THE SUCCESS AND TRENDS OF HYDRODYNAMIC STABILITY RESEARCH

Jiachun Li
Institute of Mechanics, CAS
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In the memory of Professor C.C. Lin

Professor C.C. Lin

(1916~2013)

Fluid Dynamicist

Astrophysicist

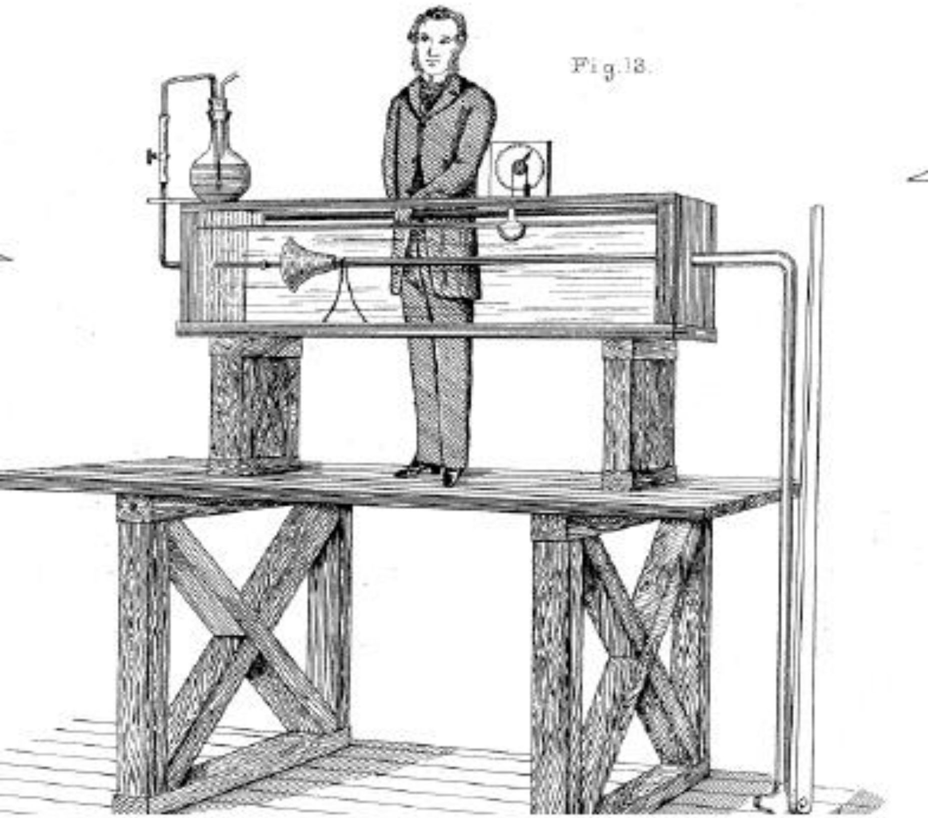
Applied Mathematician

One of the successor of Applied Mechanics school

Applied Mechanics School

- Background
- Lin’s contributions to hydrodynamic stability
- Nonlinear theory
- Research trends
- Concluding remarks

1) Background: O. Reynold’s experiment and paper in Phil. Trans. Roy. Soc. (1883)



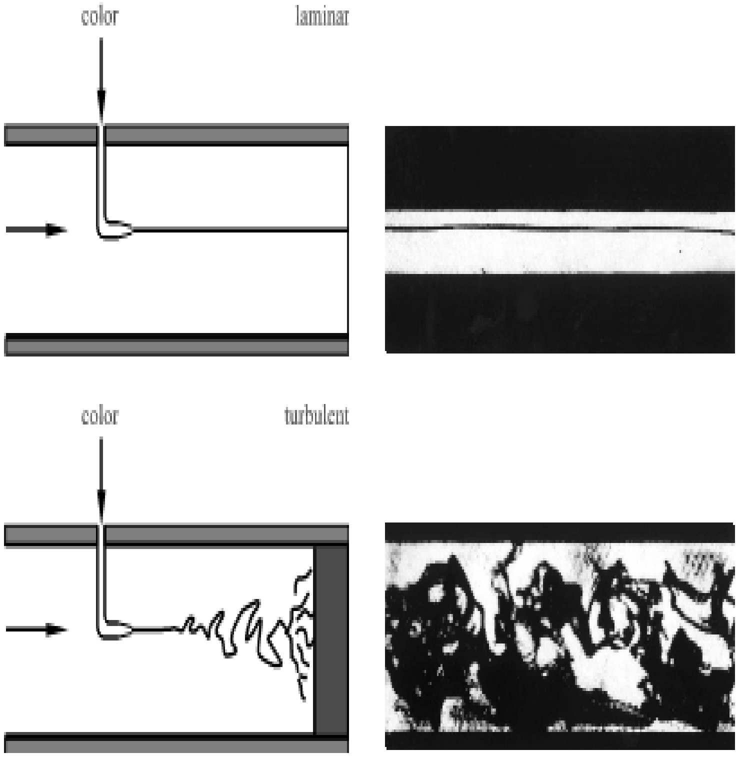


Fig. 4.52. Laminar and turbulent pipe flow, *O. Reynolds* 1883

As a milestone at the turn of 19-20 century

- Reynold’s significant discoveries:
  - Two states of motions: laminar or turbulent flows with direct or sinuous streamlines
  - The non-dimensionless parameter known as Reynolds number is decisive;
  - Remarkably difference in resistance at two states;
- 100 year’s exploration of the following questions:
  - What is the mechanism of transition from laminar flow to turbulence;
  - What is the process or route of such transition ;
  - How do you describe the fully developed turbulence and calculate turbulent flows;

2) Lin's contributions to hydrodynamic stability theory

- Comprehensive study on the stability of plane Poiseuille flow
- Exploration of two roles of viscosity
- Extended WKB method in solving Orr-Sommerfeld Eq.
- The theory was applied in boundary layer problem with parallel flow approximation;

Hydrodynamic Stability Theory (linear)

- Assumptions:
  - transition is attributed to hydrodynamic instability;
  - Unavoidable small disturbances are consisting of normal modes;
  - Transition occurs when disturbance grows;
- Queries:
  - This kind of transition is spontaneous or dependent on external forcing?
  - The solution of the Orr-Sommerfeld equation contradicts with experiments;
  - People didn't see TS wave of linear theory in experiments;

Orr-Sommerfeld Equation (1907-1908)

$$(D^2 - \alpha^2)^2\phi = i\alpha R[(U - c)(D^2 - \alpha^2) - D^2U]\phi$$

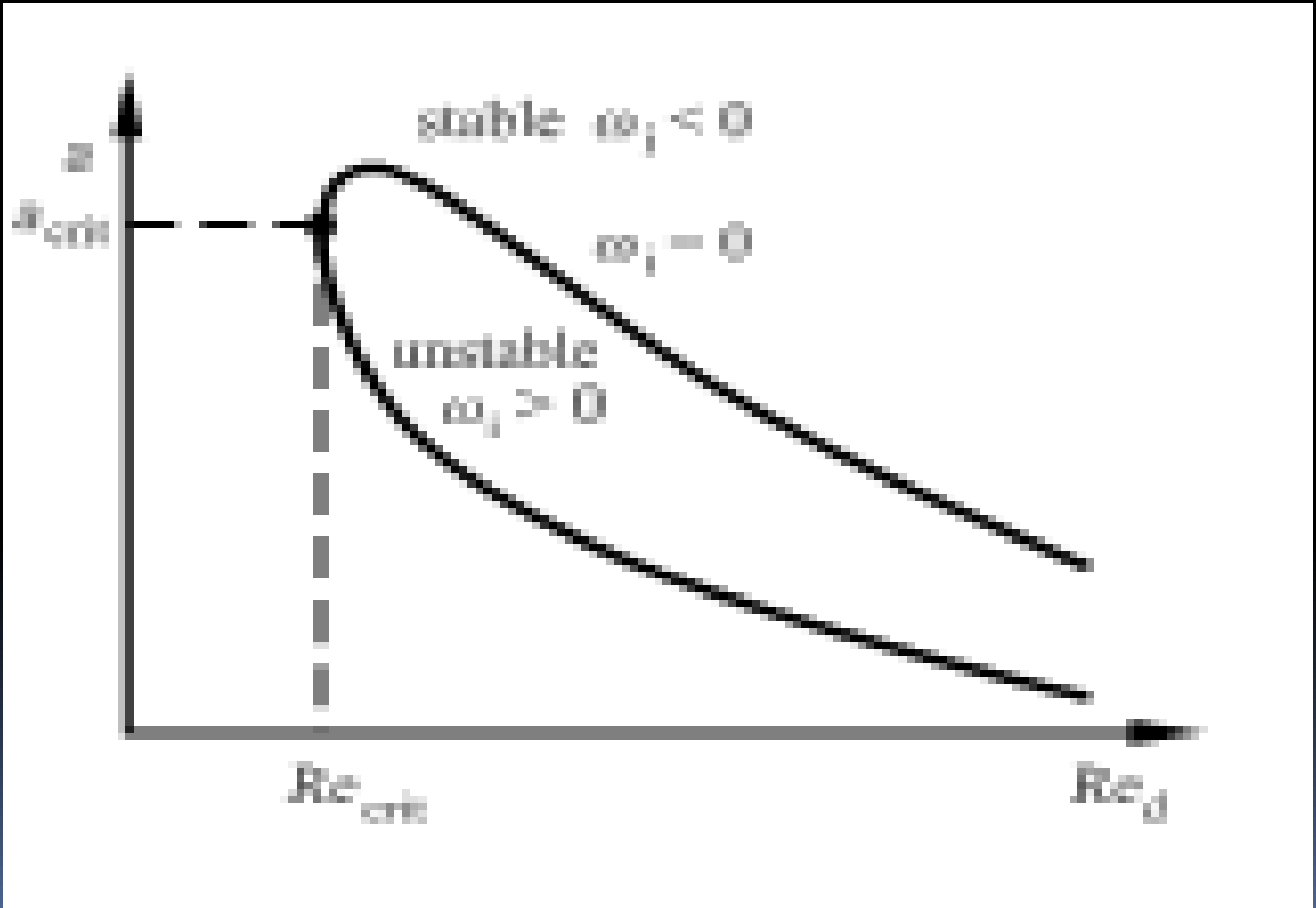
$$\left.\begin{aligned} \phi(y_1) &= \phi(y_2) = 0 \\ \phi'(y_1) &= \phi'(y_2) = 0 \end{aligned}\right\}$$

- Eigenvalue problem
  - Stable: if all of the eigenvalues are with negative imaginary part;
  - Unstable: if only one eigenvalue with positive imaginary part

Asymptotic method in hydro- dynamic stability

- There was no computers available;
- WKB method was successfully applied in quantum mechanics including turning point by Matched asymptotic method
- How to use asymptotic method in stability analysis
  - Fourth order ODE ;
  - There are two boundary layers (a Stokes layer at the wall and a critical layer nearby the turning point where main flow speed equals phase speed of disturbance waves);

Neutral stability curve



The roles of viscosity (by C.C. Lin)

- Viscosity usually dissipates the energy of disturbance to stabilize the flow;
- However, when phase difference is suitable, disturbance might extract energy from the main flow by positive work due to Reynolds stress;
- Analysis was given by C.C.Lin

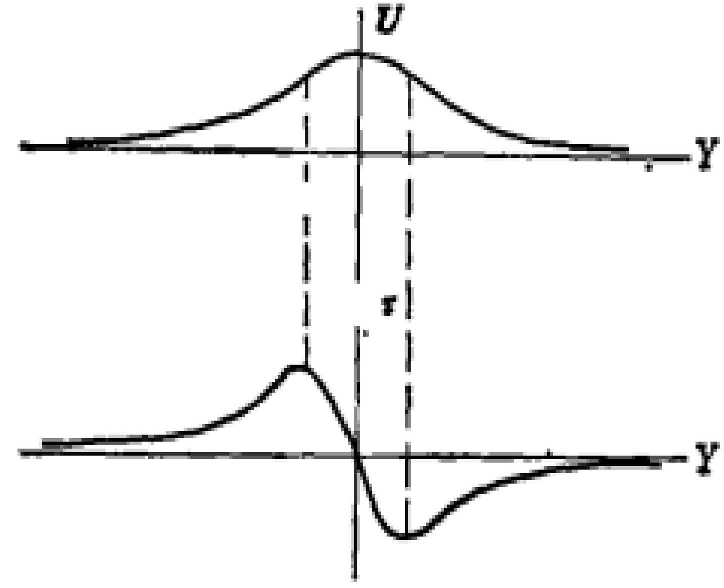


图 4 自激对称射流的速度和 Reynolds 应力分布

$$P=\int \tau \frac{dU}{dy} dy>0$$

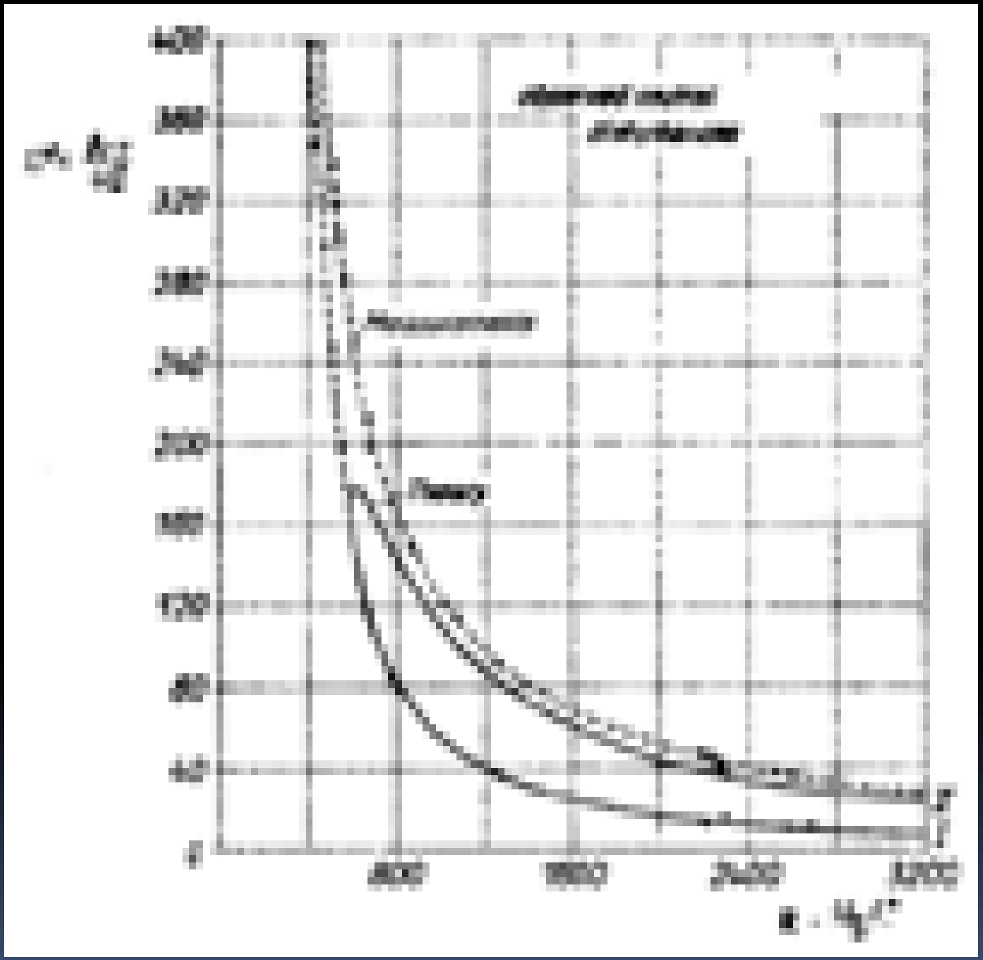
Lin’s studies in this field

- Plane Poiseuille flow (Lin 1946)
- Boundary layer over a flat plate (Shen & Lin, 1954)
- Compressible boundary layer flow (Lin & Lees 1958)

Comparison with experiments

- By Schubauer and Scramatad at NBS for BL Flow
  - In a wind tunnel of low turbulent (intensity less than 0.02%)
- TS wave was identified

By Nishioka (1975) for the plane poiseulille flow



Numerical computation verification

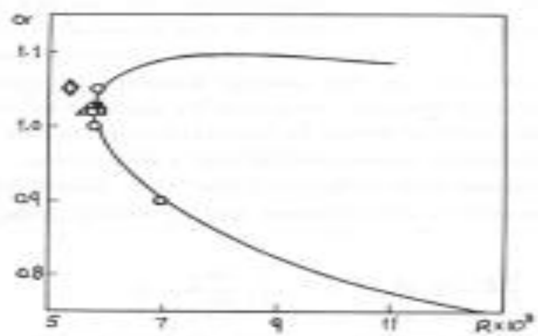


Fig. 3 Critical parameters
◇ Lin, Shen (1954)
● Thomas (1953)
□ Orszag (1971)
△ Lakin (1978)
○ Li, Zhao (1982)

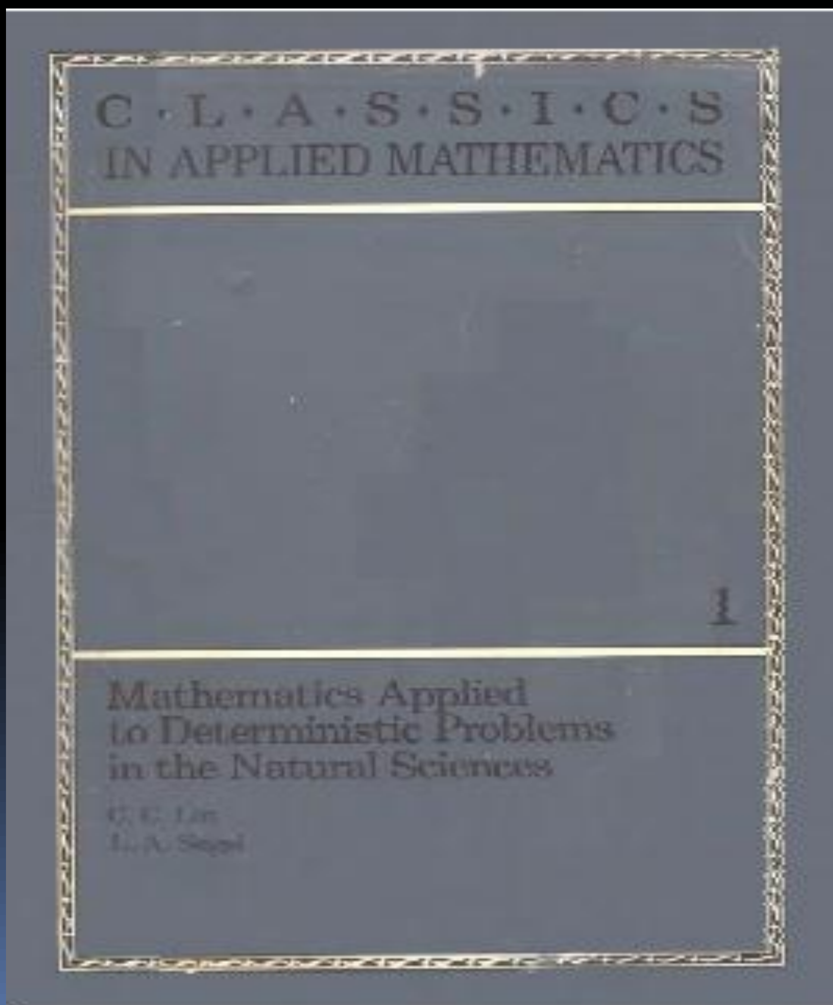
Table 2. Critical parameters

Lin, Shen (1954)	1.05	5360
Thomas (1953)	1.026	5780
Grosch (1968)	1.025	5750
Orszag (1971)	1.020	5772
Lakin et al. (1978)	1.0207	5769.7
Li, Zhao (1982)	1.018	5770

CONCLUDING REMARKS

- FDM (Thomas)
- OFE (Orszag)
- Green Function (Li & Zhao)

Contributions to applied mathematics (C.C. Lin and T.A. Segel)



Mathematics Applied to Deterministic Problems in the Natural Sciences

C. C. Lin *Massachusetts Institute of Technology*
L. A. Segel *Weizmann Institute of Science*

with material on elasticity by
G. H. Handelman *Rensselaer Polytechnic Institute*

siam

The nature of applied math. by C.C. Lin

The purpose of applied mathematics is to elucidate scientific concepts and describe scientific phenomena through the use of mathematics, and to stimulate the development of new mathematics through such studies. The process of using mathematics for increasing scientific understanding can be conveniently divided into the following three steps:

(i) The **formulation** of the scientific problem in mathematical terms.

(ii) The **solution** of the mathematical problems thus created.

(iii) The **interpretation** of the solution and its empirical verification in scientific terms.

There is widespread misunderstanding that the second step is the most important and that manipulative skill is the most valued asset of an applied mathematician. Generally speaking, however, all three steps are equally important. In a given class of problems, one step might stand out as more important or more difficult than another.

Theodore Von Karman:

- It is a rare example of cooperation between "the men of mathematics"--- as my friend Eric T. Bell called them ---and creative engineers. Mathematical theories from happy haunting ground of of pure mathematics were found suitable to describe the airflow produced by aircraft with such excellent accuracy that they could be directly applied in airplane design.

3) Nonlinear theory (Landau equation)

$$\frac{d|A|^2}{dt} = 2\sigma|A|^2 - l|A|^4$$

$$\sigma = k(R - R_l) + O\{(R - R_l)^2\}$$

$$|A|^2 = A_0^2 \Big/ \left\{ \frac{l}{2\sigma}A_0^2 + \left(1 - \frac{l}{2\sigma}A_0^2\right)e^{-2\sigma t} \right\}$$

（1）当 $l>0$ 时，若 $R<R_L$，则 $|A|\to 0 \quad (t\to\infty)$；

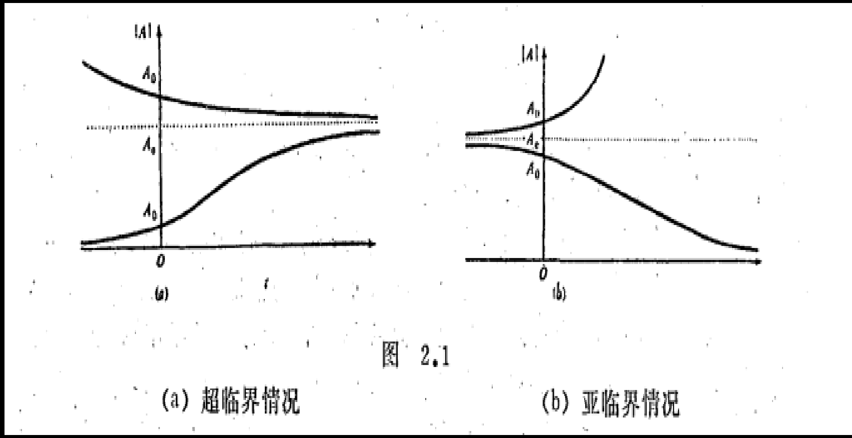
若 $R>R_L$，则 $|A|\to\left(\frac{2\sigma}{l}\right)^{\frac{1}{2}} \sim \left[\frac{2k(R-R_L)}{l}\right]^{\frac{1}{2}} \quad (t\to\infty)$

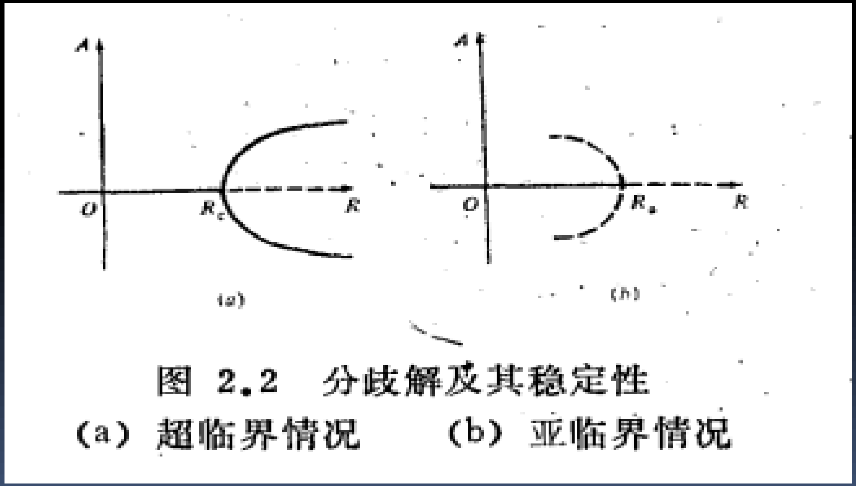
（2）当 $l<0$ 时，若 $R>R_L$，则 $|A|\to\infty$；

若 $R<R_L$，$A_0<A_T=\left(\frac{2\sigma}{l}\right)^{\frac{1}{2}}$，则 $|A|\to 0 \ (t\to\infty)$

$A_0>A_T=\left(\frac{2\sigma}{l}\right)^{\frac{1}{2}}$，则 $|A|\to\infty \ \left(t\to\frac{1}{2\sigma}\ln\left(1-\frac{2\sigma}{lA_0^2}\right)\right)$

Super- & sub- critical transition modes





- Supercritical mode:
  - Continuous bifurcation leads to turbulence
  - In reality, only finite bifurcations happen
- Subcritical mode
  - Finite amplitude disturbance leads to by-pass transition

Transition process

- Five stages:
  - Receptivity stage: the disturbance out of BL (vortex, sound, entropy waves) is perceived
  - Linear instability stage: T-S wave growth to saturation:
  - Secondary instability stage: 3D instability, triad resonance, K type & N (H & C) type instability;
  - Varieties of eddies, strong shear layer, high and low speed streaks, turbulent spots;
  - Break down and fully developed turbulence

4) Research trends

Basic researches:

- Non-normal mode approach (Initial value problem) with continuous spectrum
- By-pass transition;
- Mechanism of Hagen-Poiseuille and plane CoUette flows

- Applications in science and engineering
  - Location of transition point (for drag prediction)
  - Flow stability of swept wing with transverse flow (for flow control)
  - Stability of stokes layer (for sediment movement)
  - Benjamin-Feir instability of Stokes wave (for wind wave spectrum evolution)
  - R-T instability (for ICF)
  - S-T instability (for EOR)

Hagen –Poiseuille flow-an open classic problem

- Stable for infinitesimal disturbance
- we may have several critical Reynolds numbers by theory or observations:
  - Re(E)~81.5 disturbances monotonically decay
  - Re(G)~1250, disturbances finally decay
  - Re(ext) ~2000, small probability of transition to turbulence;
  - Re(m)_40000, the maximal Re at which the laminae flow can be maintained;
- Coherent structures such as vortices, streaks, slug and puff as turbulent sections ?
- The mechanism were explained by finite amplitude disturbance?
- Incorporation in dynamic system theory?

Full developed turbulence

- Turbulent studies have made great achievements in the last century
  - Reynolds EQ. & Prandtl mixing length theory
  - Taylor's statistical theory and Kolmogorov scale law for isotropic homogeneous turbulence
  - Coherent structure since 1970s
  - Turbulent modeling applied in engineering design and the development of LES ;
  - Wide applications in engineering such as multiphase flow, CAA, combustion and natural flows

Concluding remarks

- Professor C.C. Lin made contributions of fundamental importance and was among the great masters ,who laid the foundation of hydrodynamic stability theory;
- When linear theory become mature, a great many issues concerned with nonlinear theory are still challenging for us;

- Research in turbulence has got great achievements in the past century. With supercomputers and advanced measurement devices, potential progresses in theory, simulation and applications can be expected;
- We should continue to push forward the advances of applied mathematics and its practical applications to engineering, natural and biological sciences in the spirit of C.C. Lin as he insisted on.

Thank you !