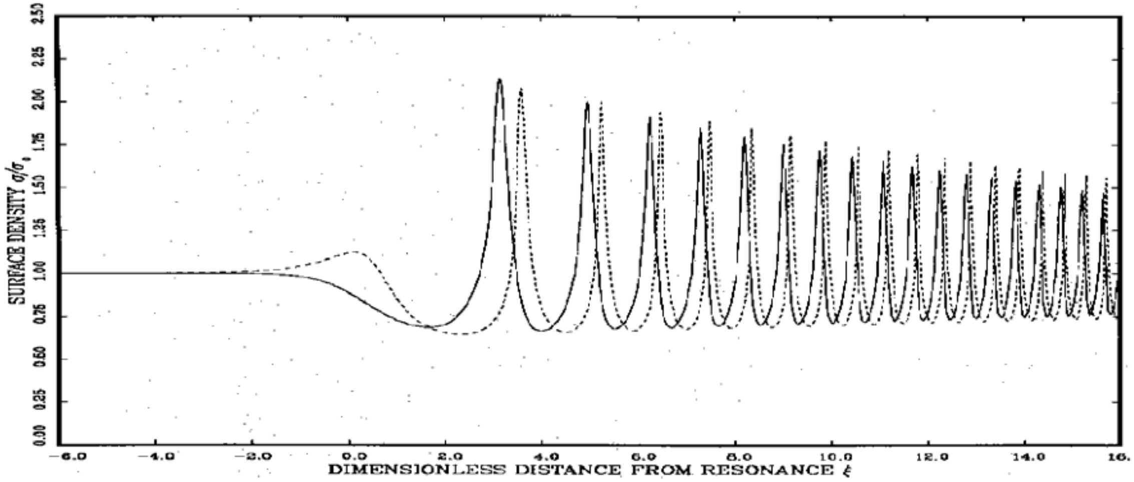


NASA: Cassini

Damping of Nonlinear SDWs by Inelastic Collisions

Mimas 5:3 SDWs





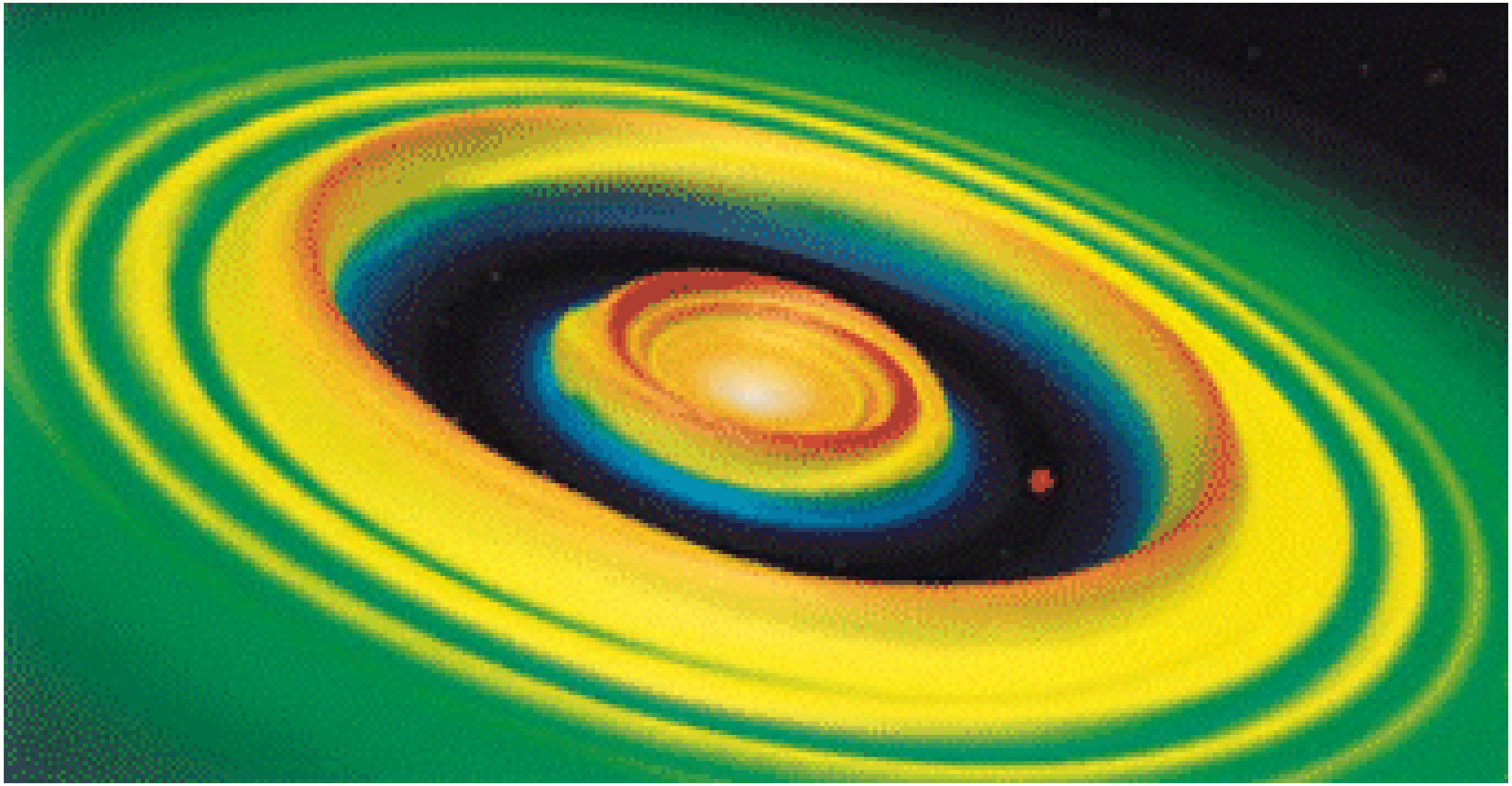
Shu, Dones, Lissauer, Yuan, Cuzzi (1985)

Disk truncation at stronger resonance (e.g., Cassini Division at Mimas 2:1): Resolution of Goldreich-Tremaine paradox: Reversal of viscous torque at large forcing amplitude.

Multiton? (Constrained dispersion of Fourier components balances nonlinear steepening?)

Gap Opening in Protoplanetary Disk and Planet Migration

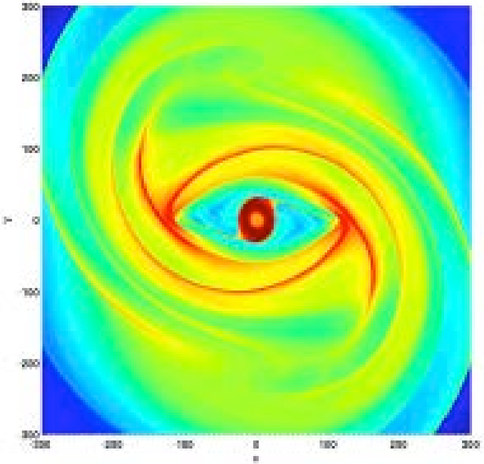
Lin & Papaloizou (1979), Papaloizou/Lin/Zhu (this symp)

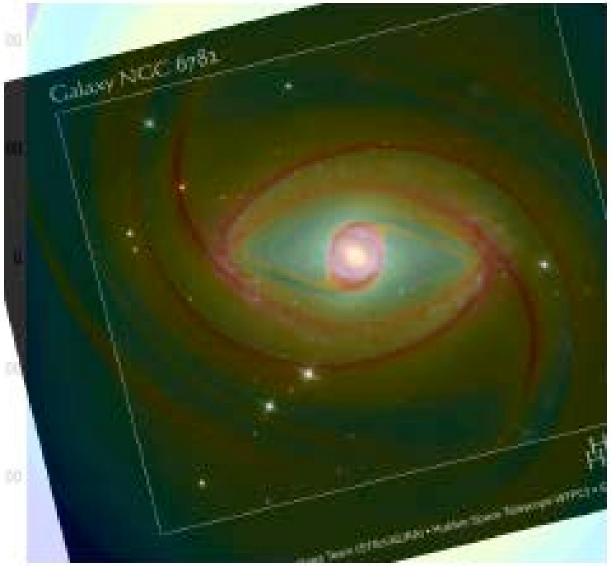


Geoff Bryden

- Impulse approx = sum of resonances for small perturber.
- Large perturber: much richer, vortex street, edge modes, planets in stacked res

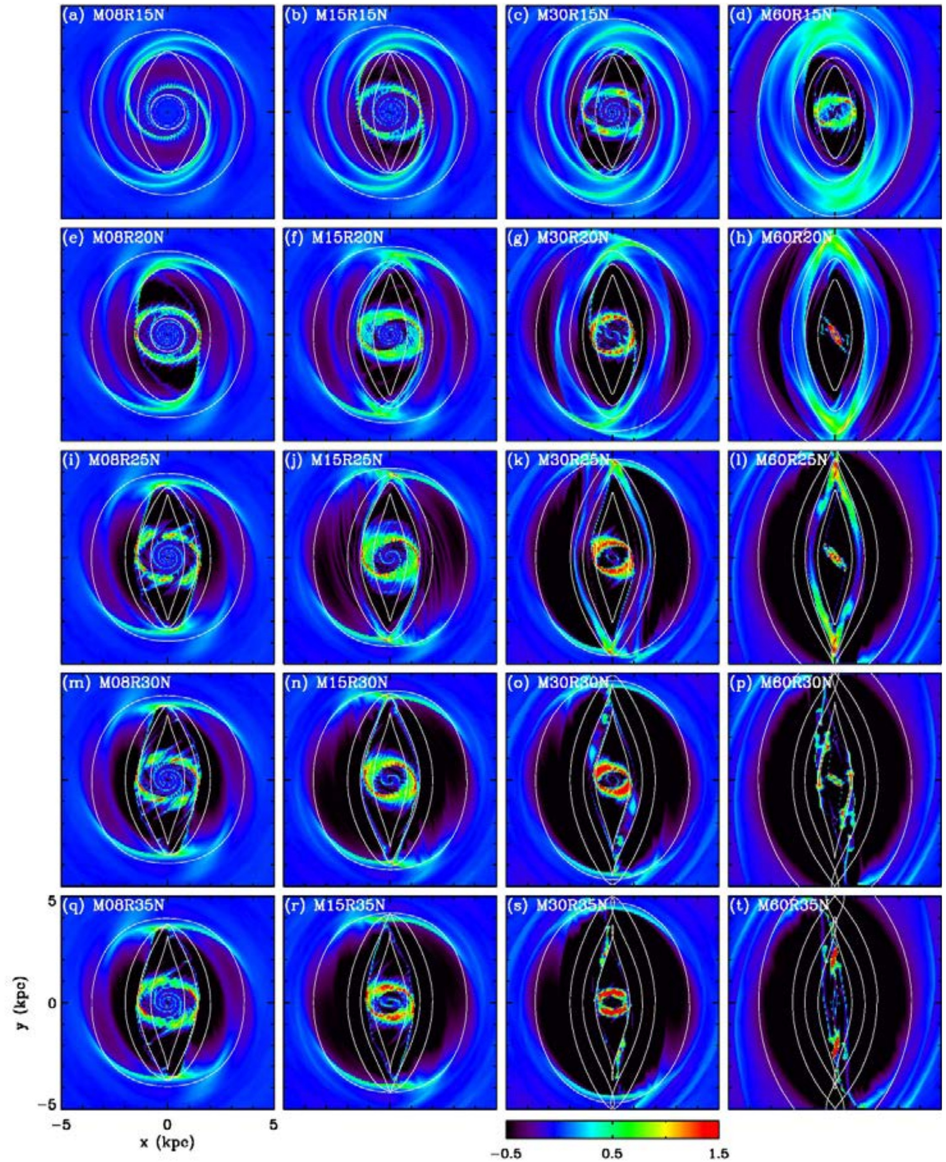
Bars & Inner Resonances
NGC 6782





Lin Lien-Hsuan, Li & Qin, de Grijs (this Symposium)

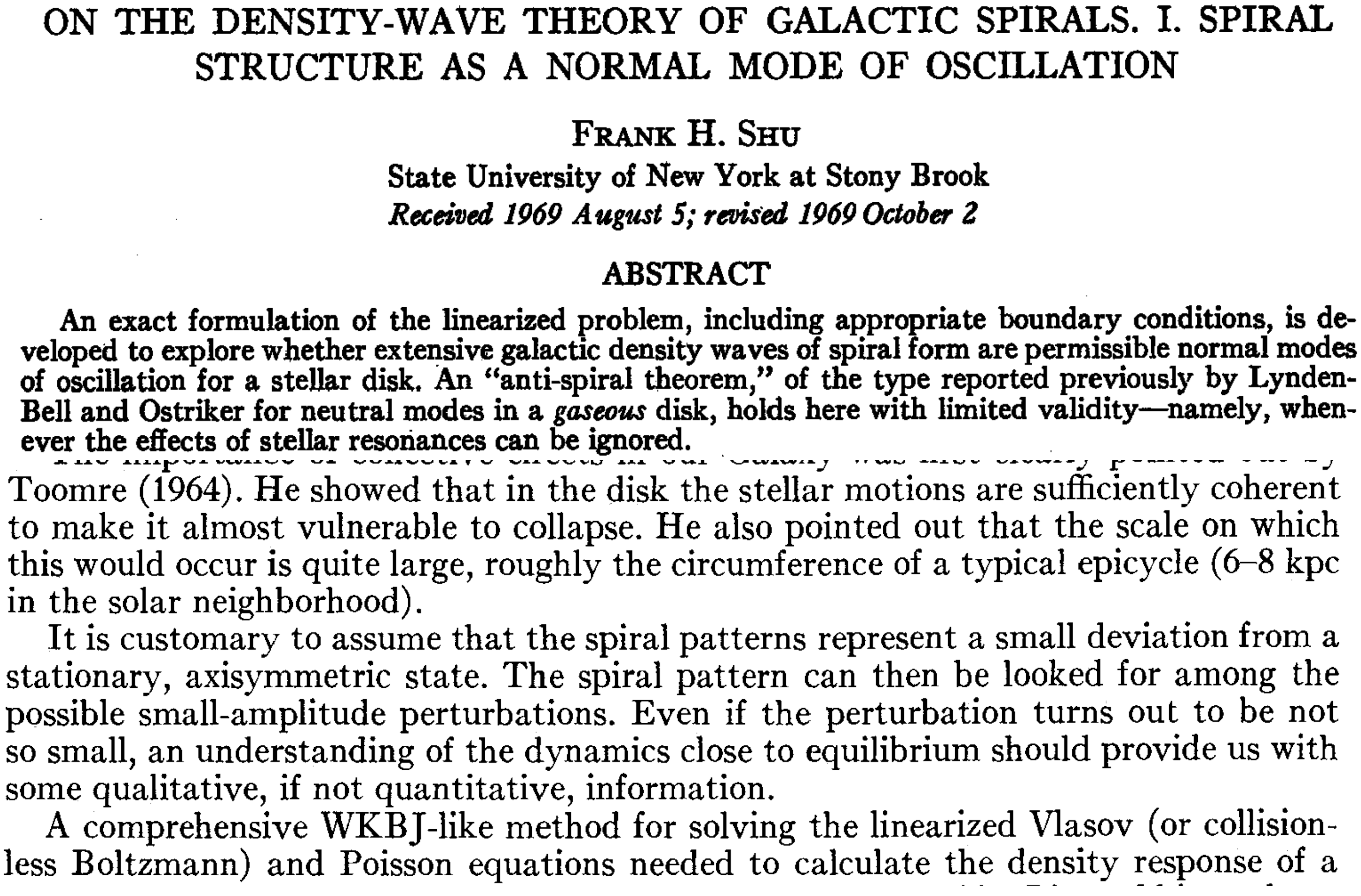
Bars, Rings, & Orbit Families



Kim, Seo, Kim (2012)

Growing Normal Modes

Stellar Dynamical Formulation as Integral Equation



ON THE DENSITY-WAVE THEORY OF GALACTIC SPIRALS. I. SPIRAL STRUCTURE AS A NORMAL MODE OF OSCILLATION

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Received 1969 August 5; revised 1969 October 2

ABSTRACT

An exact formulation of the linearized problem, including appropriate boundary conditions, is developed to explore whether extensive galactic density waves of spiral form are permissible normal modes of oscillation for a stellar disk. An "anti-spiral theorem," of the type reported previously by Lynden-Bell and Ostriker for neutral modes in a *gaseous* disk, holds here with limited validity—namely, whenever the effects of stellar resonances can be ignored.

Toomre (1964). He showed that in the disk the stellar motions are sufficiently coherent to make it almost vulnerable to collapse. He also pointed out that the scale on which this would occur is quite large, roughly the circumference of a typical epicycle (6–8 kpc in the solar neighborhood).

It is customary to assume that the spiral patterns represent a small deviation from a stationary, axisymmetric state. The spiral pattern can then be looked for among the possible small-amplitude perturbations. Even if the perturbation turns out to be not so small, an understanding of the dynamics close to equilibrium should provide us with some qualitative, if not quantitative, information.

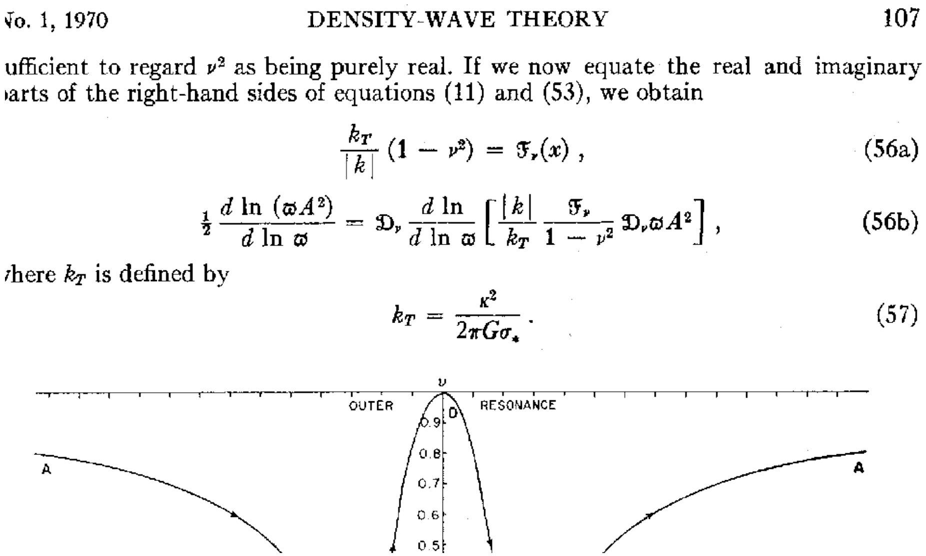
A comprehensive WKBJ-like method for solving the linearized Vlasov (or collisionless Boltzmann) and Poisson equations needed to calculate the density response of a

Group Velocity & Wave Action
Toomre (1969); Shu(1970b); Kalnajs (priv commun)

Dispersion Relation:

$$D(\omega, k; \varpi) = \frac{|k|}{k_T}\frac{\mathcal{F}_\nu(x)}{1-\nu^2} = 1.$$

$$c_g = -\left(\frac{\partial \omega}{\partial k}\right)_{\varpi} = \frac{(\partial D/\partial k)_{\omega,\varpi}}{(\partial D/\partial \omega)_{k,\varpi}}$$



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ufficient to regard ν^2 as being purely real. If we now equate the real and imaginary arts of the right-hand sides of equations (11) and (53), we obtain

$$\frac{k_T}{|k|}(1-\nu^2) = \mathfrak{F}_\nu(x)\,, \tag{56a}$$

$$\frac{1}{2}\frac{d\ln(\varpi A^2)}{d\ln\varpi} = \mathfrak{D}_\nu \frac{d\ln}{d\ln\varpi}\left[\frac{|k|}{k_T}\frac{\mathfrak{F}_\nu}{1-\nu^2}\mathfrak{D}_\nu \varpi A^2\right], \tag{56b}$$

here k_T is defined by

$$k_T = \frac{\kappa^2}{2\pi G\sigma_*}\,. \tag{57}$$

Amplitude Relation:

$$\varpi k A^2 \left(\frac{\partial D}{\partial k}\right)_{\omega,\varpi} = \text{const},$$

i.e., $\varpi c_g k A^2 \left(\frac{\partial D}{\partial \omega}\right)_{k,\varpi} = \text{const},$

where wave energy density

$= \Omega_p \cdot$ angular-momentum density

is proportional to $kA^2\left(\frac{\partial D}{\partial \omega}\right)_{k,\varpi}$

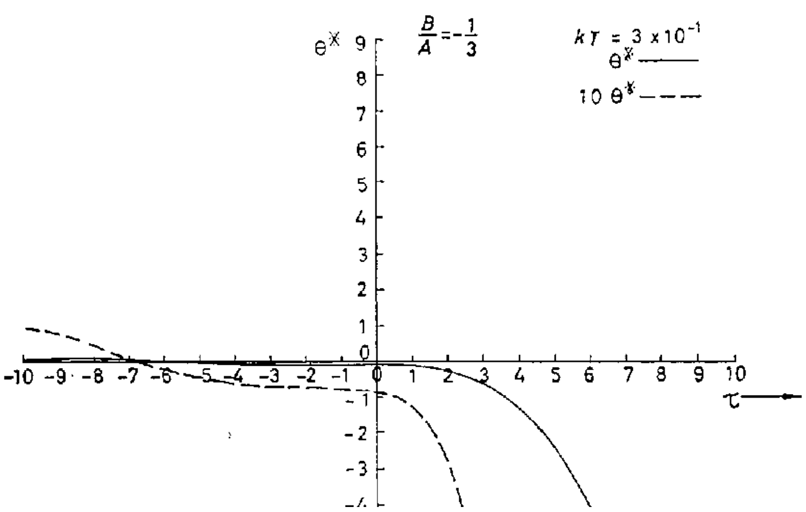
Energy/ang-mom densities are negative inside CR, positive outside CR

Amplification Across CR

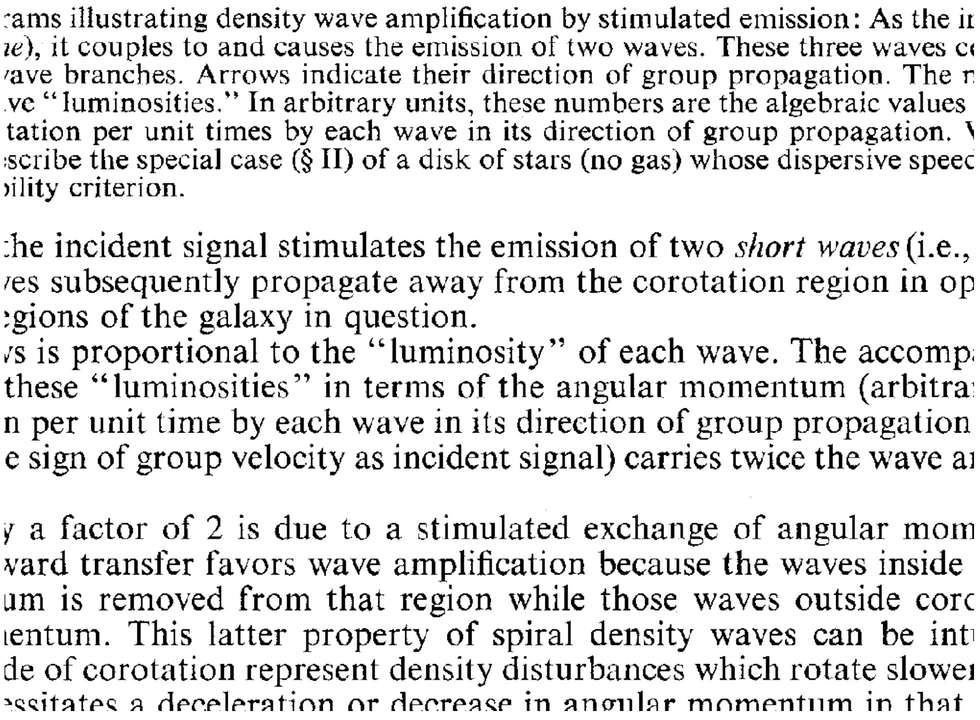
Goldreich & Lynden-Bell (1964), Mark (1976)

Swinging Packets

P. Goldreich and D. Lynden-Bell



WASER



ams illustrating density wave amplification by stimulated emission: As the i
e), it couples to and causes the emission of two waves. These three waves c
ave branches. Arrows indicate their direction of group propagation. The
ve "luminosities." In arbitrary units, these numbers are the algebraic values
tation per unit times by each wave in its direction of group propagation.
scribe the special case (§ II) of a disk of stars (no gas) whose dispersive spee
ility criterion.

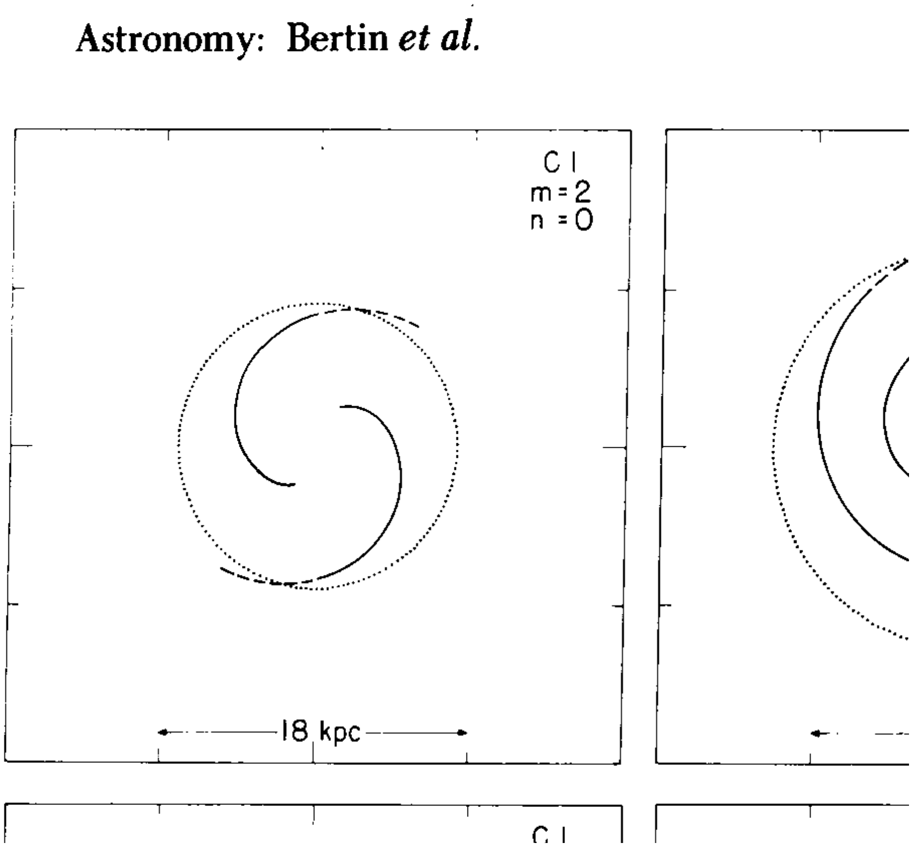
he incident signal stimulates the emission of two *short waves* (i.e.,
es subsequently propagate away from the corotation region in op
gions of the galaxy in question.
s is proportional to the "luminosity" of each wave. The accomp
these "luminosities" in terms of the angular momentum (arbitra
n per unit time by each wave in its direction of group propagation
e sign of group velocity as incident signal) carries twice the wave a

y a factor of 2 is due to a stimulated exchange of angular mom
ward transfer favors wave amplification because the waves inside
um is removed from that region while those waves outside cor
entum. This latter property of spiral density waves can be int
de of corotation represent density disturbances which rotate slowe
ssitates a deceleration or decrease in angular momentum in that

Mestel Disk: Zang (1976); Goldreich & Tremaine (1979); Goodman & Evans (1999);
Shu, Laughlin, Lizano, Galli (2000); m = 1 SLING: Adams, Ruden, & Shu (1989); STAR (1990)

Discrete Modes (Fluid Dynamical)

Bertin, Lau, Lin, Mark, Sugiyama (1977)



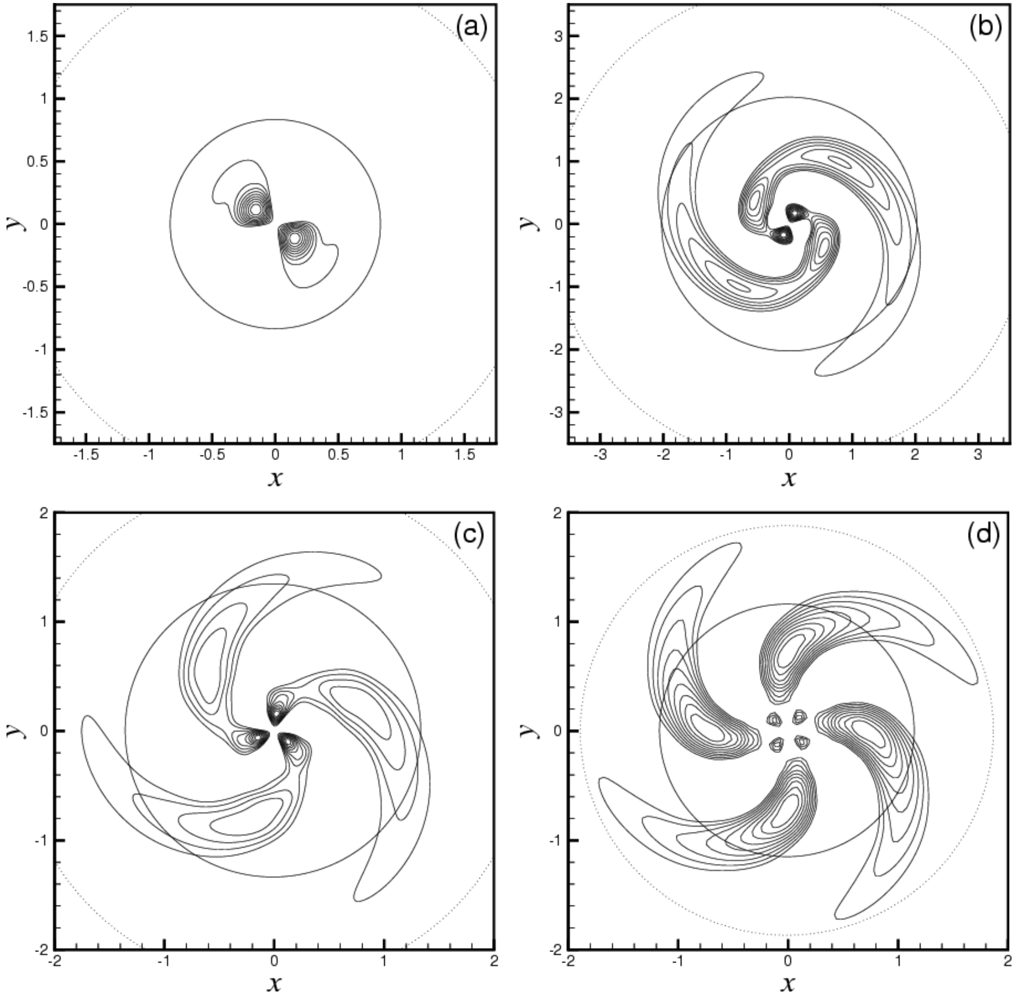
Nature of normal mode depends not only on quantum numbers *m* & *n*, but also on feedback from central regions (short leading vs. long trailing).

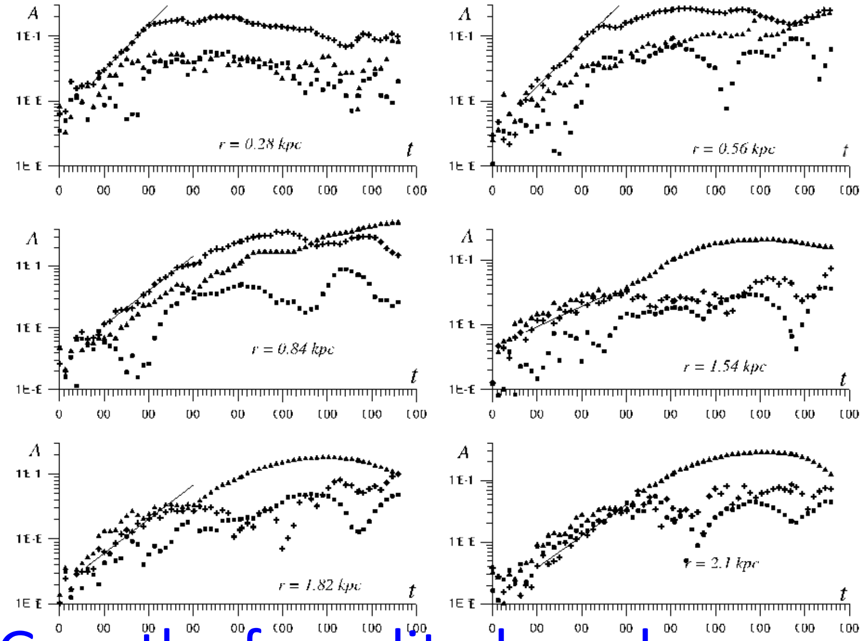
Role of (pseudo) bulge (Shen & Jin)

Unification of theory of barred and normal spirals.

Discrete Modes (Stellar Dynamical)

Khoperskov, Just, Korchagin, Jalali (2007)





Growth of amplitude and saturation at finite amplitude (QSSS!) No gas in model!

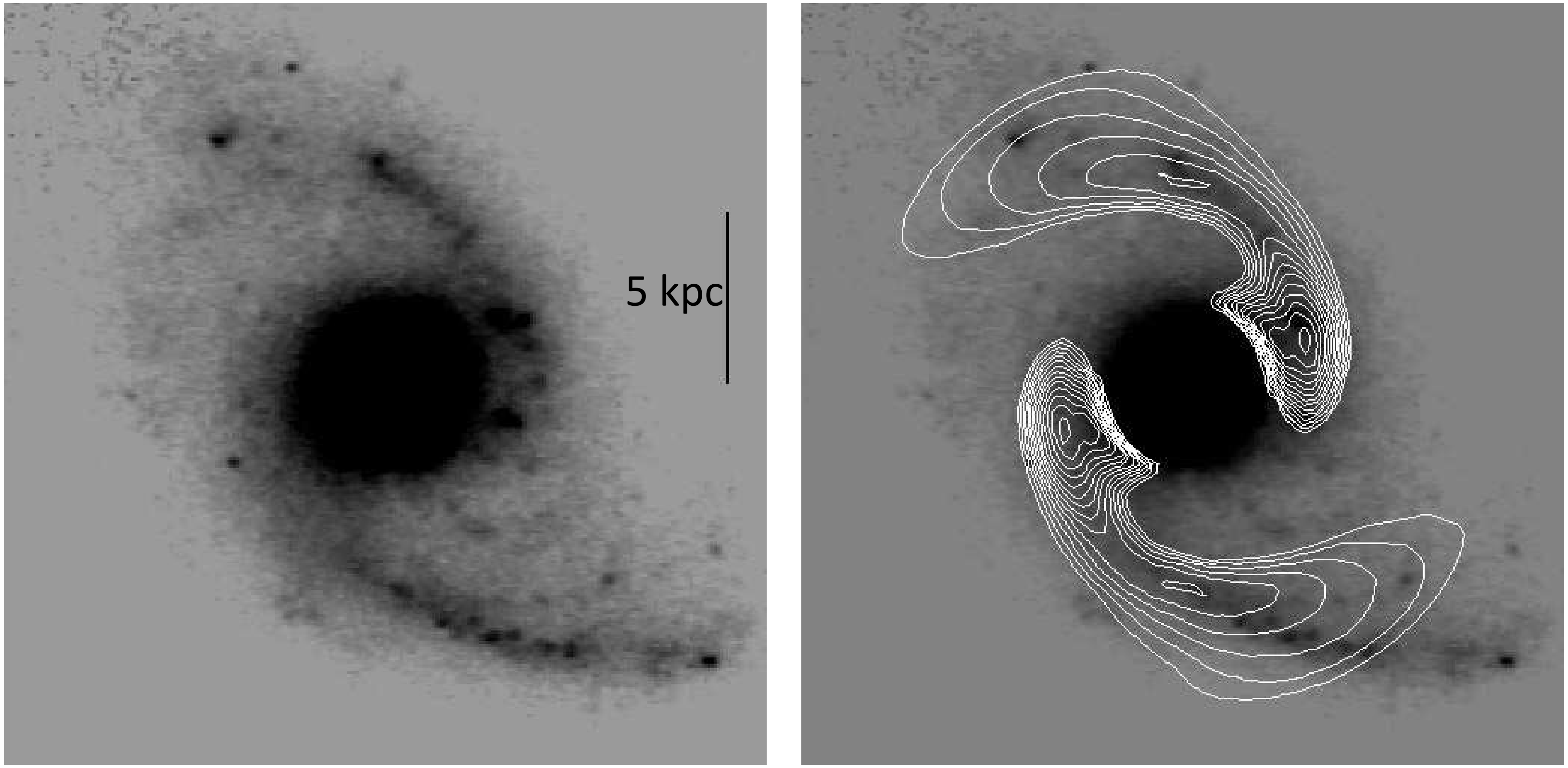
Need *N* > 3 x 10^7 to achieve good agreement for low-growth modes

Jilali & Hunter (2005); cf. Polychenko

Radial orbits for no ILR in *m* > 2 (fluid: $\overleftrightarrow{P}$)

Constructive/Destructive Interference in K-Band = Signature of Stellar Normal Mode

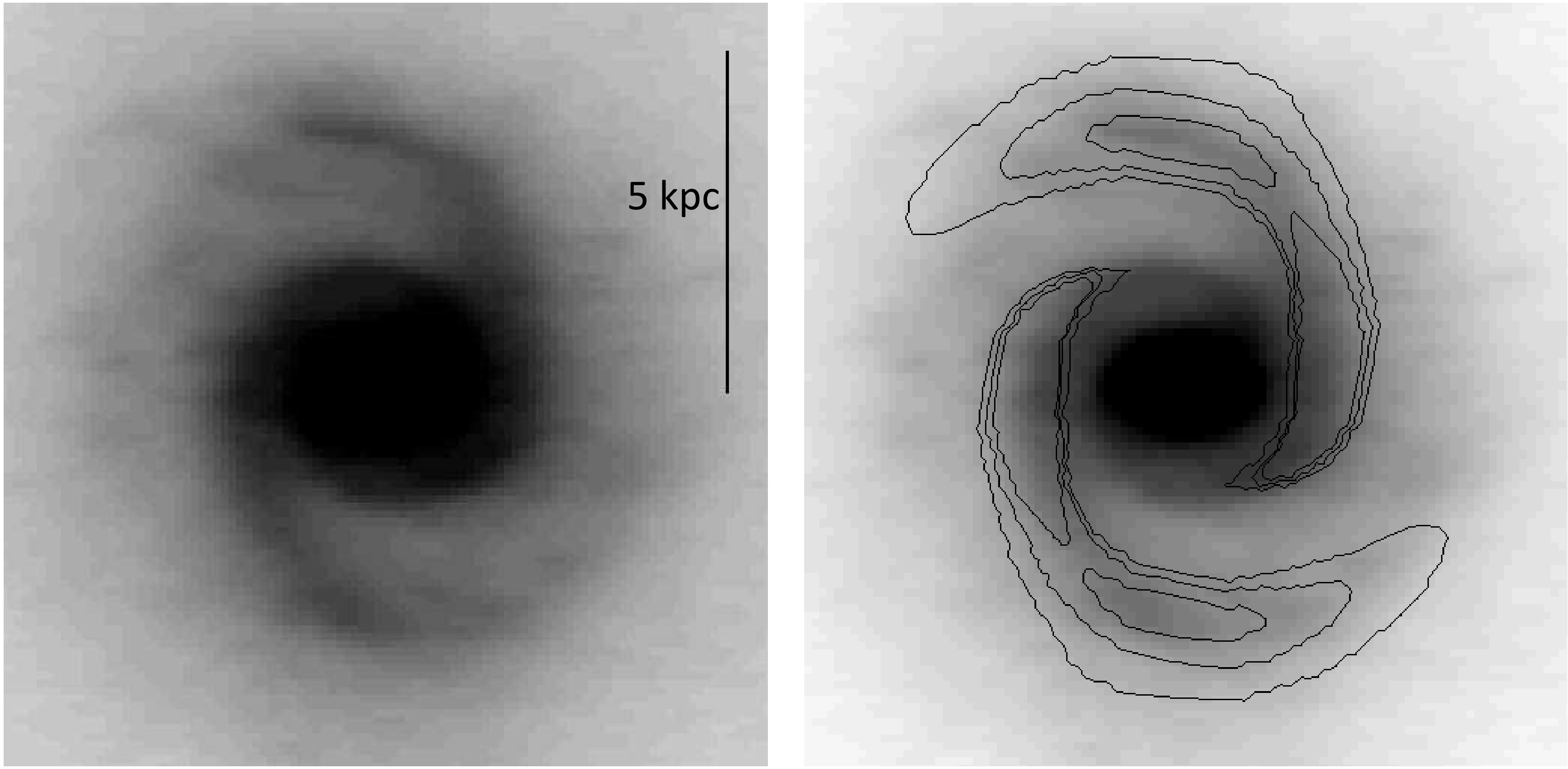
Puerari, Block, Elmegreen, Frogel, Eskridge (2000)



NGC 5248 – Optically Grand Design

Modulations of K-Band Amplitude Order/Disorder

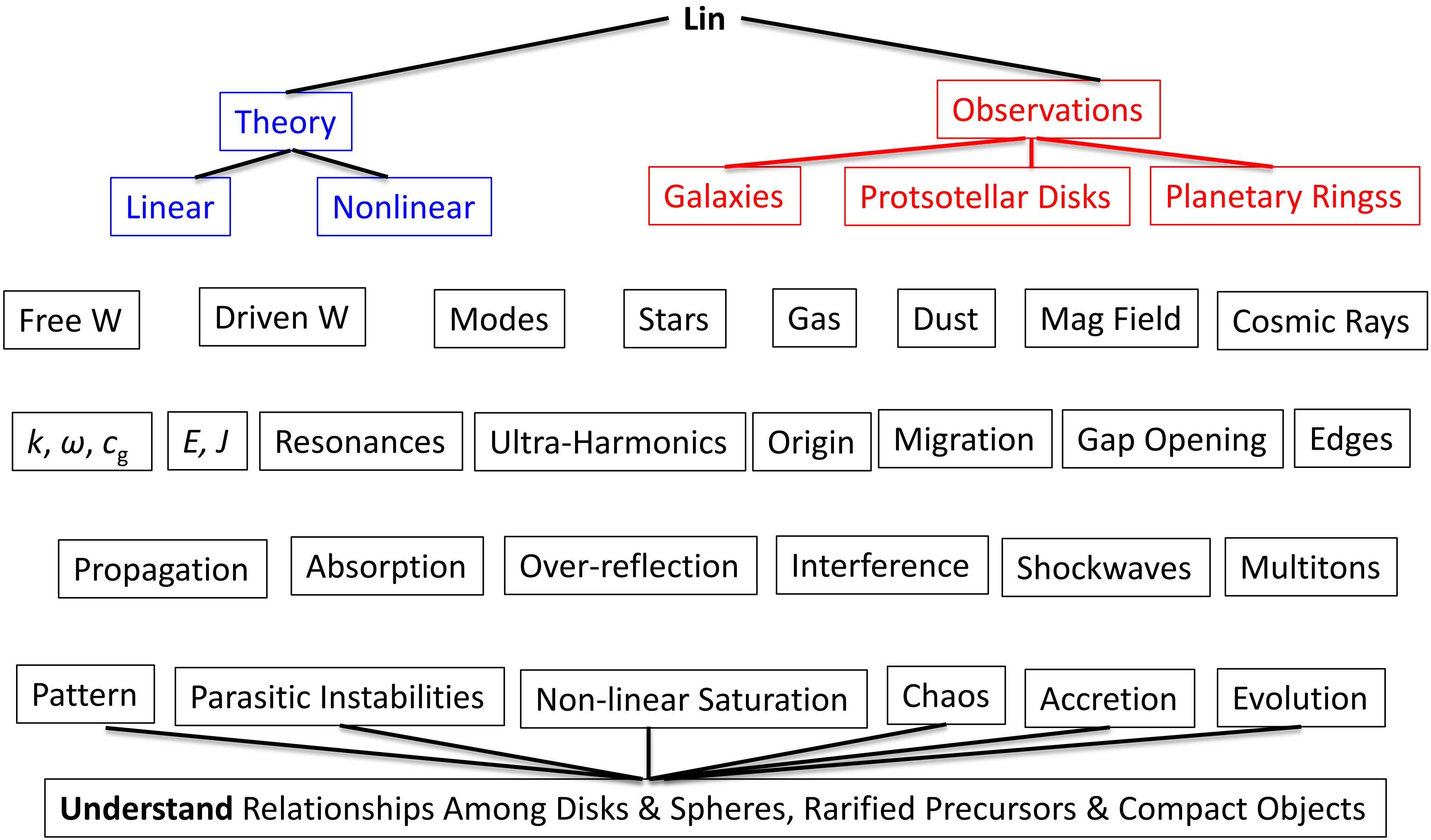
Puerari, Block, Elmegreen, Frogel, Eskridge (2000)



NGC 4062 – Optically Flocculent

Debate should be over. Four Stages of Acceptance.

Drawing Connections Among Branching *Lin*eages



Lin

Theory

Linear

Nonlinear

Observations

Galaxies

Protsotellar Disks

Planetary Ringss

Free W

Driven W

Modes

Stars

Gas

Dust

Mag Field

Cosmic Rays

k, ω, c_g

E, J

Resonances

Ultra-Harmonics

Origin

Migration

Gap Opening

Edges

Propagation

Absorption

Over-reflection

Interference

Shockwaves

Multitons

Pattern

Parasitic Instabilities

Non-linear Saturation

Chaos

Accretion

Evolution

Understand Relationships Among Disks & Spheres, Rarified Precursors & Compact Objects

Happy Birthday SDWT!

Thank You Everyone, especially
Shang Hsien, Yong Wen-An, & Pu Yi-Kang.