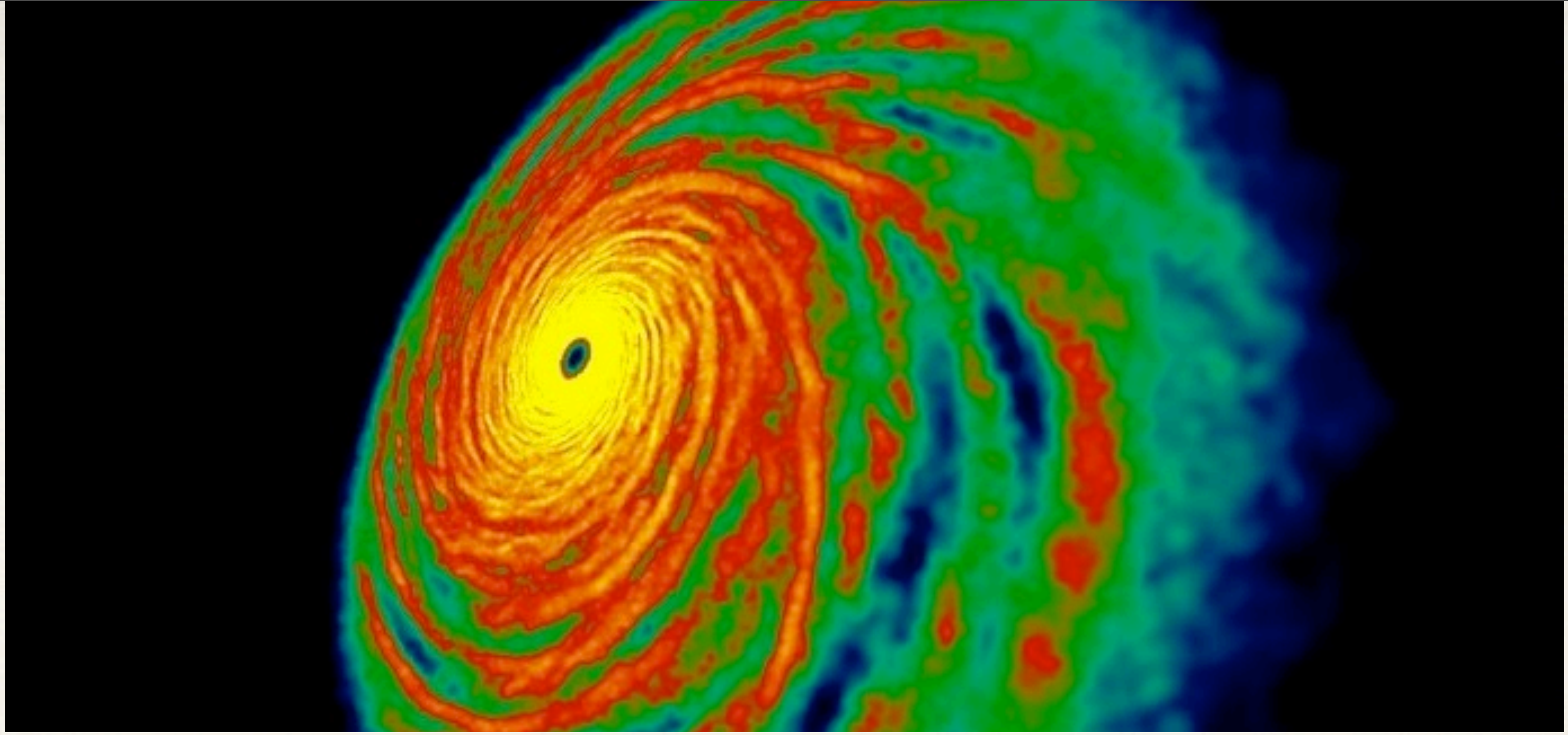

Spiral structure and gravitational instabilities in protostellar discs

Giuseppe Lodato - Università degli Studi di Milano

27 June 2013 - Beijing - Lin-Shu Symposium

Gravitational instabilities in protostellar discs

- Conditions for instability
- Dynamics of self-gravitating discs:
  - Self-regulation
  - Local vs global behaviour
- Numerical unceretainties - convergence, fragmentation
- Observations of density waves in protostellar discs

Gaseous vs stellar discs

- Protostellar discs are mostly gaseous
- Potentially significant differences with respect to the case where the structure is determined mostly by a dissipationless component
    - Role of Lindblad resonances?
    - Shocks in the gas might play a significant role (gas is not merely a "passive" component that responds to the structure)

Linear stability criterion

- Linear dispersion relation

$$(\omega - m\Omega)^2 = c_\mathrm{s}^2 k^2 - 2\pi G\Sigma|k| + \kappa^2$$

- Well known axisymmetric instability criterion:

$$Q = \frac{c_\mathrm{s}\kappa}{\pi G\Sigma} < \bar{Q} \approx 1$$

- Equivalent form of the instability criterion

$$\frac{M_\mathrm{disc}(R)}{M_\star} \gtrsim \frac{H}{R}$$

- Need the disc to be cold and/or massive

- What are the masses and aspect ratio in actual protostellar discs?

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Are protostellar discs linearly unstable?

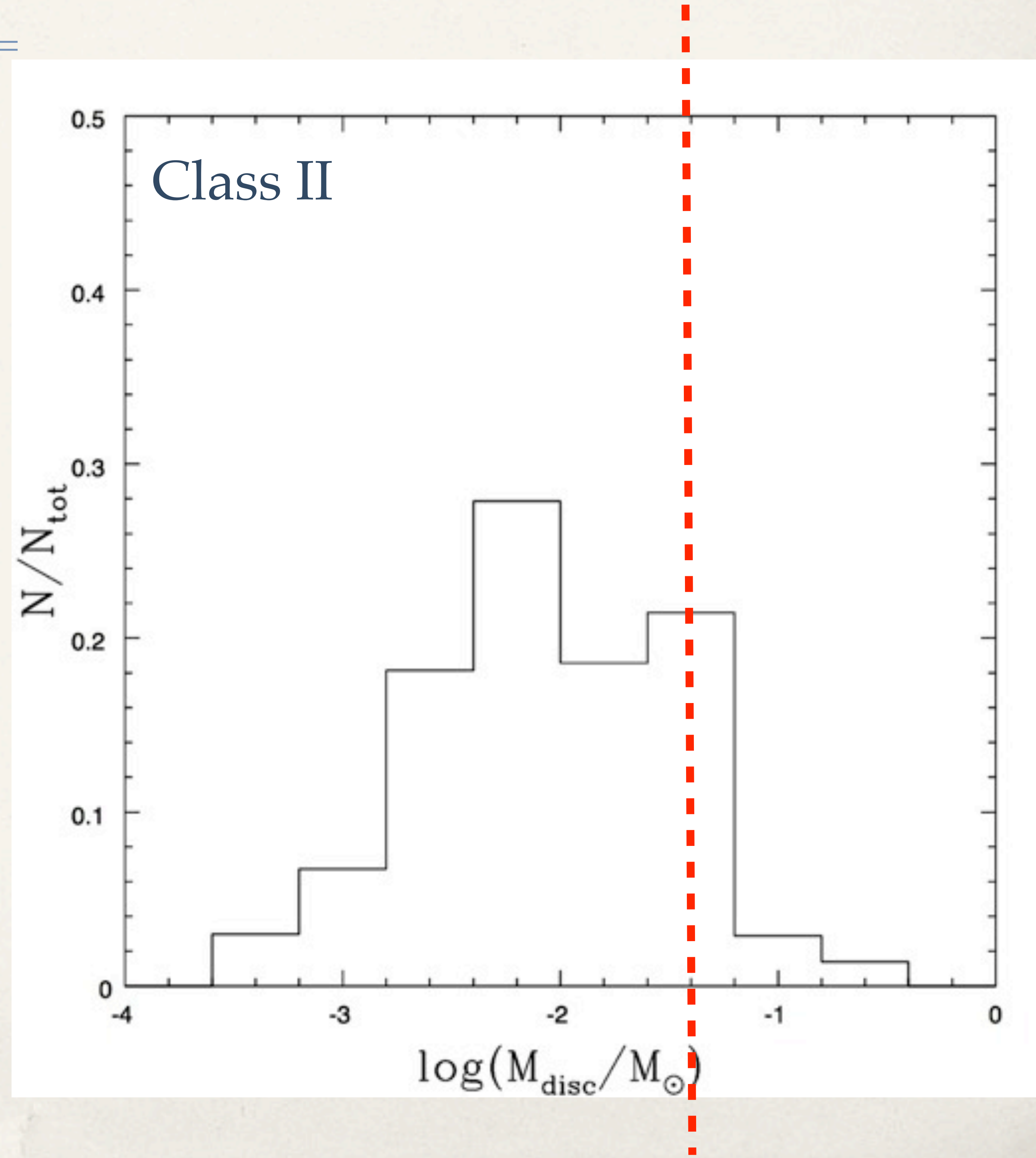
- Midplane temperature for irradiated discs (Chiang & Goldreich 1997, Chiang & Youdin 2009) gives:

$$\frac{H}{R} \simeq 0.02 \left(\frac{R}{\mathrm{AU}} \right)^{2/7}$$

- Therefore H/R varies from **0.02** at 1AU to **0.06** at 100 AU
- Need disc masses of order 5% of the stellar mass to be unstable
- Protostellar disc masses difficult to measure (see Hartmann et al 2006)

Are protostellar discs linearly unstable?

- Disc masses in Taurus and Ophiucus by Andrews and Williams (2005, 2007)
- Disc masses might be underestimated significantly (Hartmann et al 2006)
- Uncertainties in dust opacities
- If density profile steep, most of the mass might be hidden in optically thick inner parts (Hartmann 2009)



Are protostellar discs linearly unstable?

- Class II (T Tauri) discs are relatively evolved. Can we infer the masses at early stages?
- Simple (simplistic?) approach:
- Take all objects with measured M and Mdot
- Apply similarity solutions (Lynden-Bell & Pringle 1973)
- Find "initial' disc mass and evolutionary timescale

$$M_0 = M_\mathrm{d}(t)\left(\frac{t_\mathrm{d}}{t_\mathrm{d}-t}\right)^{1/2(2-\gamma)}$$

$$t_\mathrm{d} = \frac{M_\mathrm{d}(t)}{2(2-\gamma)\dot{M}(t)}$$

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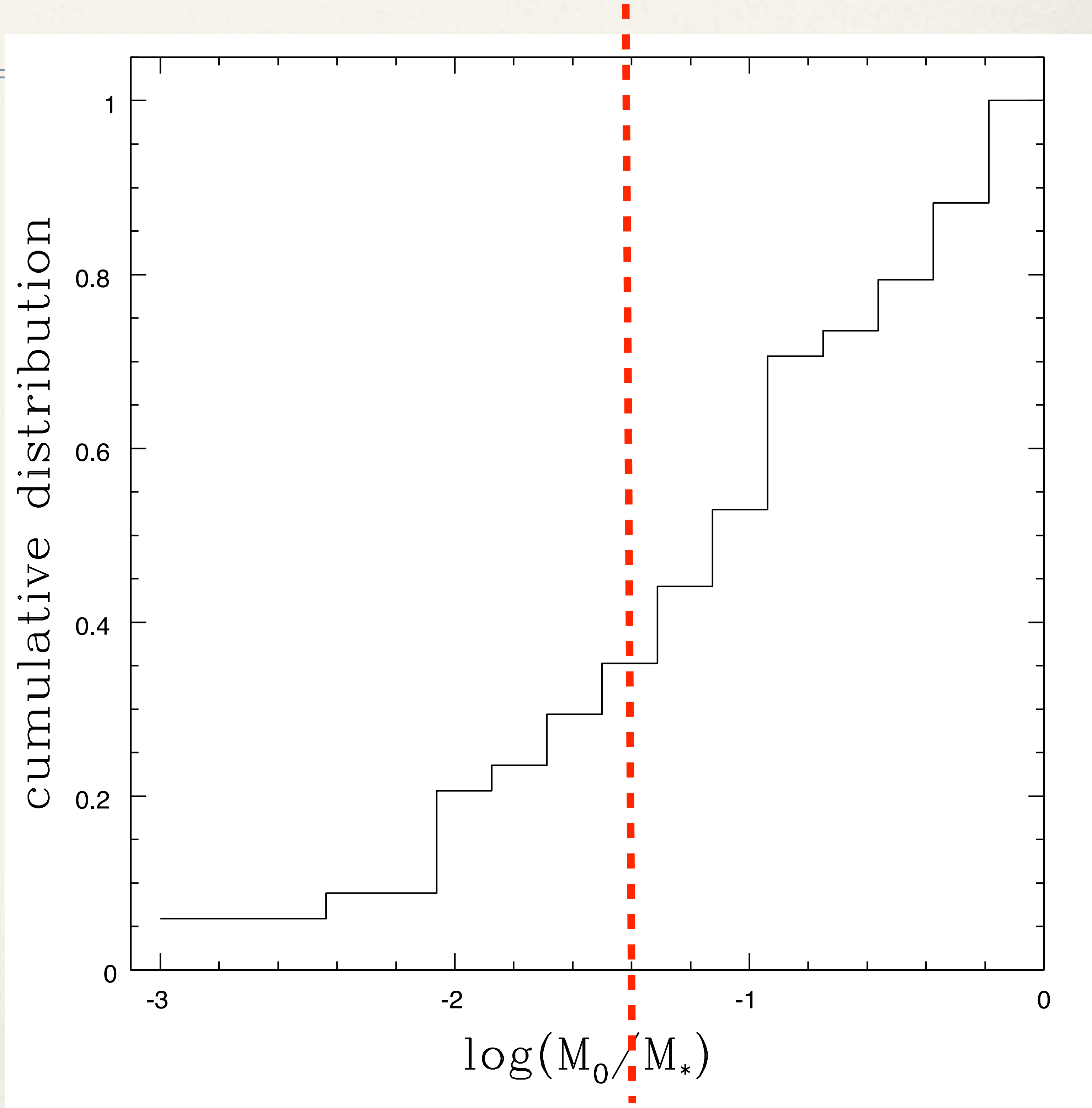

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Non linear evolution of GI

- Investigated numerically in the last decade by several authors (Laughlin & Bodenheimer 1994, Laughlin et al 1998, Pickett et al 2000, Boss 2000, Gammie 2001, Mayer et al 2002, Lodato & Rice 2004, 2005, Mejia et al 2005, Boley et al 2006)

- Early simulations used an isothermal or polytropic equation of state (Laughlin & Bodenheimer 1994, Mayer et al 2002)

- Starting from Gammie (2001) it has become clear that the evolution is strongly dependent on the cooling time t_{cool}

- Introduce a cooling parameter as the ratio of cooling to dynamical timescale

$$\beta = t_{\rm cool}\Omega$$

Thermal self-regulation of GI

- Role of cooling time clear if one thinks at the form of the stability parameter Q

$$Q = \frac{c_{\rm s}\kappa}{\pi G \Sigma} \propto T^{1/2}$$

- Development of the instability feeds energy back onto the equilibrium and stabilizes the disc

- Works as an effective thermostat for the disc

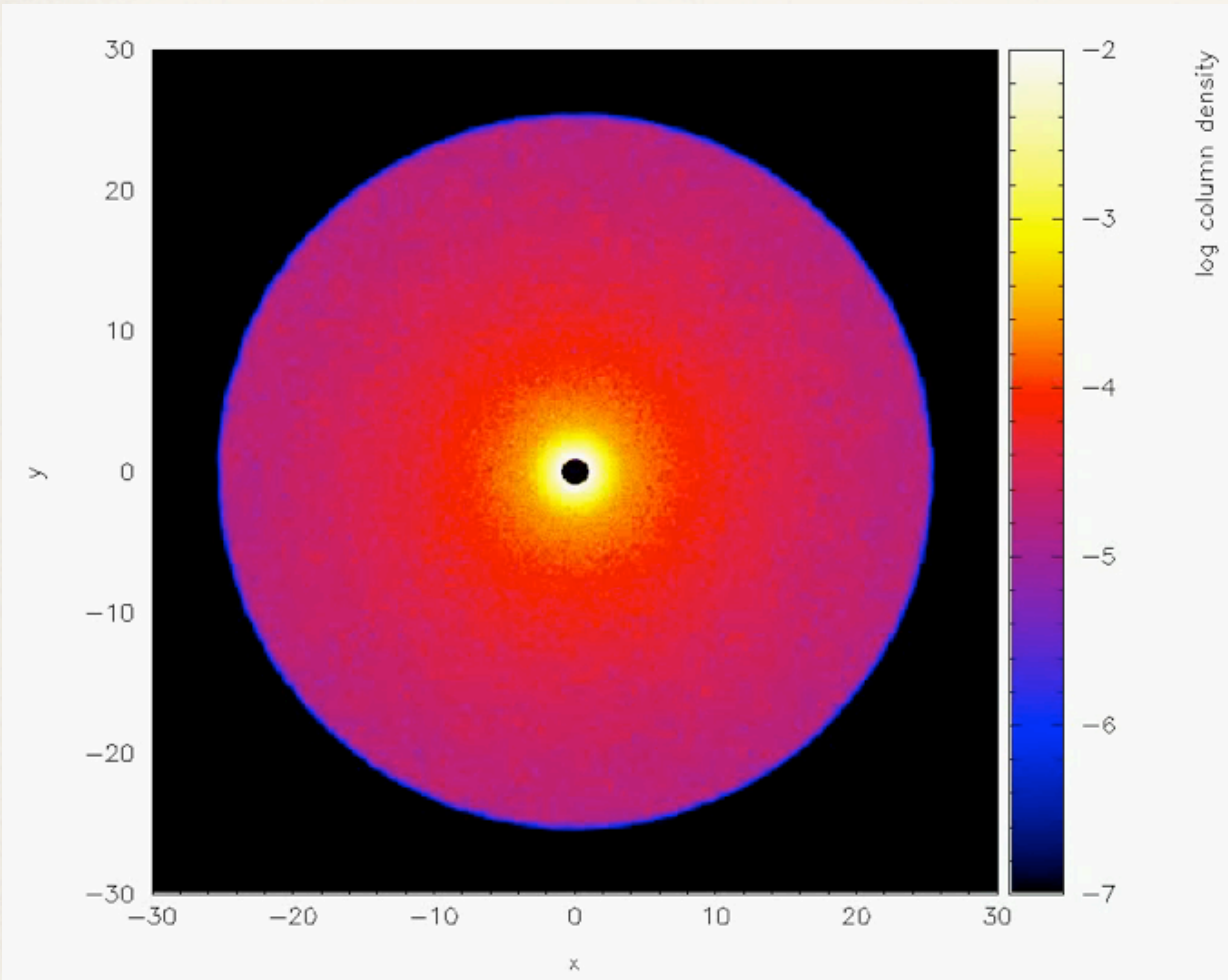
- Expect the disc to stay close to marginal stability $Q \sim 1$ *(Paczynski 1977)*

- Self-regulated discs models can be constructed (Bertin 1997, Bertin & Lodato 1999)

Long cooling time: self-regulation

Cossins, Lodato & Clarke (2009)
$\beta = 6$

Long cooling time: self-regulation



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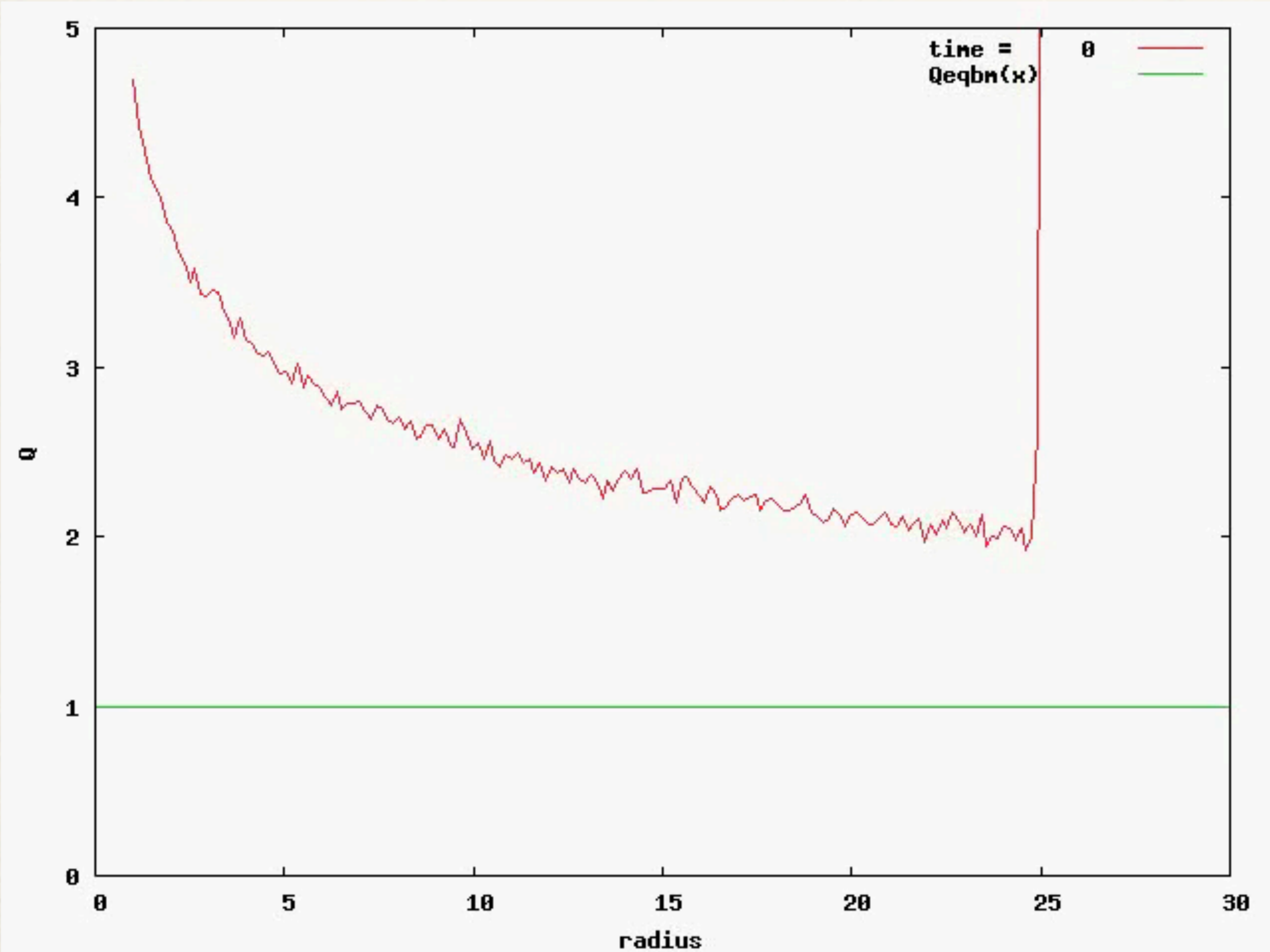

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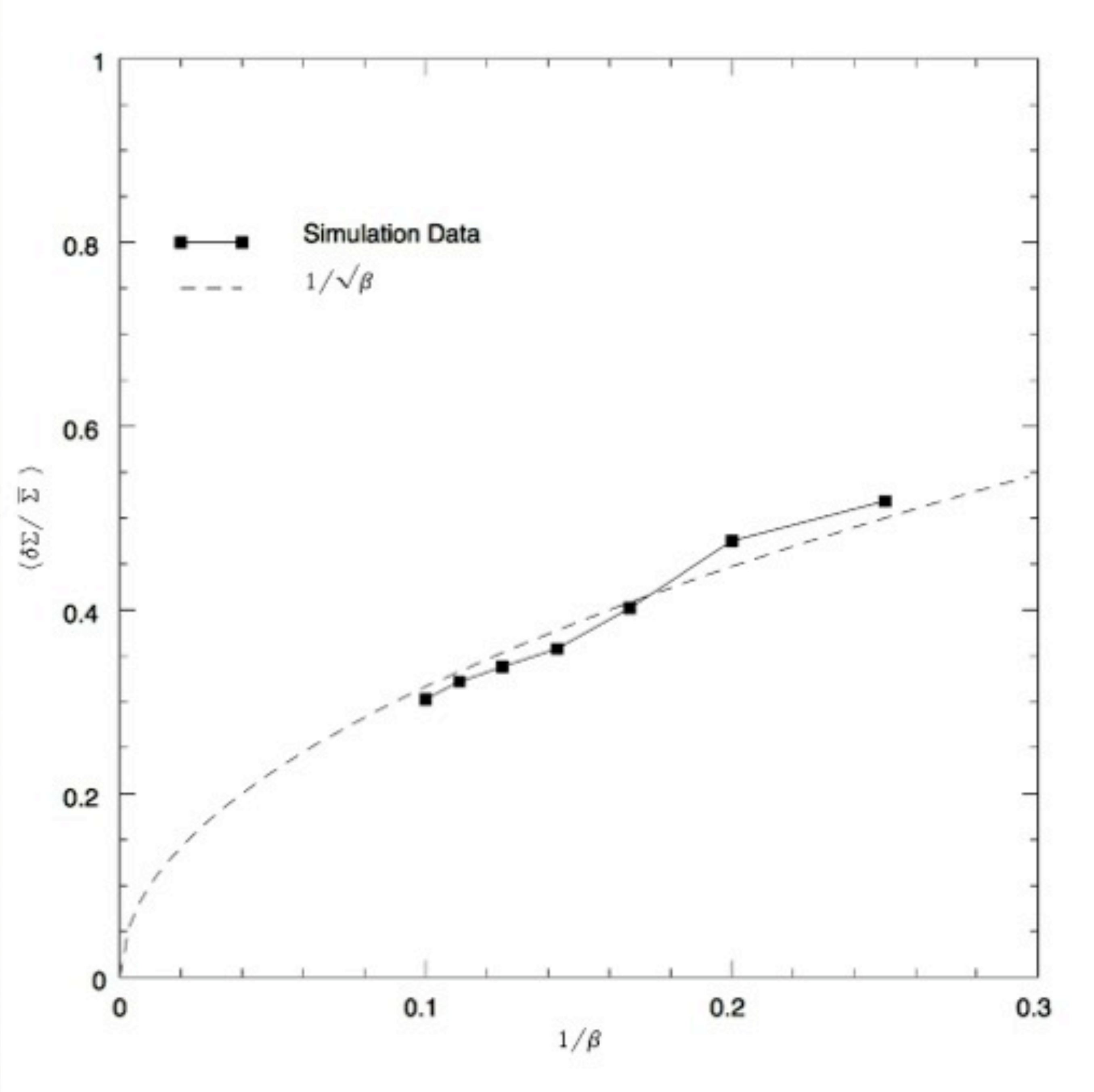
Thermal saturation of GI

Cossins, Lodato & Clarke 2009

- Self-regulation is established through thermal saturation of the spiral waves.
- Amplitude of density perturbation must be related to cooling rate
- We find that:

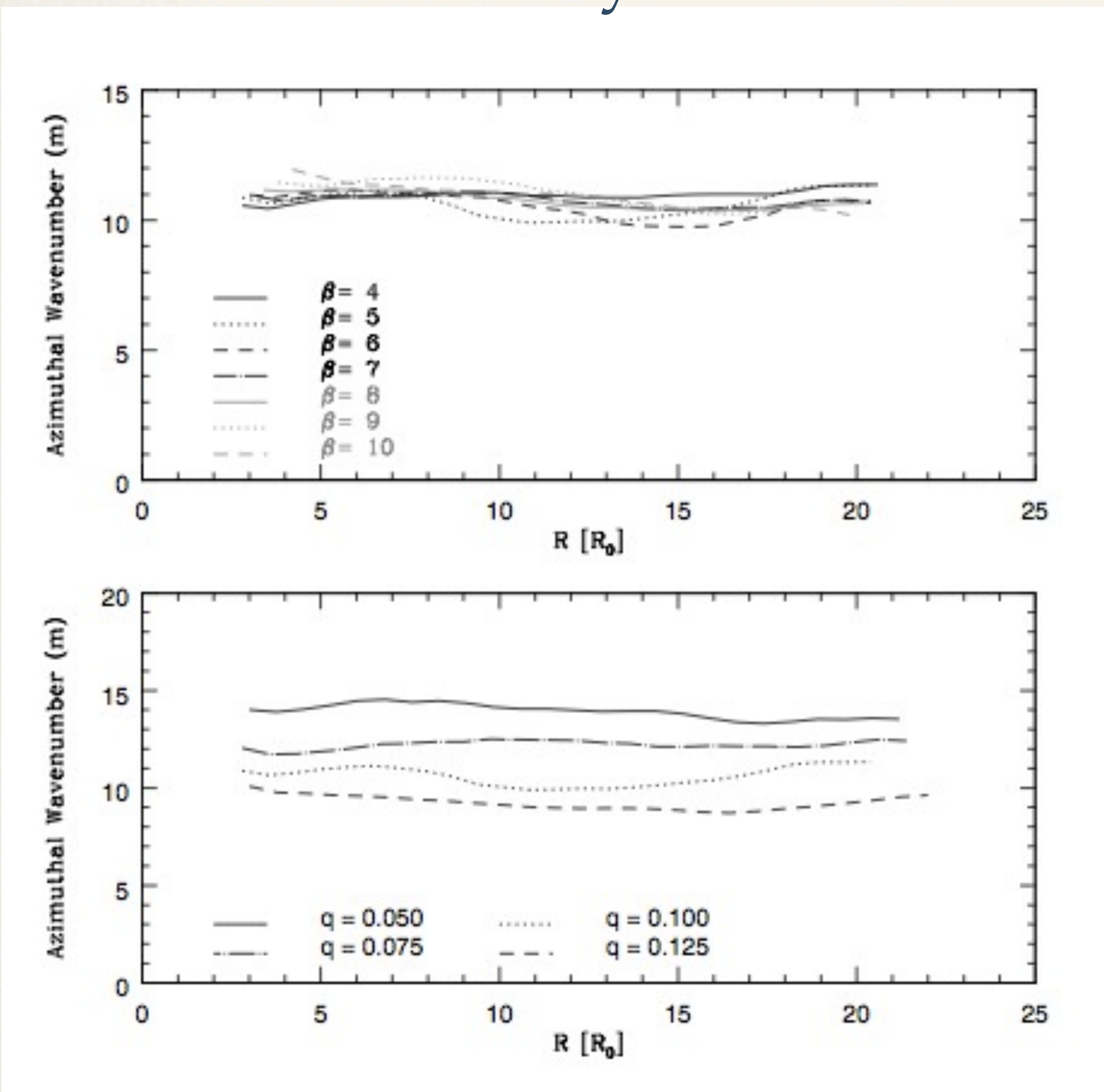
$$\frac{\Delta\Sigma}{\Sigma} \approx \frac{1}{\sqrt{\beta}}$$

- Natural if consider that energy content of waves is proportional to the square of the perturbed fields

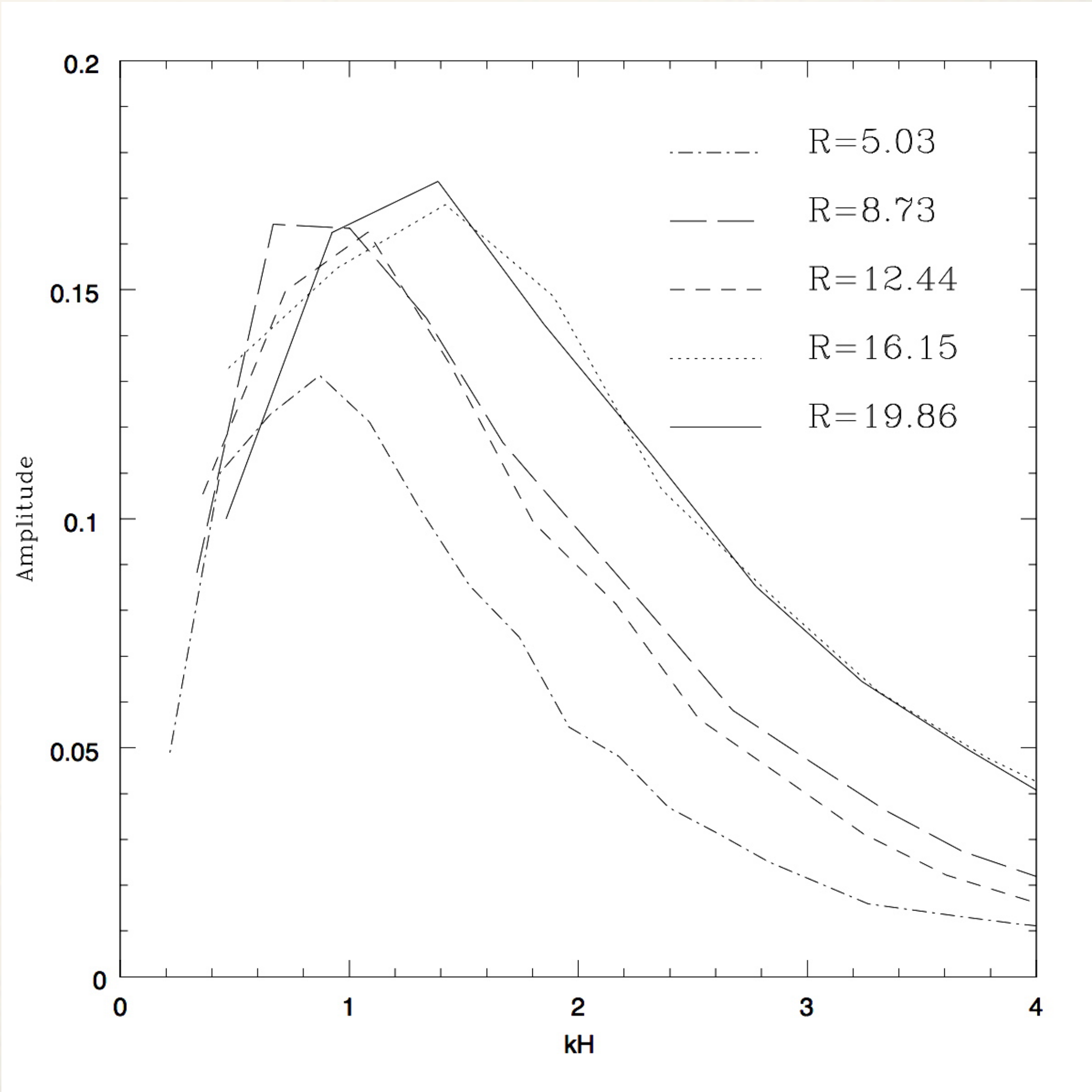


Spectrum of excited modes

Azimuthal structure: massive discs chacarcterized by small *m*



Radial structure: at all radii, *k* peaks at roughly ~*1*/*H*

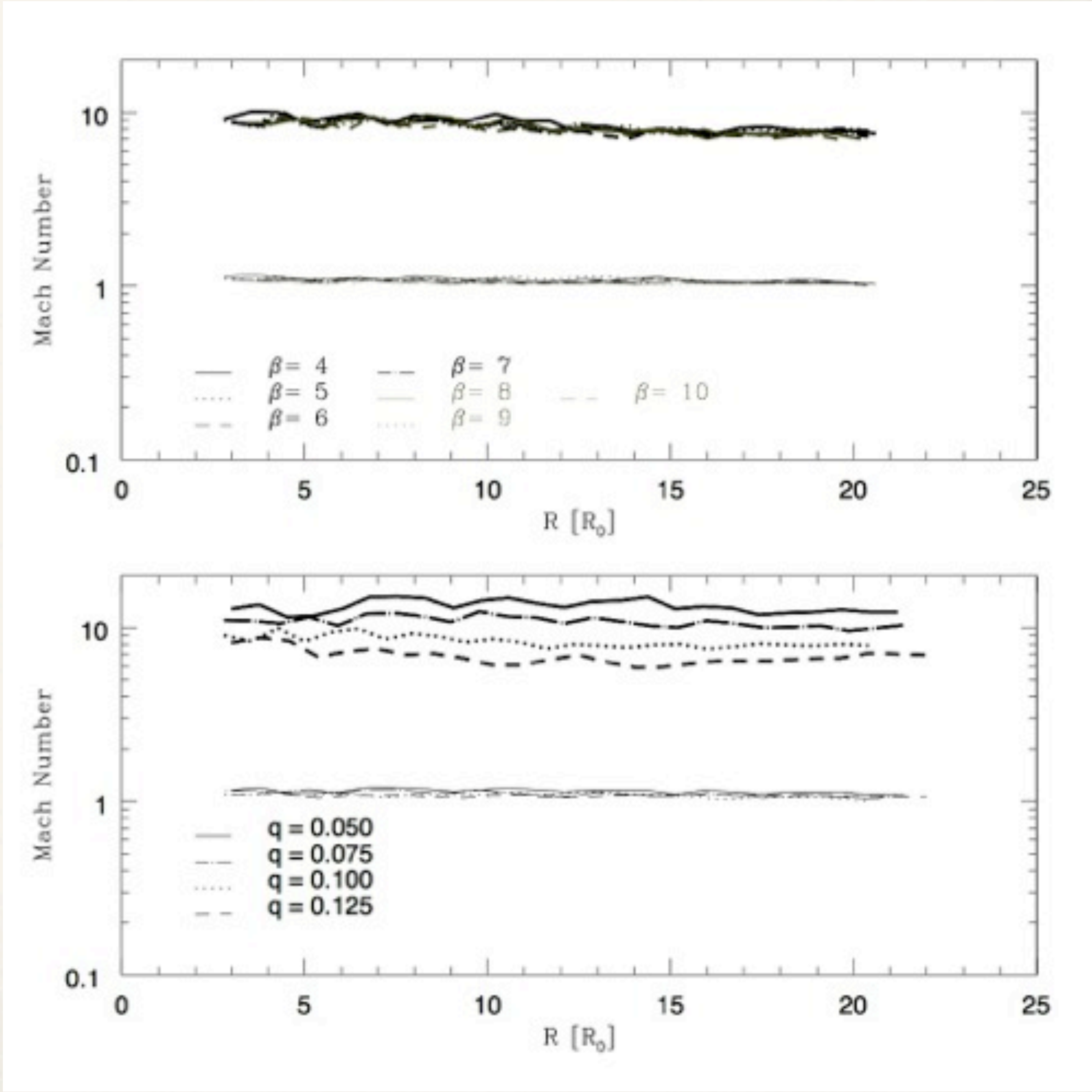


Sonic condition for spiral waves

Cossins, Lodato & Clarke 2009

- We have computed the pattern speed of the underlying spiral structure and its Mach number
- The Doppler-shifted Mach number is very close to unity, independently on radius, cooling rate, and disc mass.
- Density jump for almost sonic shocks also directly leads to

$$\frac{\Delta\Sigma}{\Sigma} \approx \frac{1}{\sqrt{\beta}}$$



Local vs global behaviour

Lodato & Rice 2004

- Can the evolution of self-gravitating discs be described within the standard, local, α-like prescription?
- Can compute gravitational + Reynolds stresses directly from simulations and compare with expectations from standard α-theory (LR04, see also Boley et al. 2006)
- **The disc adjusts so as to deliver the viscosity needed to stay in thermal equilibrium**

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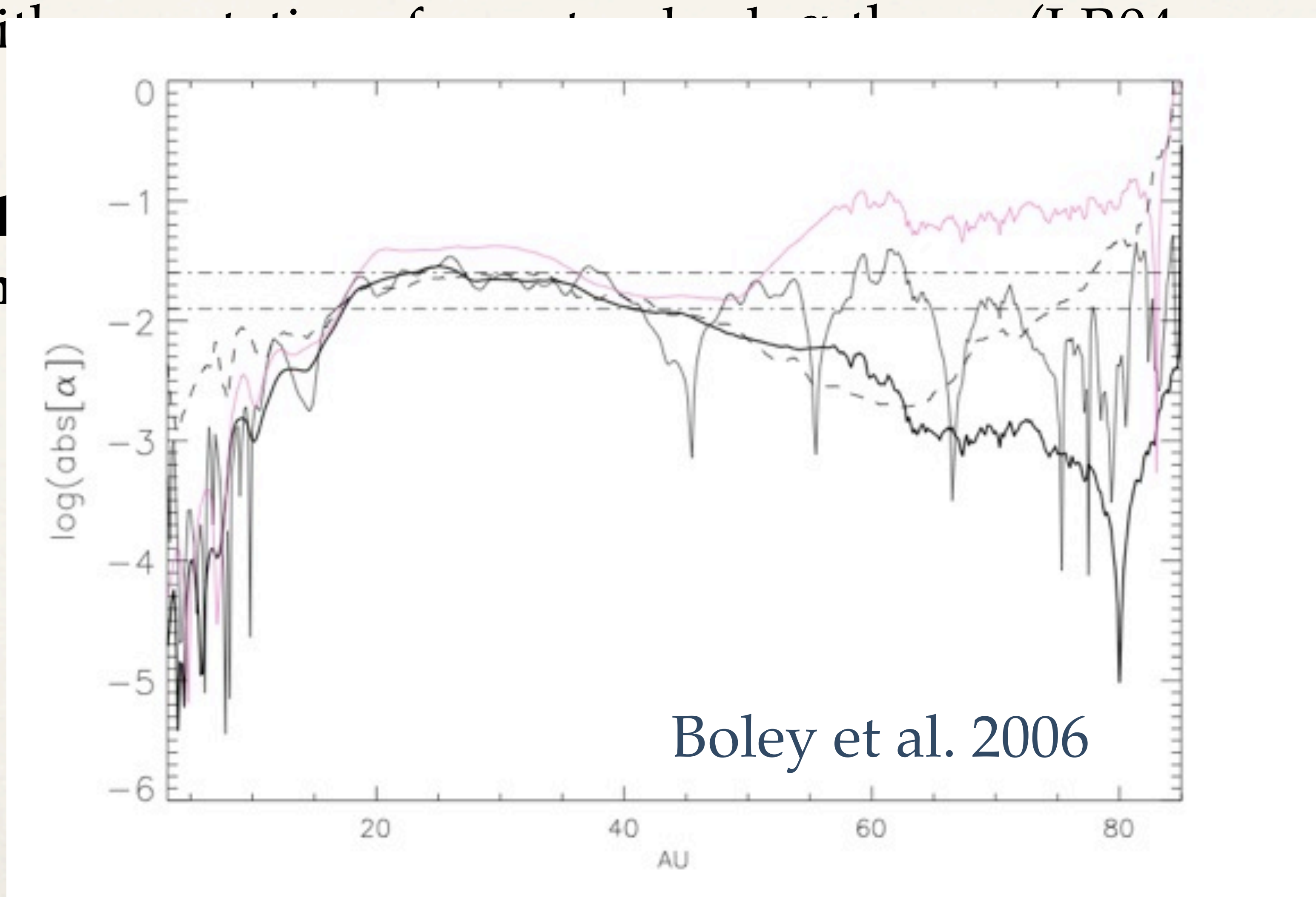


Local vs global behaviour

Cossins, Lodato & Clarke 2009

- Can the evolution of self-gravitating discs be described within the standard, local, α-like prescription?
- Described in detail by Balbus & Papaloizou (1999), recently discussed extensively by Cossins et al (2009)
- Relation between energy and angular momentum densities in a density wave

$$\mathcal{E} = \Omega_{\rm p}\mathcal{L} \longrightarrow \dot{\mathcal{E}} = \Omega_{\rm p}\dot{\mathcal{L}}$$

- Relation between power and stress due to local (viscous) processes

$$\dot{\mathcal{E}}_\nu = \Omega\dot{\mathcal{L}}_\nu$$

- If density waves dissipate far from co-rotation, behaviour is non-local

Local vs global behaviour

Cossins, Lodato & Clarke 2009

- Degree of non-locality can be measured by

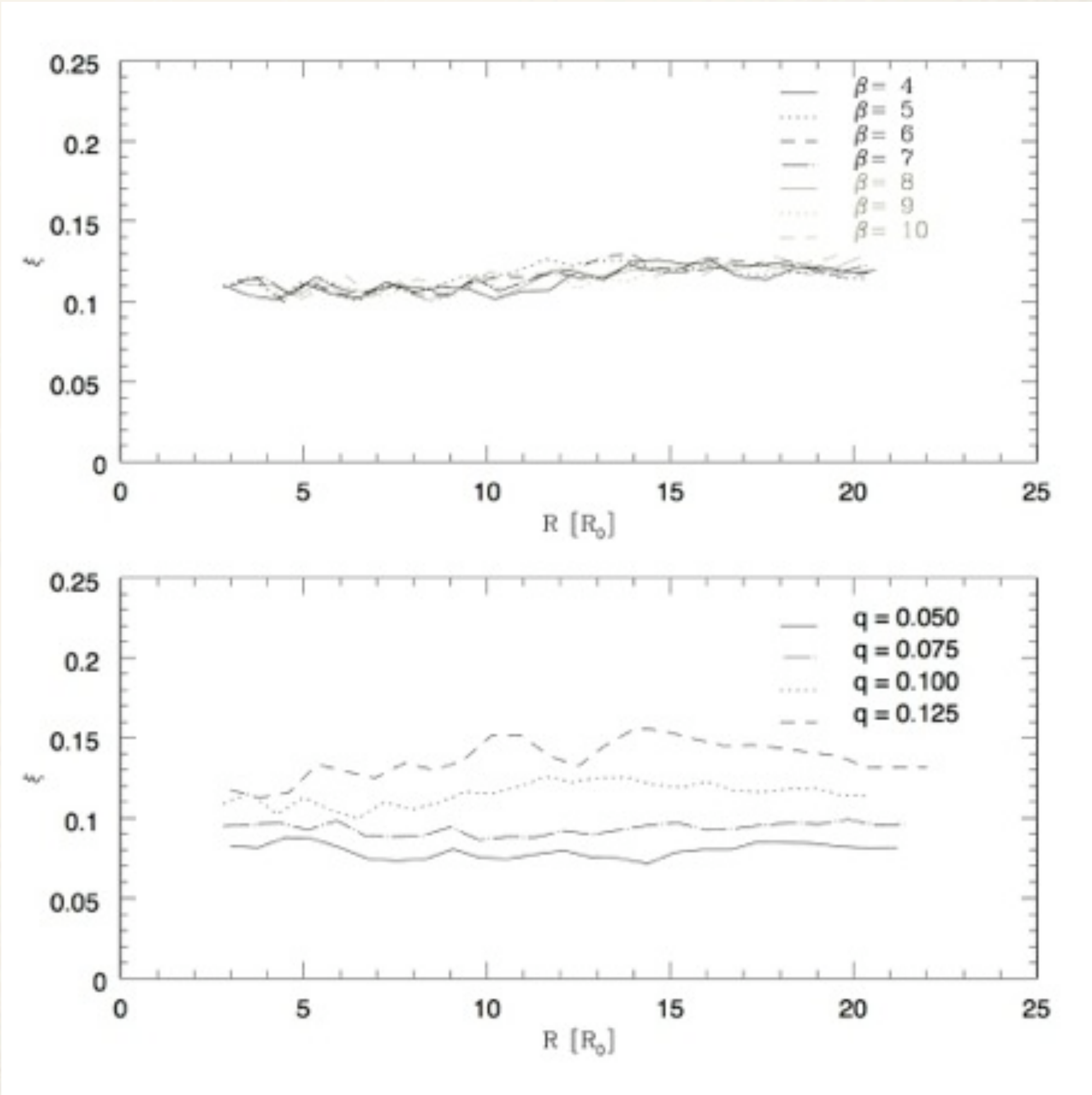
$$\xi = \left| \frac{\Omega - \Omega_{\rm p}}{\Omega} \right|$$

- Sonic condition for wave dissipation also tells us something about this:

$$\xi \approx \frac{c_{\rm s}}{v_\phi} = \frac{H}{R}$$

- To the extent that the disc is thin (*H*<<*R*), global behaviour should be negligible

- Possible to construct local, viscous models of disc evolution (Clarke 2009, Rafikov 2009)



Convergence of numerical results

- It is well known that for short cooling ties the disc is subject to fragmentation
- Meru and Bate (2011) show that such simulations are not converged
- As resolution increases fragmentation appears to become effective for longer cooling times

Simulation by Peter Cossins

$\beta = 4$

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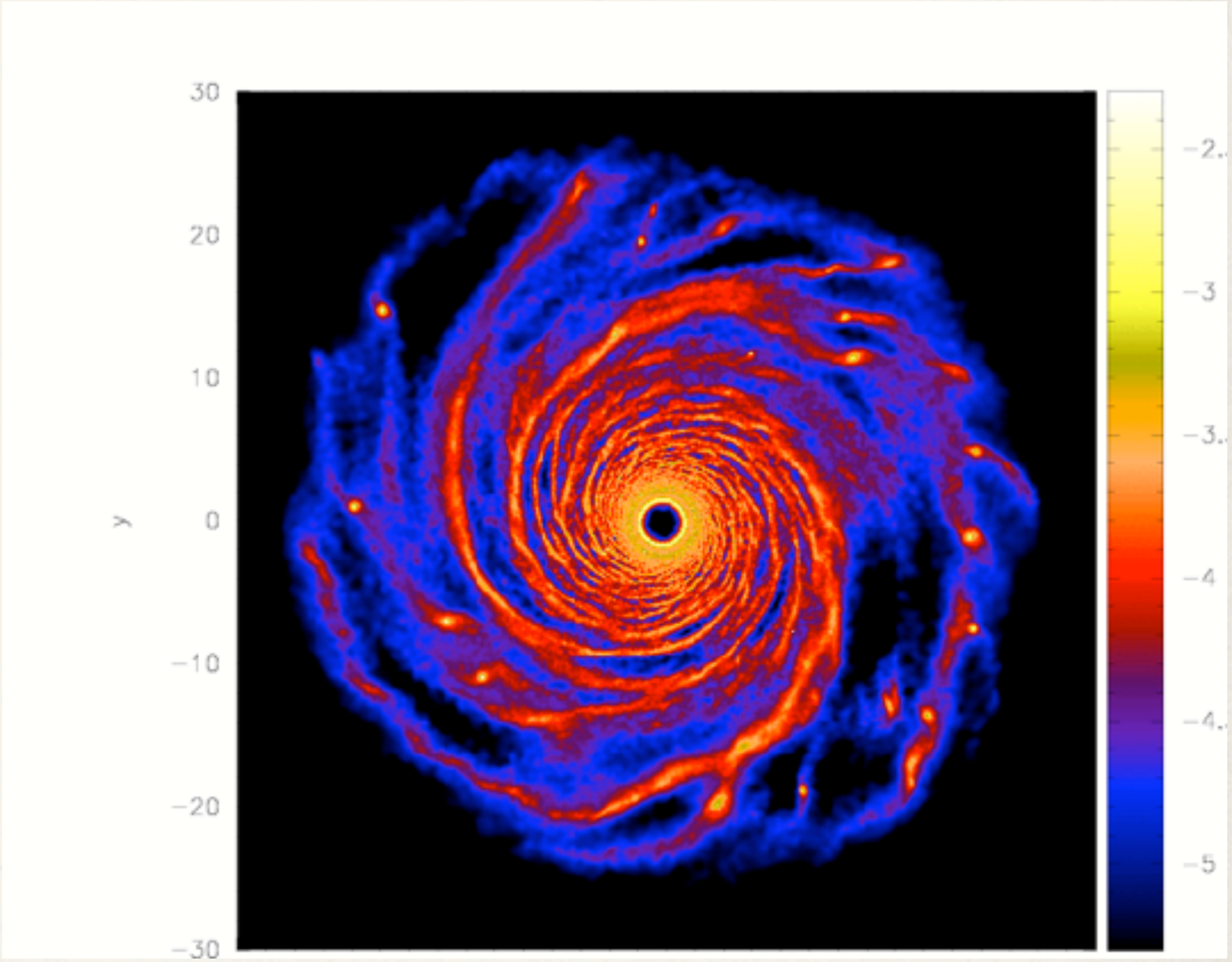

Simulation by Peter Cossins

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Convergence of numerical results

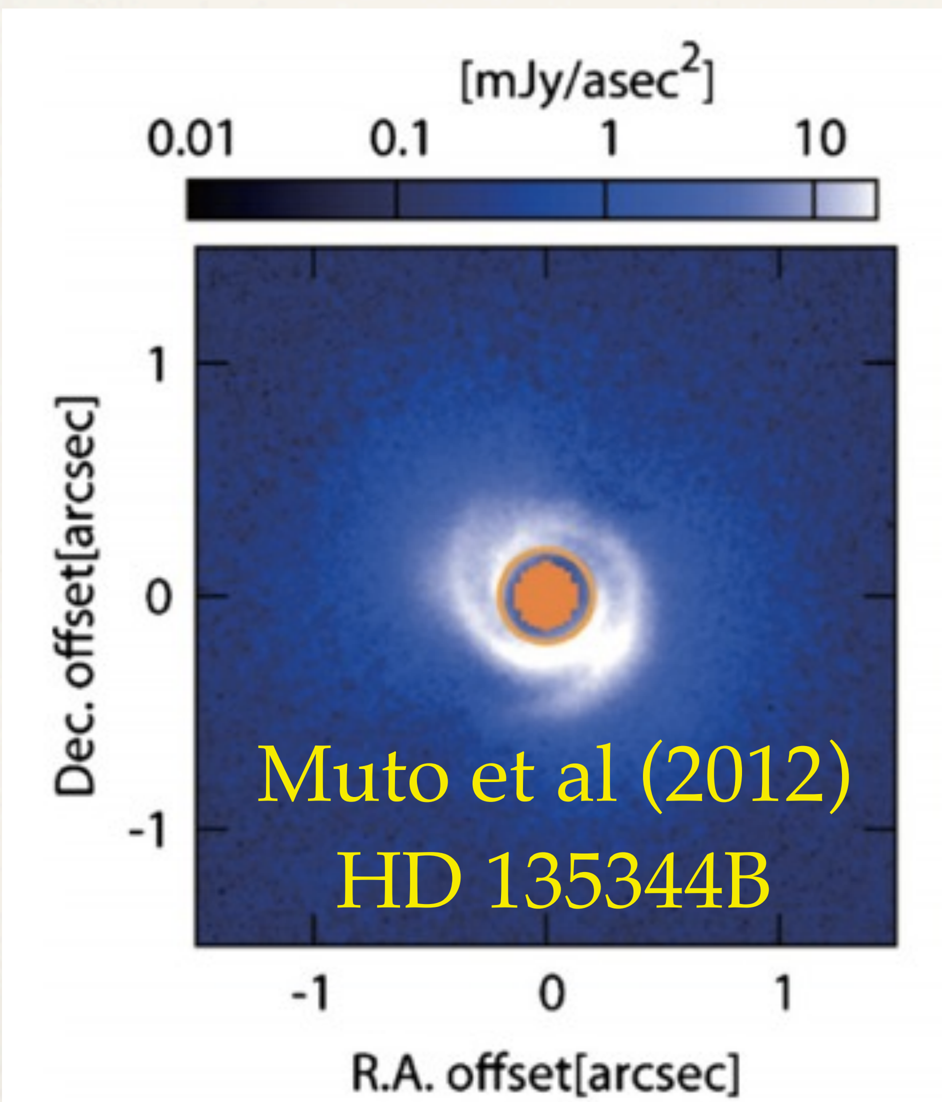
- Non-convergence most likely due to the effects of artificial viscosity in SPH simulations: additional viscosity (or any heating terms) weakens the gravitational instability (Lodato & Clarke 2011)
- Meru and Bate (2012): result converge at extremely high resolution. Fragmentation for $\beta < 20$
- Paardekooper (2012): non-convergence observed in 2D grid-based simulations. Fragmentation seen as a stochastic process
  - Essential to compute the likelihood of fragmentation in realistic protostellar discs (Hopkins and Christiansen 2013)

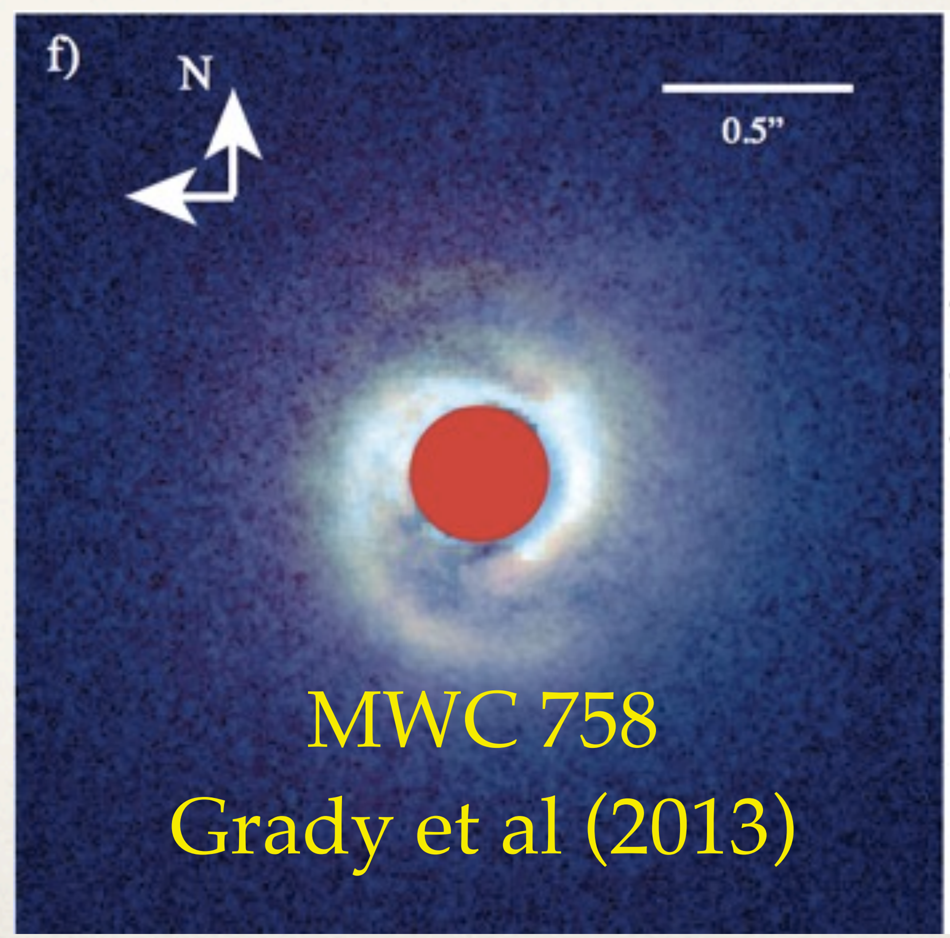
Convergence of numerical results

- What about convergence of results in the non-fragmenting limit?
- Is the stress and effective alpha converged?
- Michael et al (2012) (the Indiana group): convergence of measured stress observed in grid-based simulations
  - At low resolution alpha appears to be overestimated (more power in large scale structures ---> potentially more global transport)

Observing density waves in protostellar discs

- Several discs with spiral structures observed in scattered light
- Most of these are very evolved systems (transitional discs): most likely the origin of the spiral is not due to self-gravity

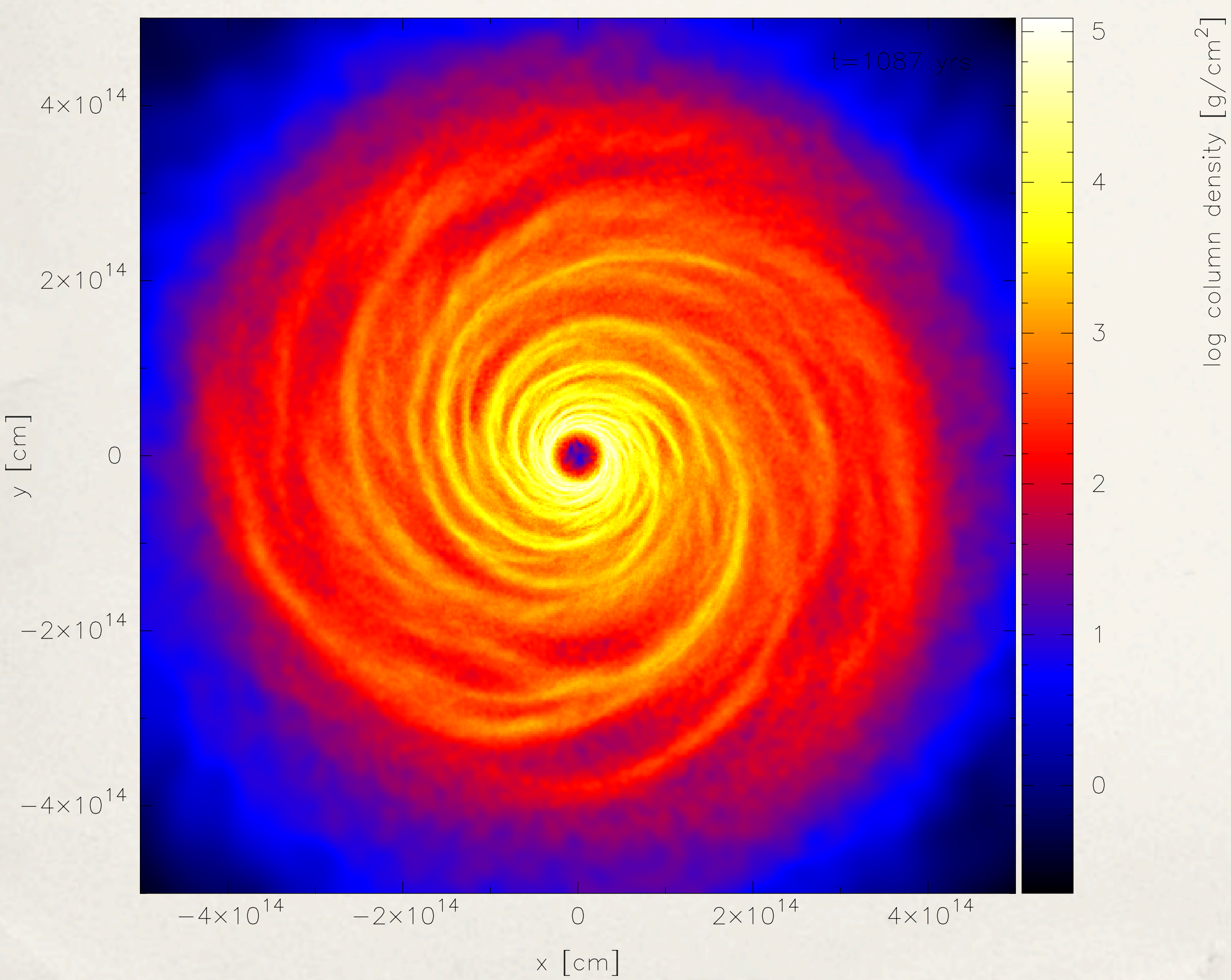


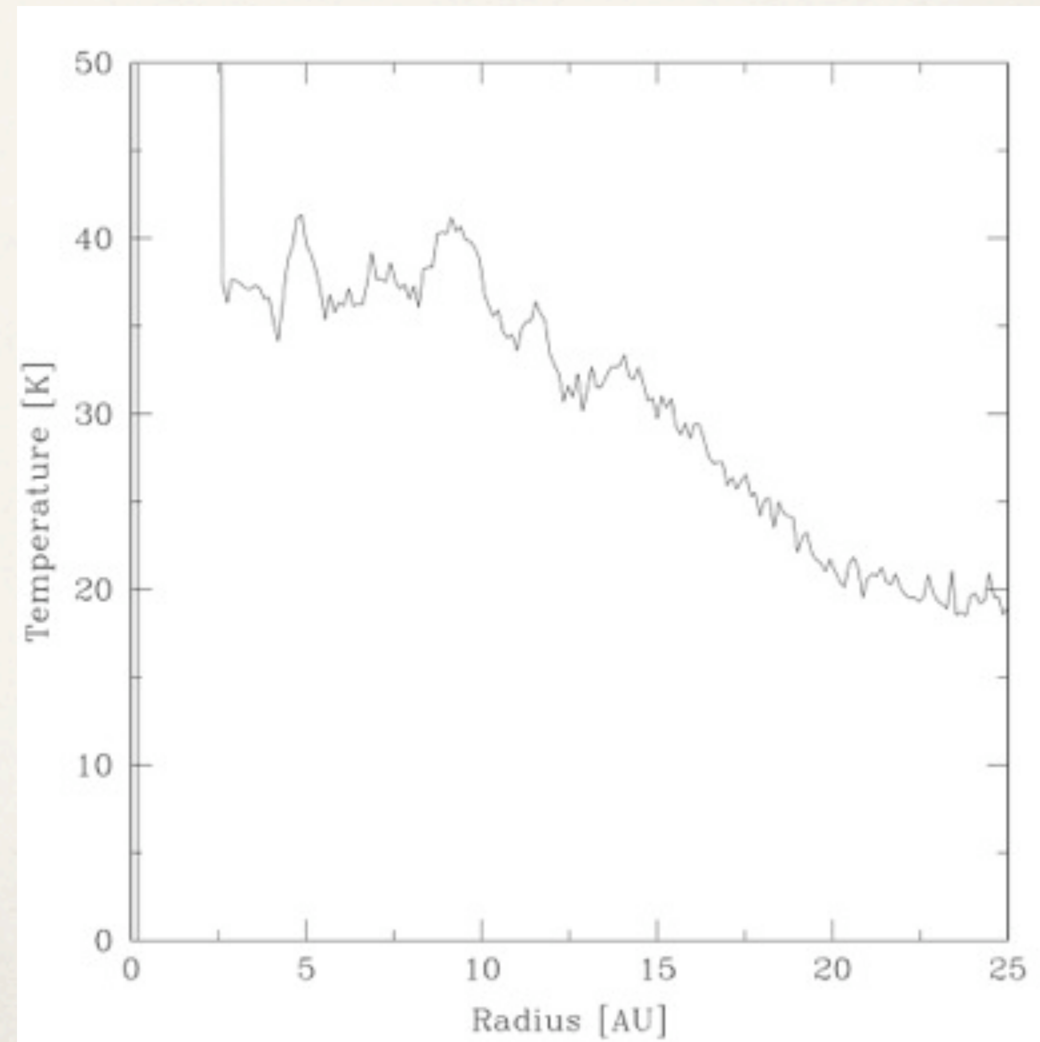


Self-gravitating discs with ALMA

Cossins, Lodato & Testi (2010)

- Start from one of our simulations, e.g. $M_* = 1M_{Sun}$, $M_{disc} = 0.2M_*$
- "Place" disc at 140pc (in Taurus) or at 50 pc (distance to TW Hya)
- Assume a "standard" opacity law





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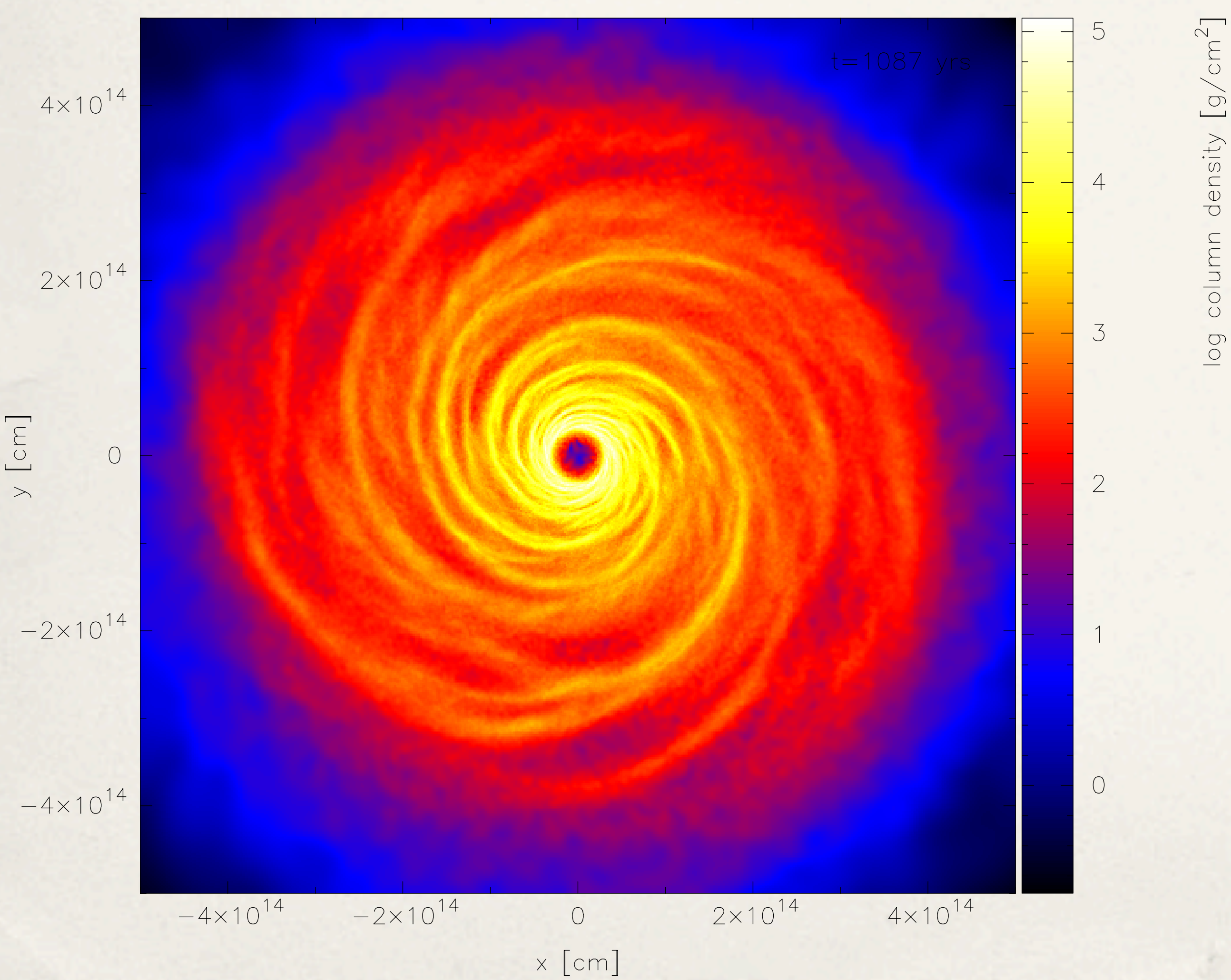

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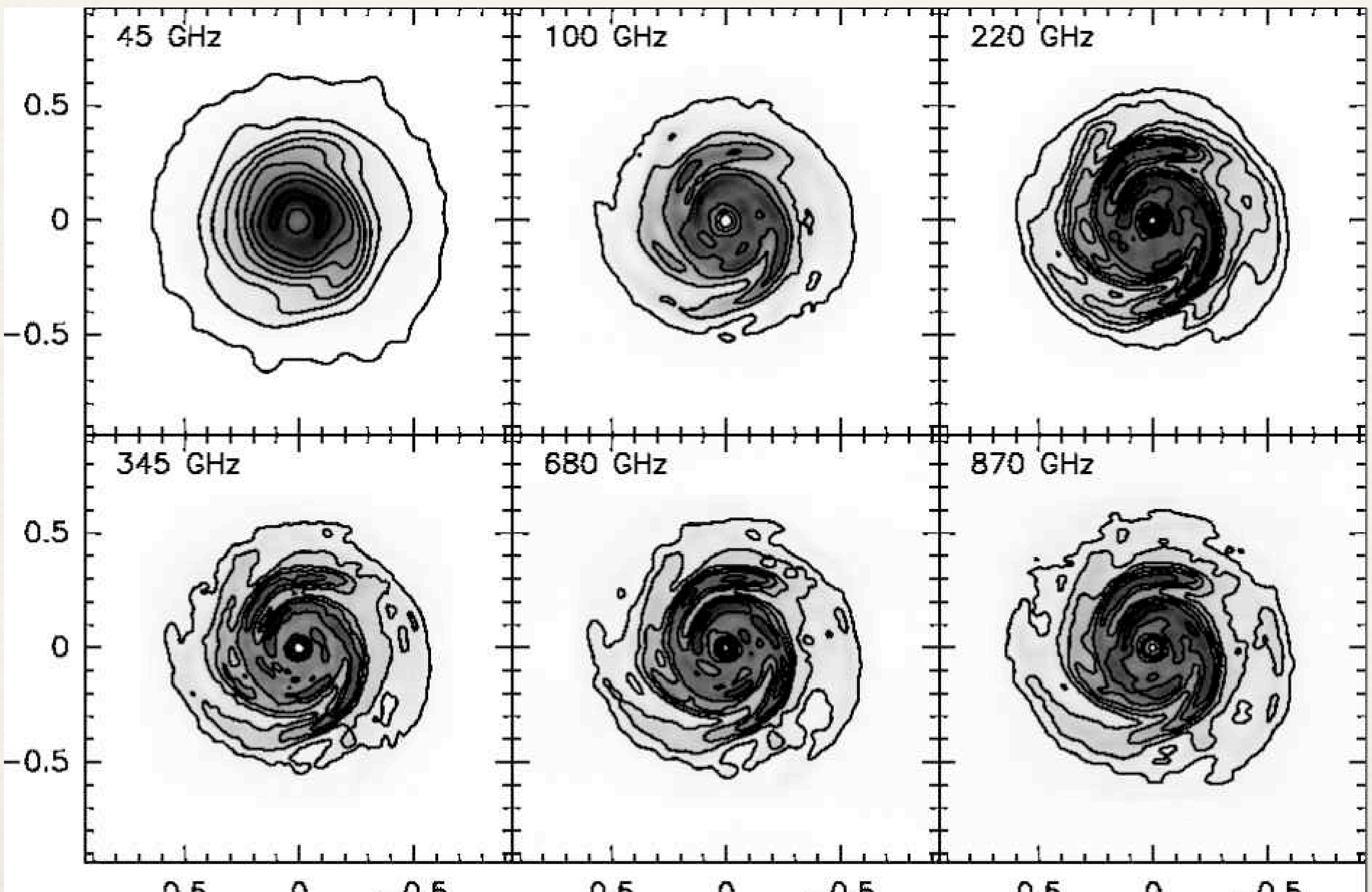

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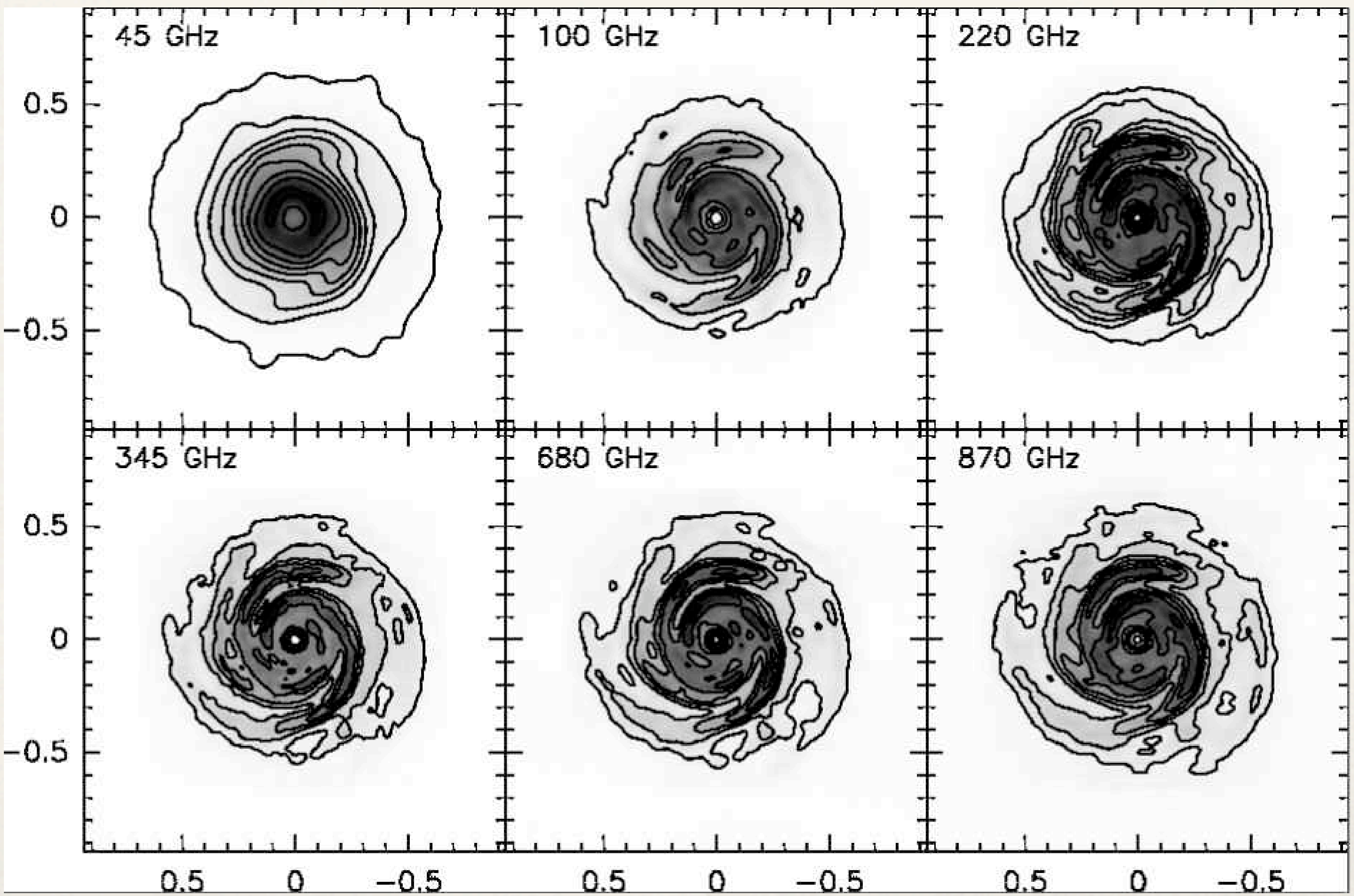
Simulated ALMA images at 50 pc



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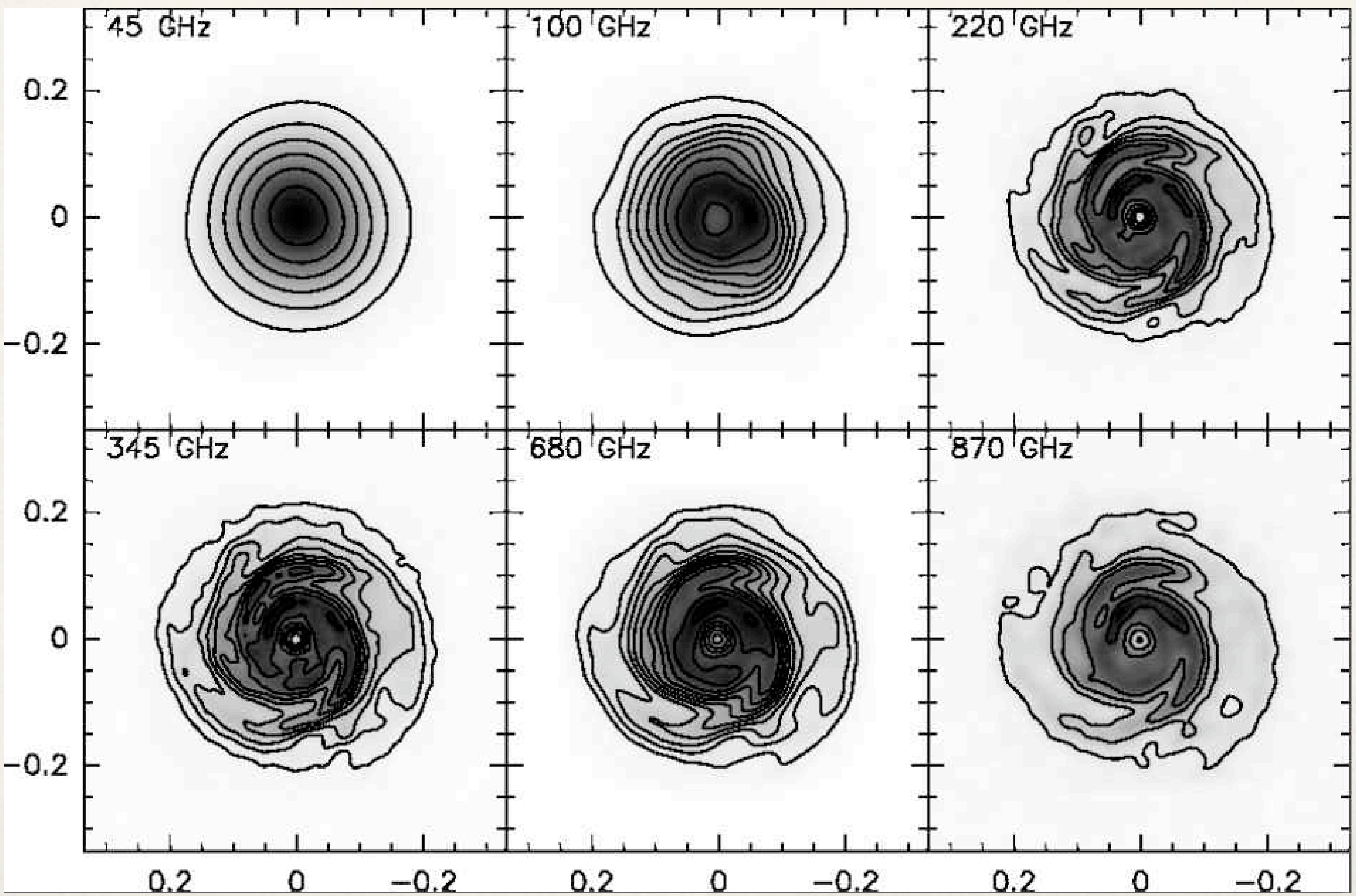
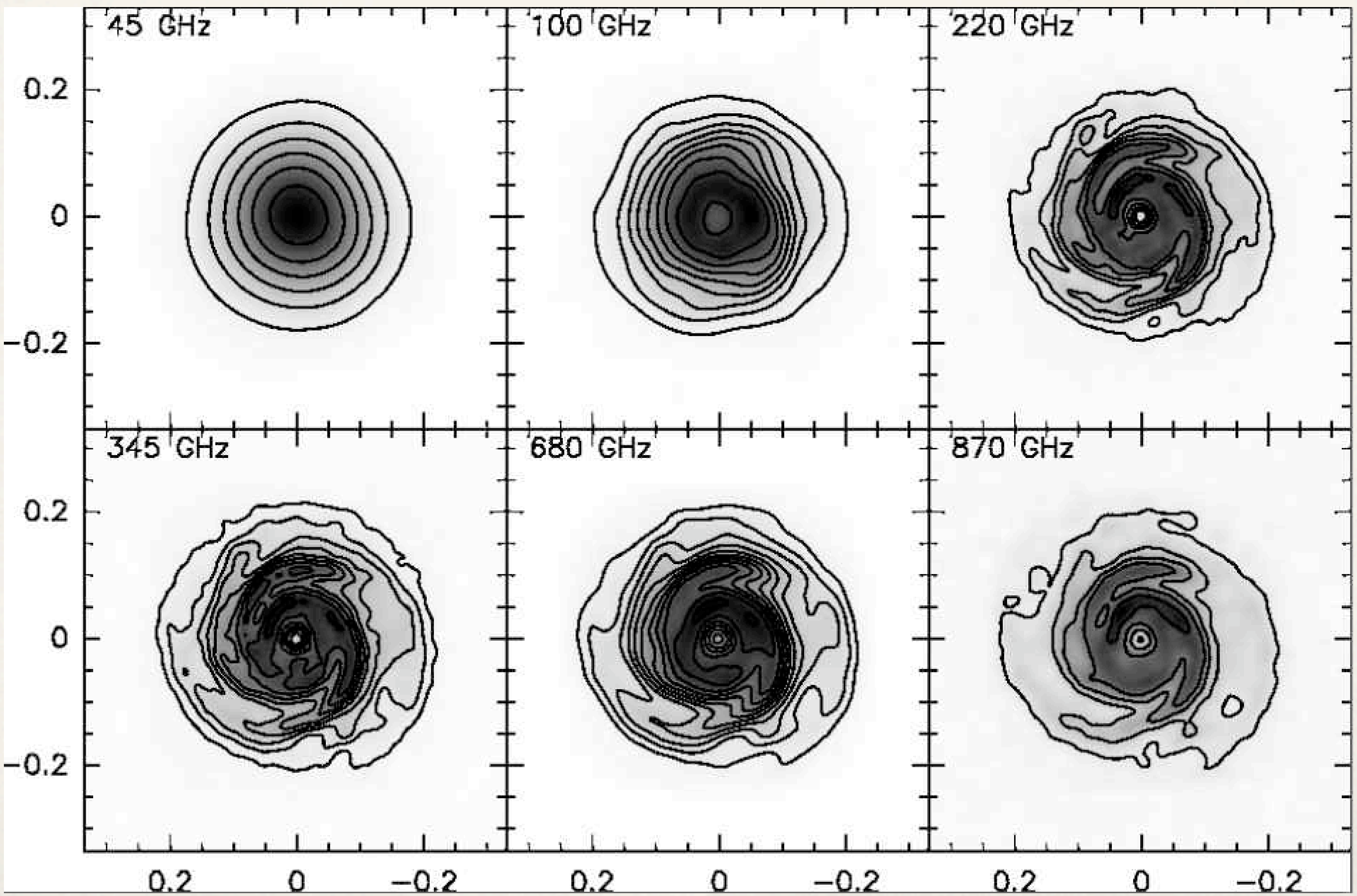


Self-gravitating discs with ALMA

Cossins, Lodato & Testi (2010)

Simulated ALMA images at 140 pc



Conclusions

- Young protostellar discs are likely to be gravitationally unstable
- Self-regulated evolution of GI leads to sustained angular momentum transport for ~ 1 Myr, bringing the disc into the T Tauri phase
- Density waves dissipate when they become sonic
- Induced transport is local IF disc is sufficiently thin
- GI could lead to fragmentation: exact fragmentation conditions unfortunately strongly affected by numerical resolution
- ALMA could be able to detect such discs very soon!