What does amplify the spirals

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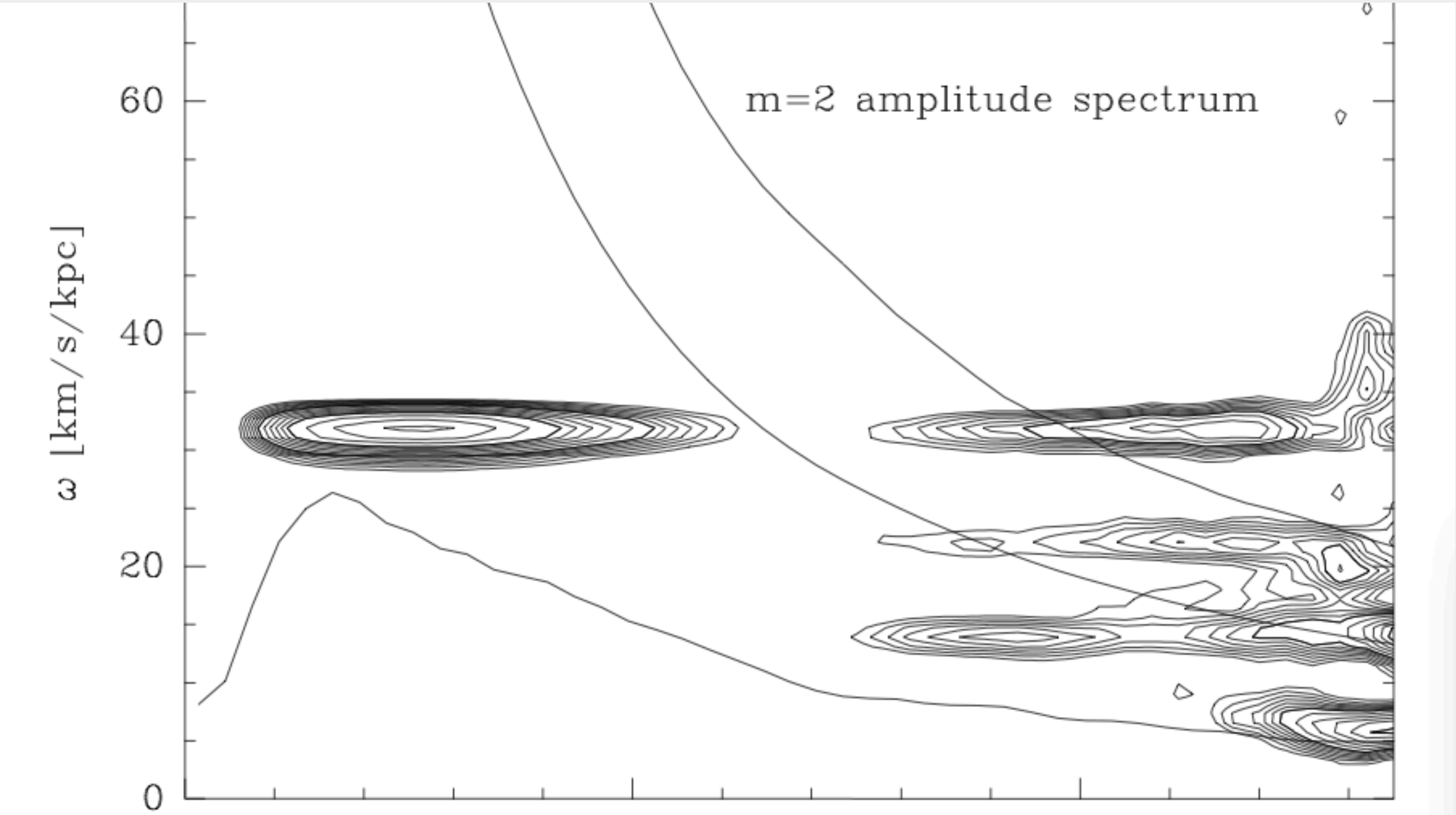
quick advertisement

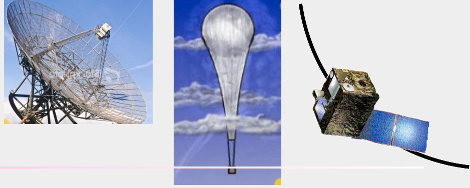


Non-linear coupling of spiral modes

Tagger et al. 1987

Sygnet&Tagger 1995

the amplification of spirals



- a very old problem!
    - Goldreich&Lynden-Bell, Toomre, Mark
    - C.C. Lin and coworkers

- discussed here: **the most basic problem,**

  reduced to its simplest expression
  - gaseous, 2D disk
  - standard shearing sheet

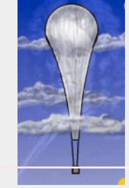
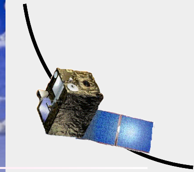
    (no profiles etc.)
  - no corotation resonance (Rossby waves)

the shearing transform

- in the very convenient form described by C.C. Lin & D. Thurstans
- separation of variables + Fourier -> goes from variables $(r,\ t)$ to k
- can describe traveling wave patterns as well as normal modes
- for detailed discussion, including Rossby waves (= corotation resonance) see Tagger 2001
- properly done in cylindrical geometry
- -> perturbations varying as $\exp i(ks + m\vartheta),\ s = \ln(r/r_0)$

and use the "horizontal" wavenumber $q = (k^2 + m^2)^{1/2}$

some mathematics…

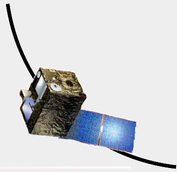
- in the shearing sheet, the problem reduces to a 2nd order ODE:

$$m^2\Omega'^2 \frac{d^2}{dk^2}\Phi + \mathcal{S}(k)\Phi = 0$$

- where the "spring constant"

$$\mathcal{S}(k) = \kappa^2 - 2\frac{\kappa c_s}{Q} q + q^2 c_s^2 + 4\Omega\Omega' \frac{m^2}{q^2} + 3\Omega'^2 \frac{m^4}{q^4}$$

the spring constant

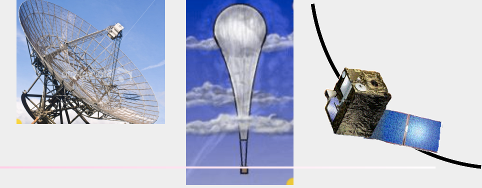
$$S(k) = \kappa^2 - 2\frac{\kappa c_s}{Q} q + q^2 c_s^2 + 4\Omega\Omega' \frac{m^2}{q^2} + 3\Omega'^2 \frac{m^4}{q^4}$$

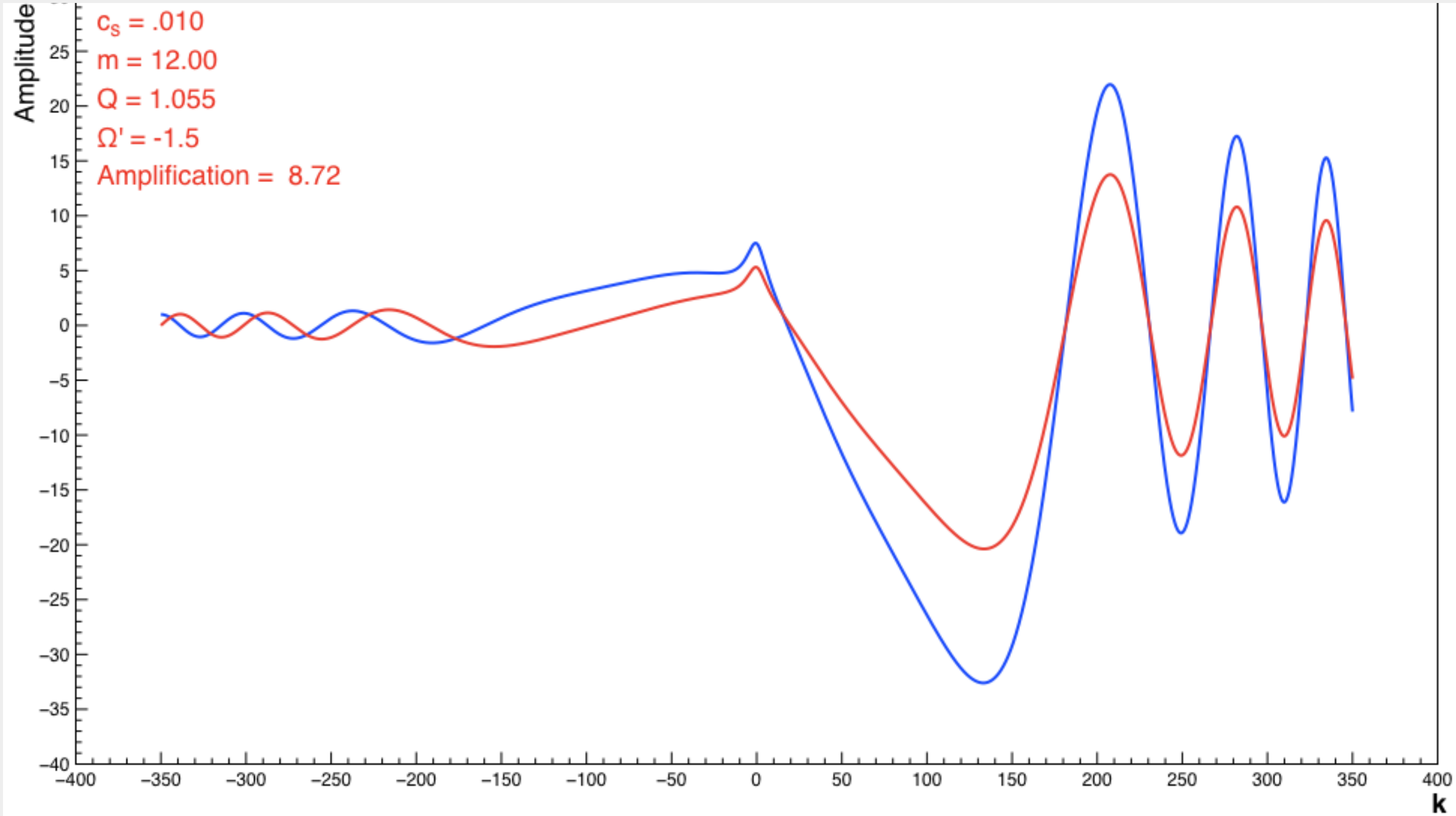
the "WKB" terms

additional dynamical terms,
small at large |k/m|
(tightly wound waves)

$\mathcal{S}$ contains all the physics of the problem;
the WKB terms describe the Lin-Shu waves

numerical integration

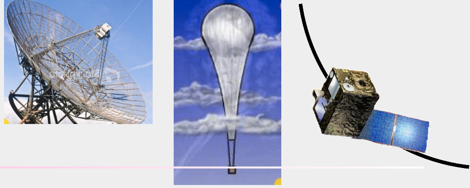


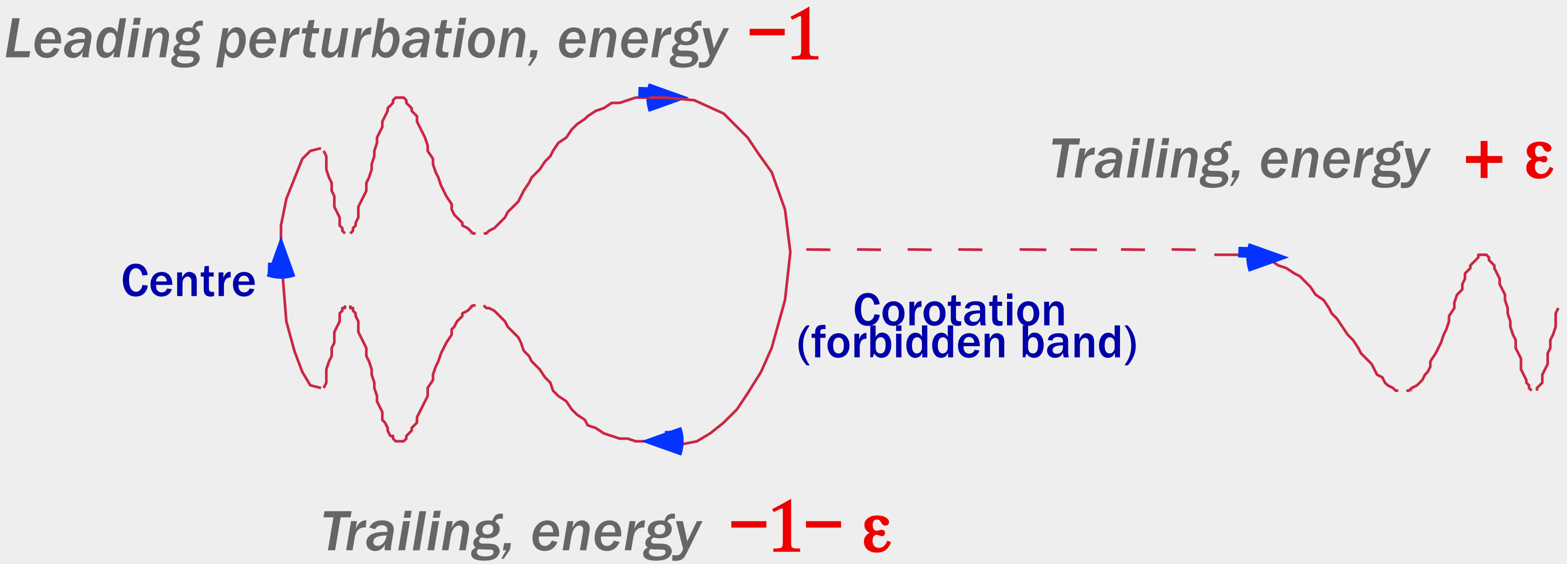


shows extremely strong amplification as

- the perturbation is sheared from leading to trailing
- the wave is reflected near corotation

amplification mechanisms



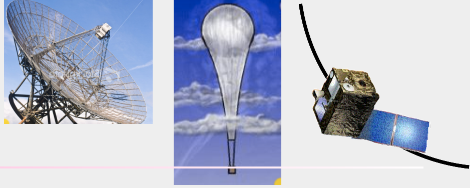


Essential : waves have **negative energy** inside corotation

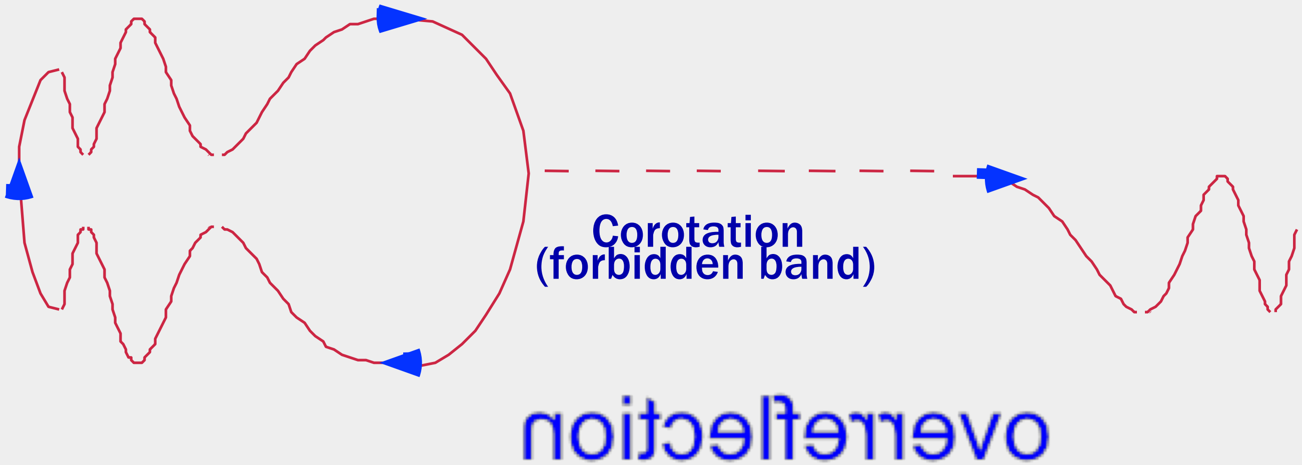
positive energy beyond corotation

→ **any transmission across corotation implies amplification**

two amplification mechanisms

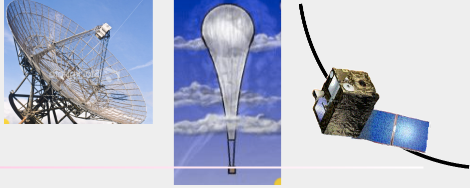


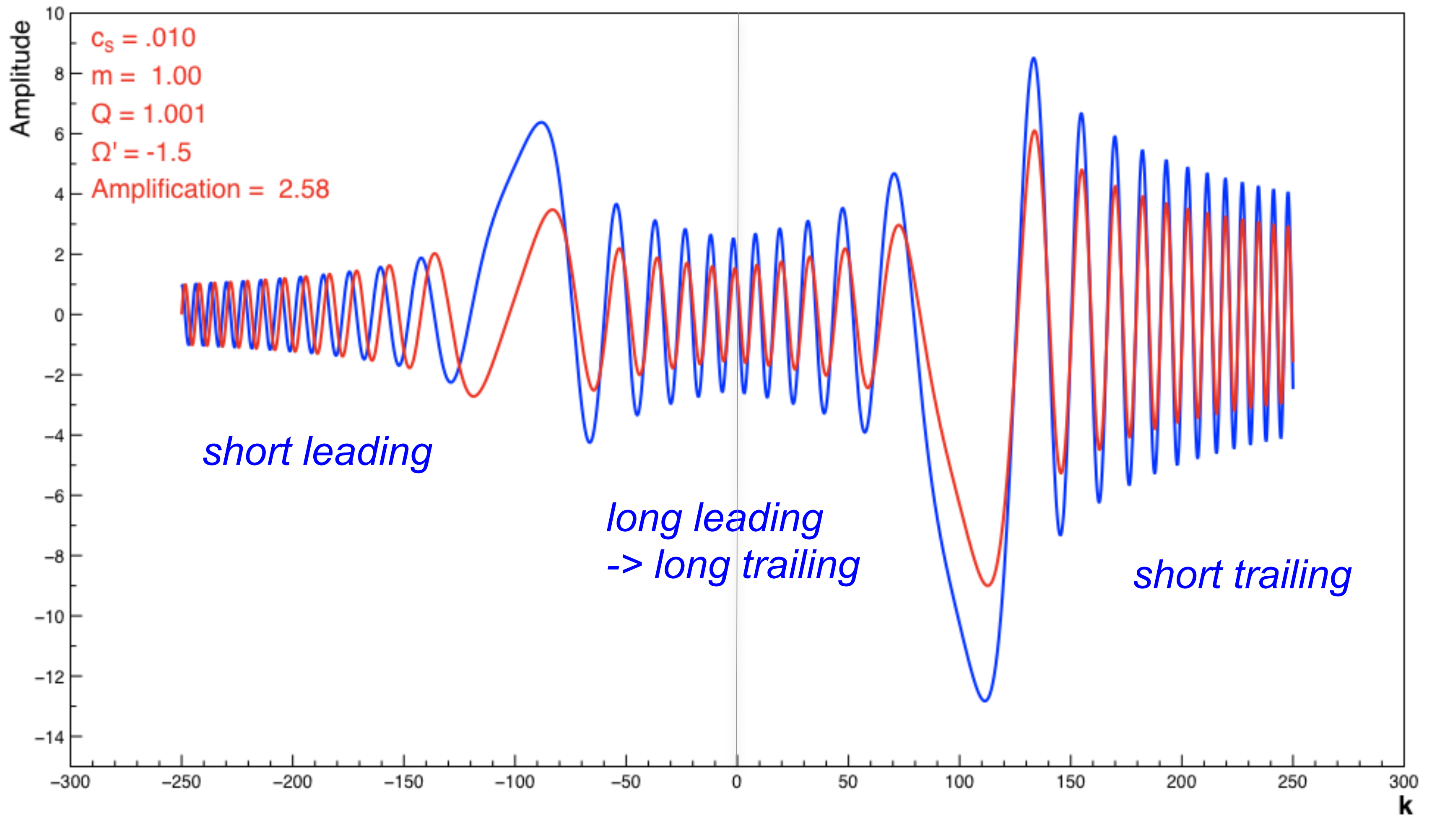
WASER (Mark ; Lin and coworkers, a few of them here) :



- as a long wave reflects into a short wave at its turning point
- maximum amplification $2^{1/2}$ for Q=1

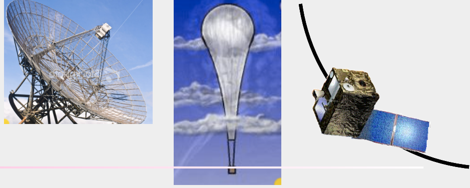
two Wasers

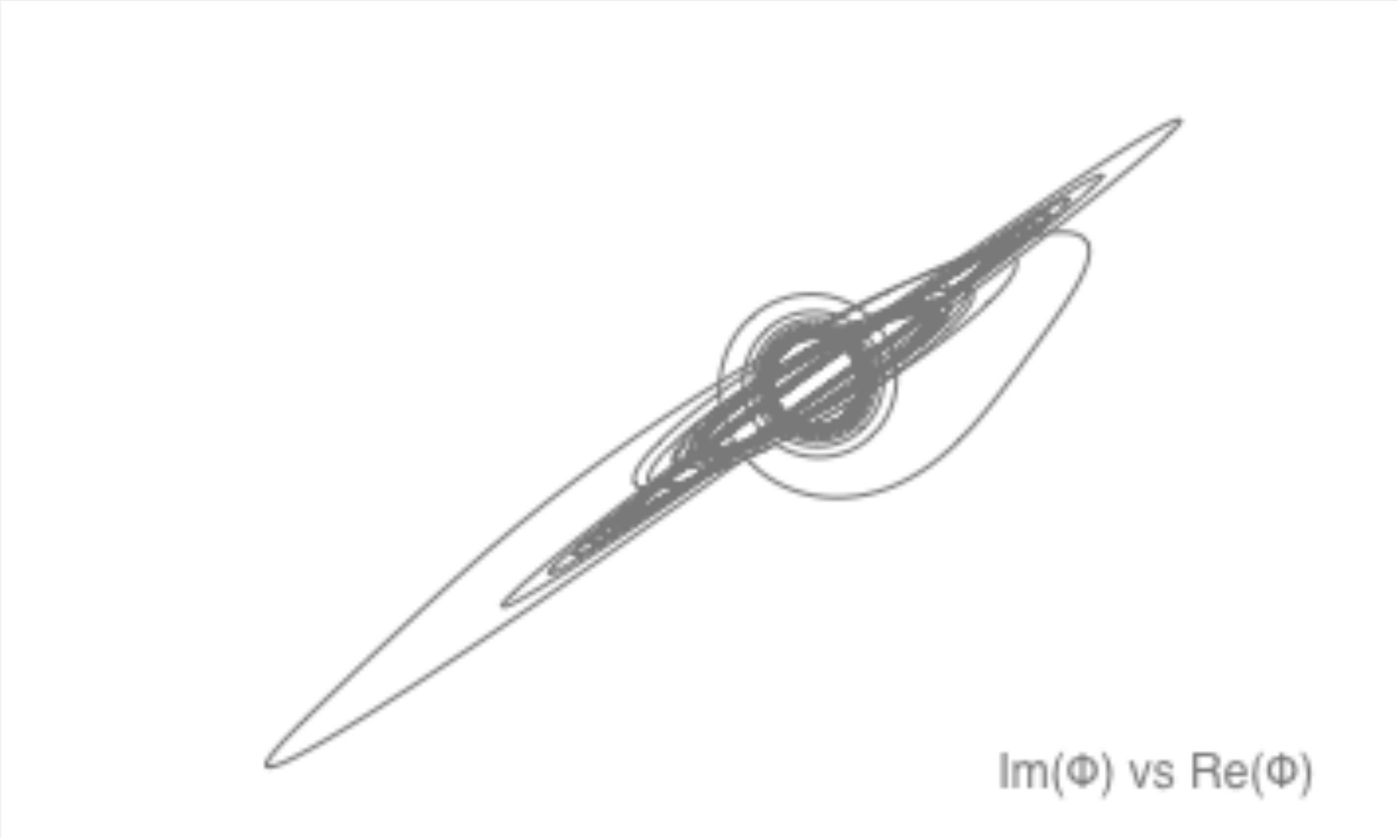




c_s = .010
m = 1.00
Q = 1.001
Ω' = -1.5
Amplification = 2.58
Amplitude
k
short leading
long leading
-> long trailing
short trailing

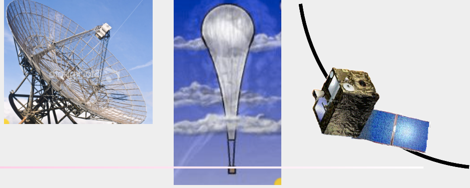
two Wasers

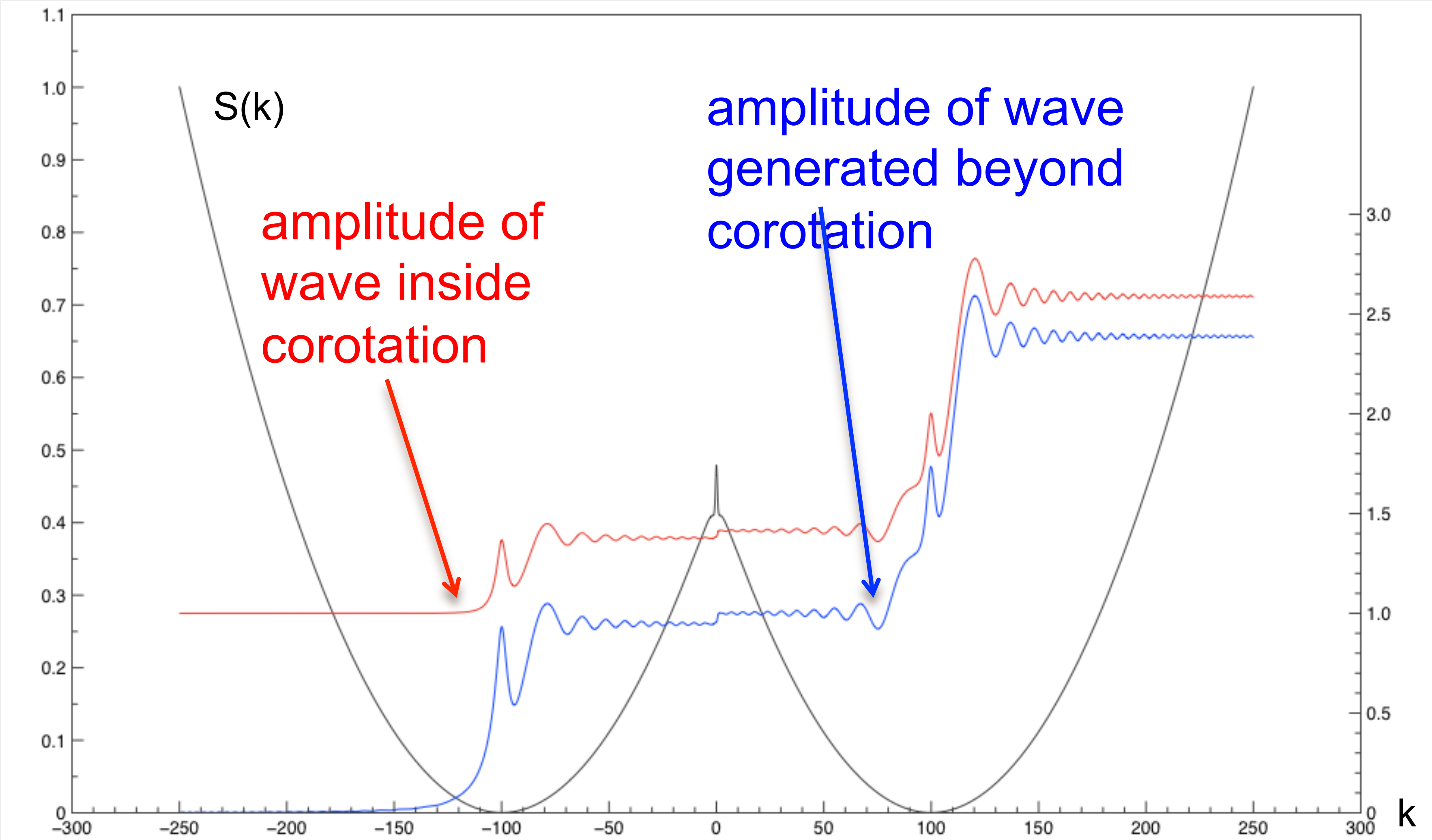




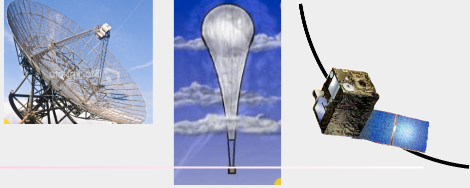
a useful plot: circle means a single propagating wave as initial condition

two Wasers



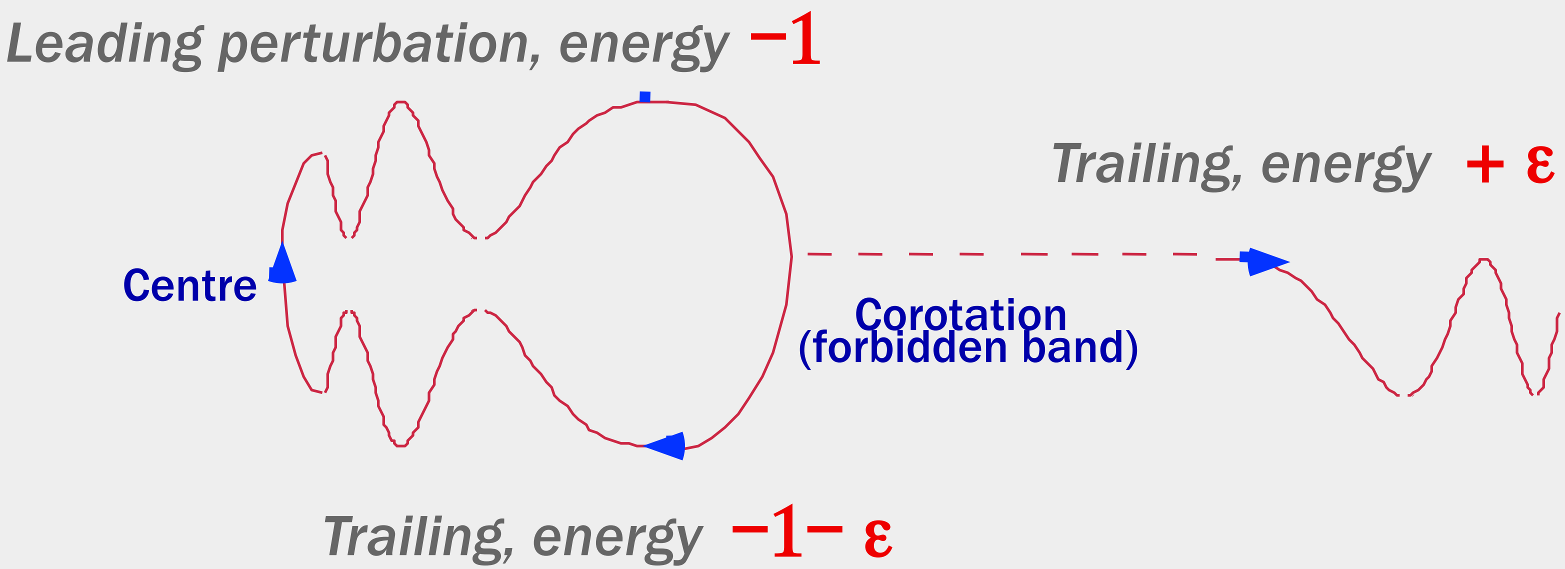


S(k)
amplitude of wave inside corotation
amplitude of wave generated beyond corotation
k
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
3.0
2.5
2.0
1.5
1.0
0.5
0
−300
−250
−200
−150
−100
−50
0
50
100
150
200
250
300



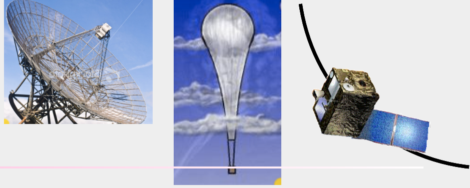
two Wasers

- WASER : great but can’t explain the extremely strong amplification obtained numerically

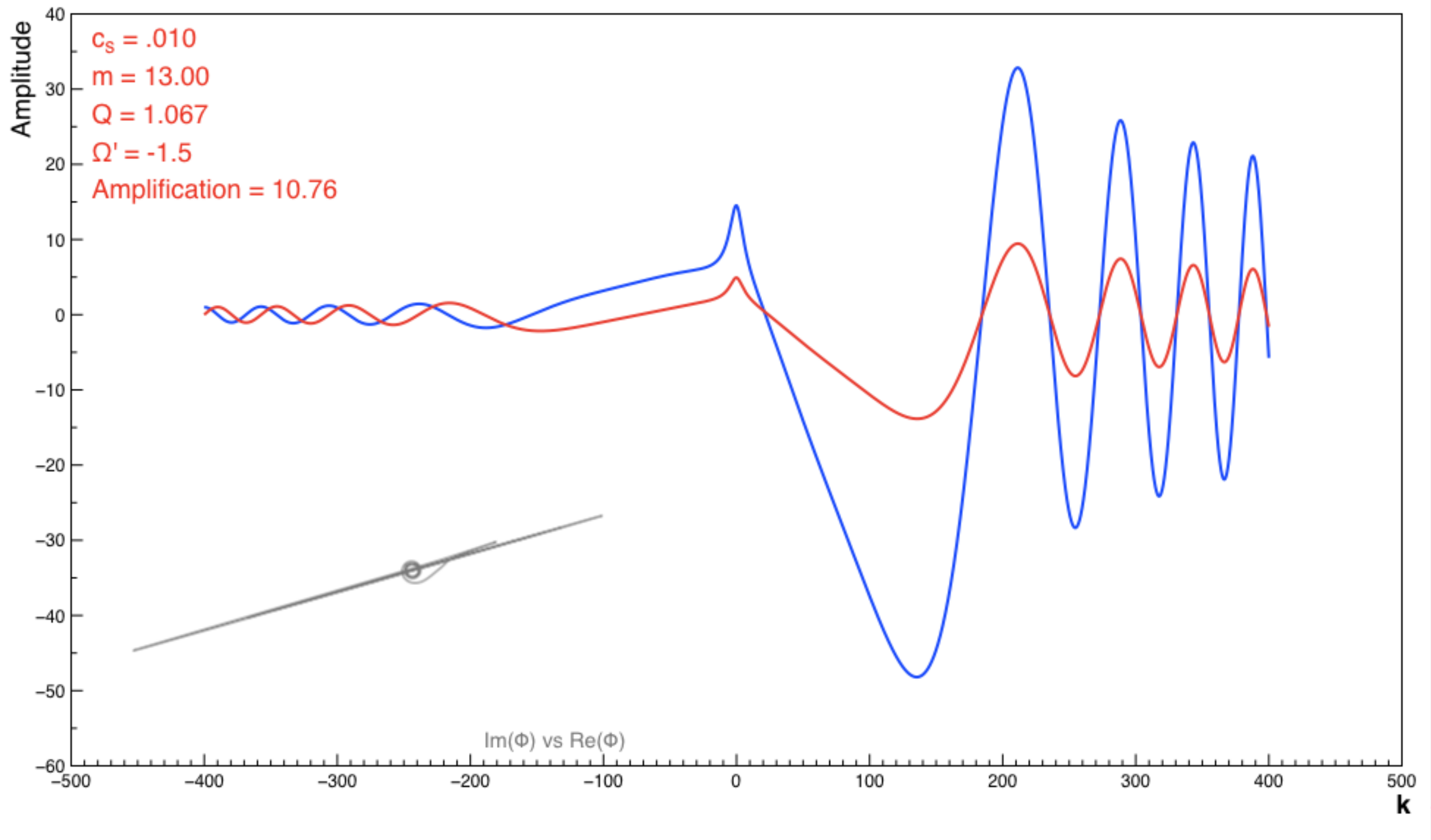


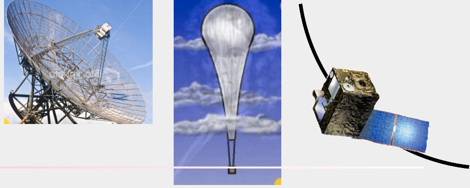
- max. amplification in principle (2 WASERs, Q=1) ~ 3

Swing amplifier

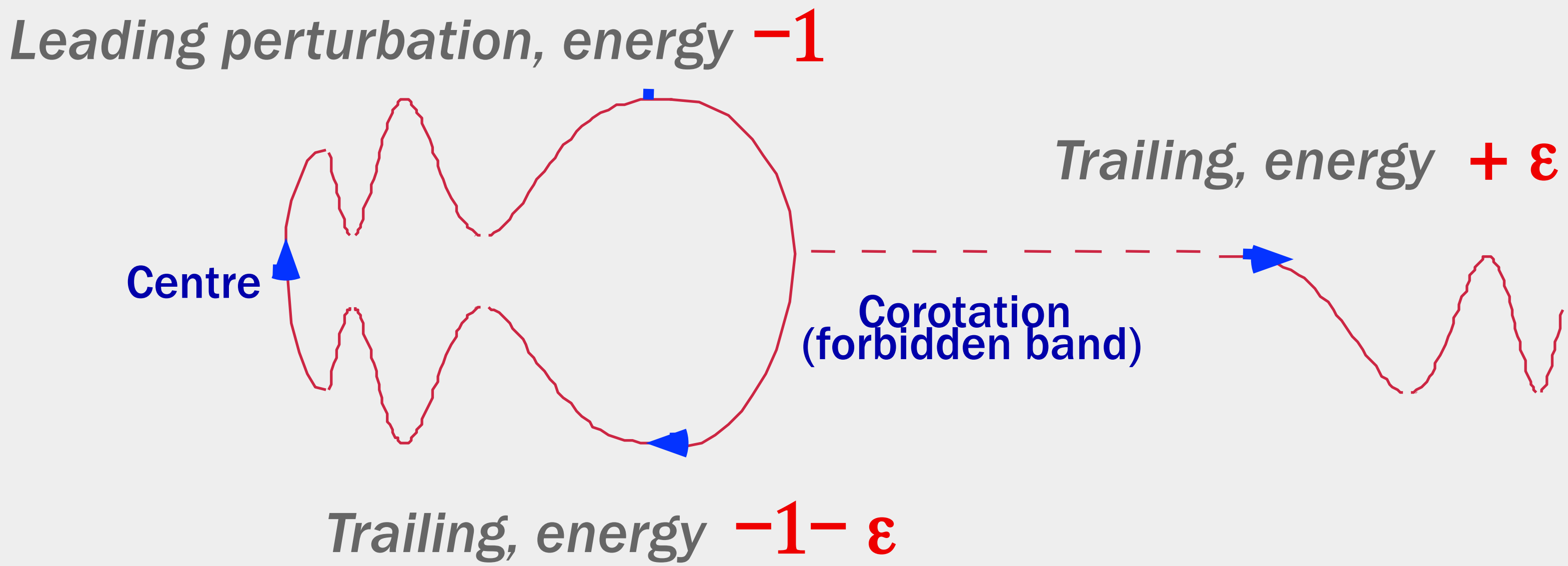


- Goldreich&Lynden-Bell 1965, … Toomre, Drury, …Toomre 1981



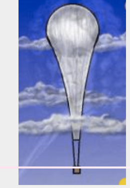
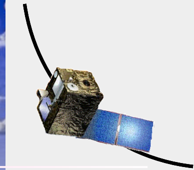


Swing amplifier



- here ε can be a few tens
- not a normal tunnel effect!

Toomre's Swing (1981)

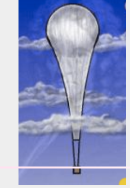
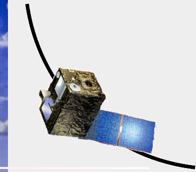




- re-derive the equation
- solve it numerically with a well-chosen (??) initial condition ("arrival phase »)

the GLB/LSK, JT and Zang models under the various circumstances. The results are summarized in Fig. 7, where the single open circle in the leftmost panel reports, for instance, that we could have obtained a net growth by factor 8.38 in Fig. 6 upon selecting the most optimal arrival phase. Well, at least my confederate and I consider these plots decisively similar! The full disk indeed re-

- explain the strong amplification by the negative spring constant
- (NB: a factor ~ 2 difference if proper boundary condition...)

Toomre's Swing

$$S(k) = \kappa^2 - 2\frac{\kappa c_s}{Q} q + q^2 c_s^2 + 4\Omega\Omega' \frac{m^2}{q^2} + 3\Omega'^2 \frac{m^4}{q^4}$$

the "WKB" terms dominant at large k

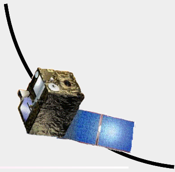
additional dynamical terms make S negative at k ~ m

- but that can't be the explanation!
- Tremaine&Goldreich, 1977 : an exact solution, with no amplification, when c_s=0 :

$$S(k) = \kappa^2 - 2\frac{\kappa c_s}{Q} q + q^2 c_s^2 + 4\Omega\Omega' \frac{m^2}{q^2} + 3\Omega'^2 \frac{m^4}{q^4}$$

so what? (Pellat, Tagger, Sygnet, 1990)



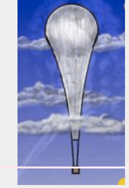
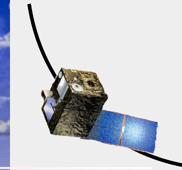


- first take hint from Tremaine's solution
- write equation for a new variable Y (related to the azimuthal velocity…)
- -> a very similar-looking 2nd order ODE…

$$\frac{d^2}{dk^2}Y + \mathcal{W}(k)Y = 0$$

- same physics but a different spring constant!
- (numerical check: does give the same amplification)

so what? (Pellat, Tagger, Sygnet, 1991)

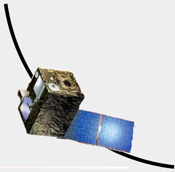
$$\frac{d^2}{dk^2}Y + \mathcal{W}(k)Y = 0$$

same physics but a different spring constant!

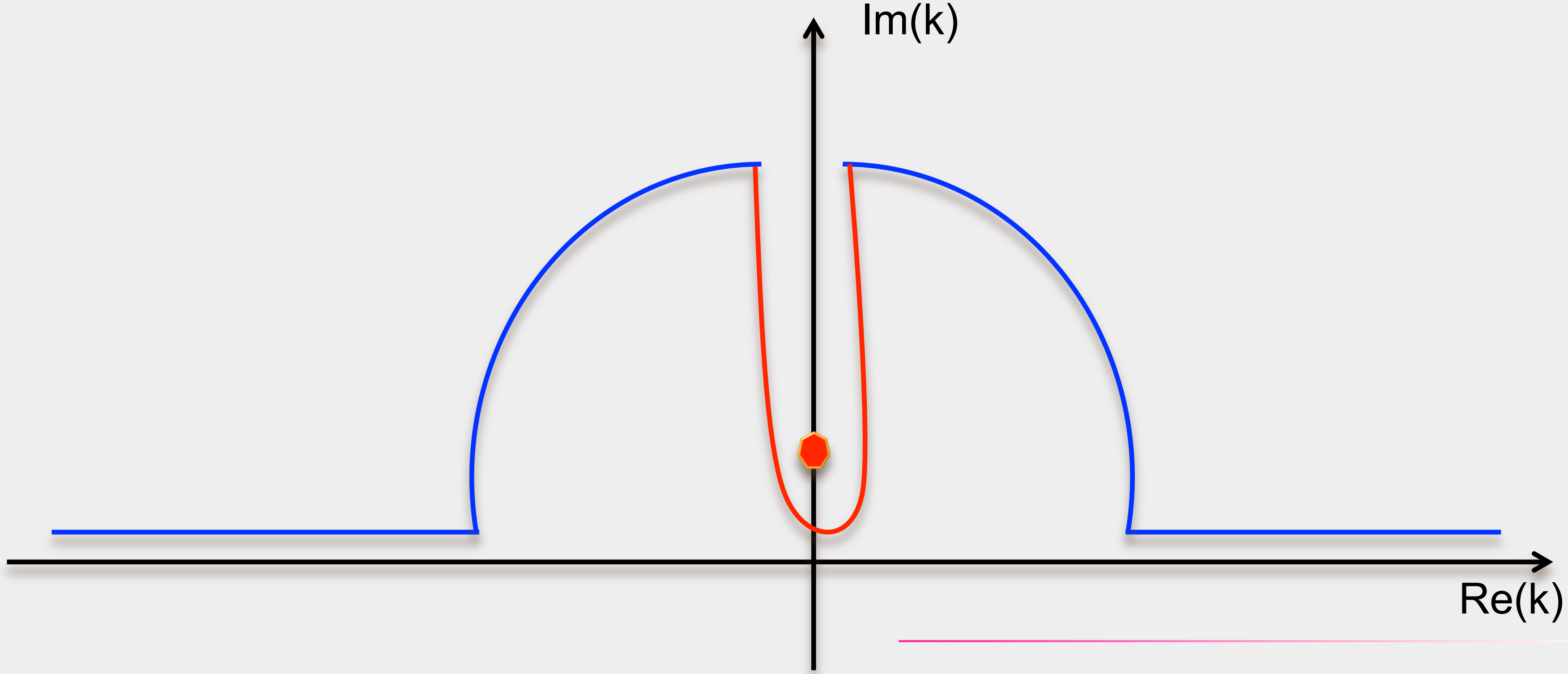
$$\mathcal{W}(k) = \kappa^2 - 2\frac{\kappa c_s}{Q}q + q^2 c_s^2 + \textit{extra terms all} \sim c_s$$

- the negative "dynamical" terms have disappeared
- Toomre's negative spring is not the explanation

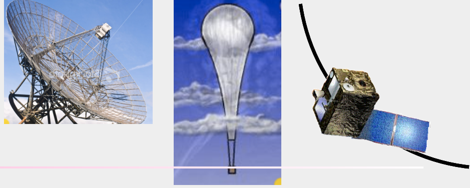
so what? (Pellat, Tagger, Sygnet, 1990)



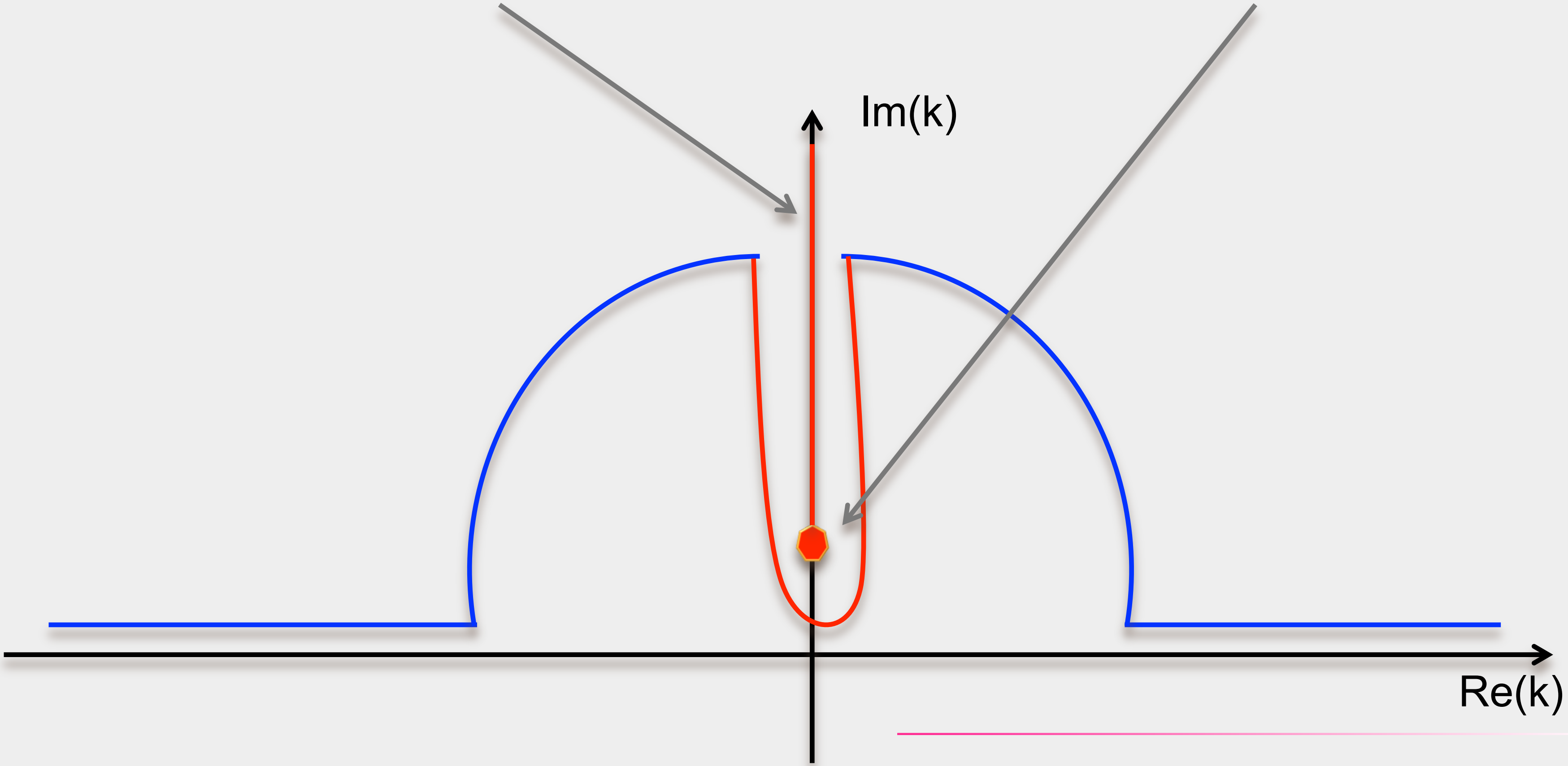
- WKB applies only at large k (tightly wound spirals)
- so use a deformed contour in complex-k plane



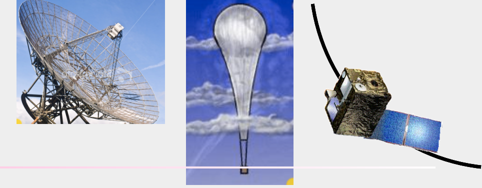
so what? (Pellat, Tagger, Sygnet, 1990)



- this contour is always at large |k|
- but hits a branch cut starting from $k = im$

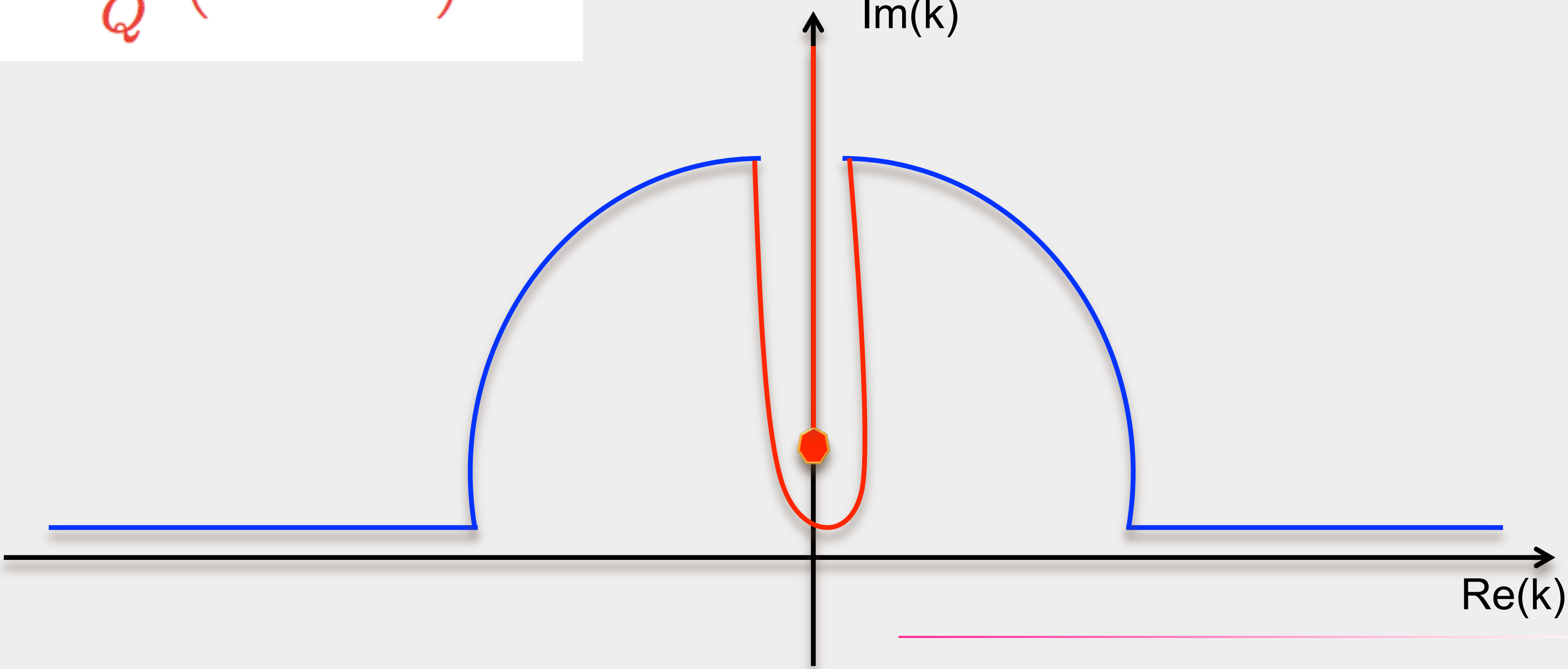


so what? (Pellat, Tagger, Sygnet, 1990)

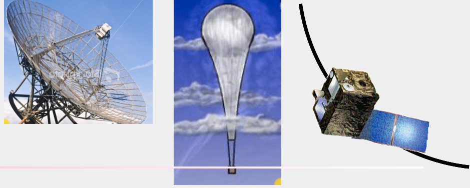


- singularity at $k = im$
- from the gravity term in the dispersion relation :

$$-2\frac{\kappa c_s}{Q}\left(k^2+m^2\right)^{1/2}$$

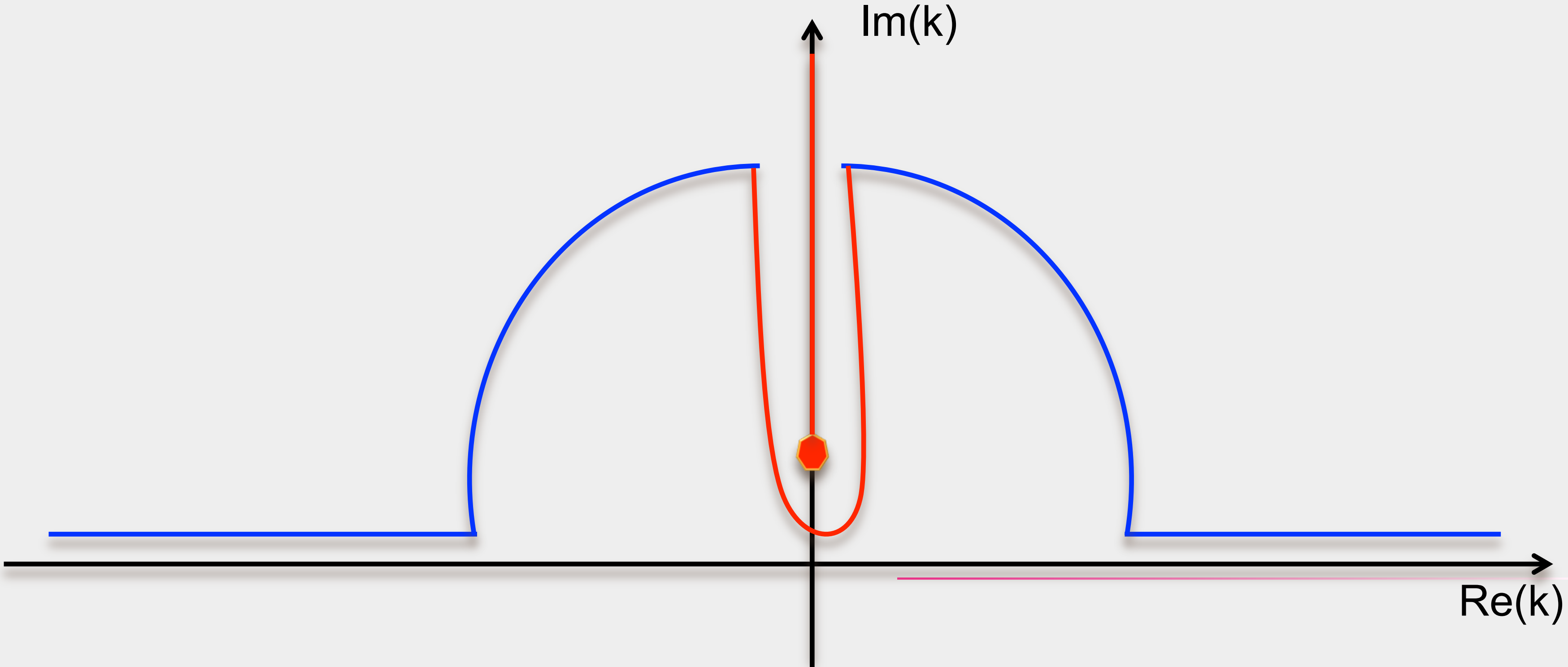


so what? (Pellat, Tagger, Sygnet, 1990)

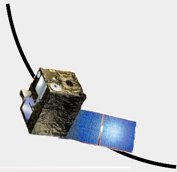


- this square root $-2\frac{\kappa c_s}{Q}\left(k^2+m^2\right)^{1/2}$
- comes from the 3D solution for the gravitational potential -> from the disk geometry

(a cylinder would NOT be unstable)







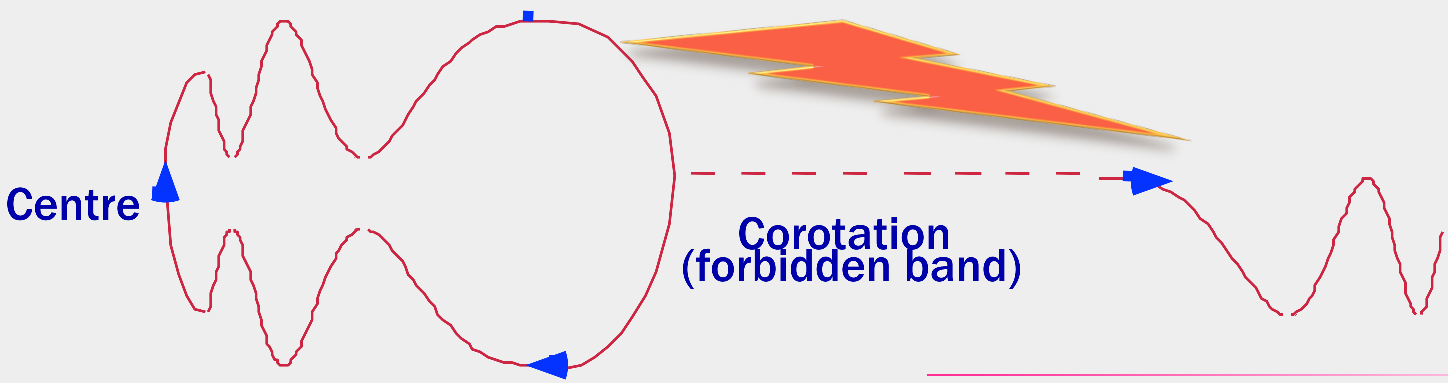
physical interpretation

- the singular point at $k = im$

introduces terms $\sim r^{-m}$

Interpretation: although the perturbation is exponentially weak in the forbidden band (tunnel effect)

the gravitational force from overdensities acts at large distance to generate directly a perturbation beyond corotation

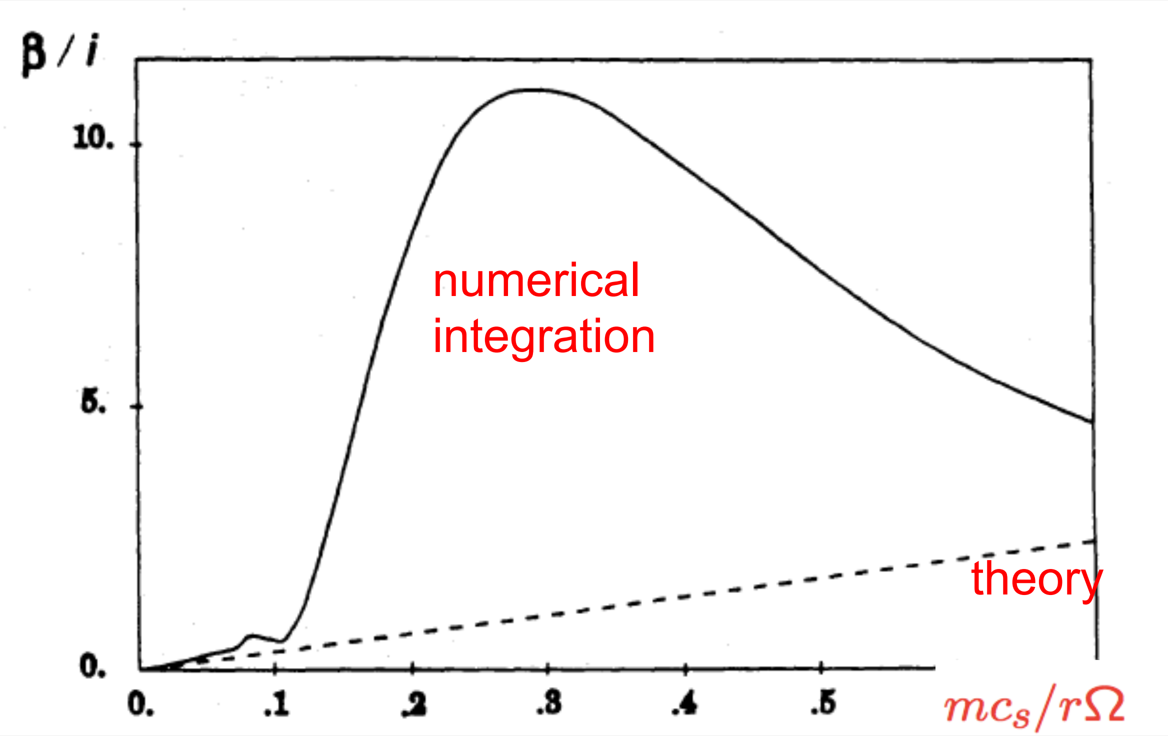


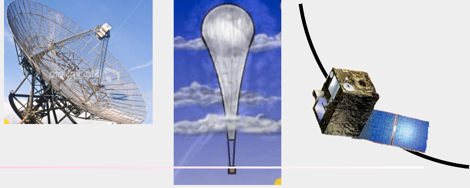
numerical result (PTS90)

computation of the amplification coefficient along the deformed contour

-> contribution only from the non-WKB region -> along the branch cut

-> OK but we miss the strong amplification!





our new work

- following the contour along the branch cut is not a good idea
- = (anti)–Stokes line because in the WKB solutions

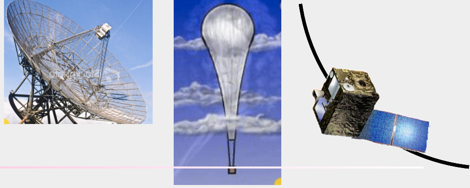
$$\sim \exp\left[\pm i \int^{k} \mathcal{W}^{1/2} dk'\right]$$

the exponent is real -> the ratio between the solutions is exponential

- -> terms of the form $\exp(-1/\varepsilon)$

not amenable to asymptotic analysis!

our new work



WKB -> osculating coefficients c and d

$$Y = c(k) \exp\left[i \int^k \mathcal{W}^{1/2} dk'\right] + d(k) \exp\left[-i \int^k \mathcal{W}^{1/2} dk'\right]$$

-> c and d constant where WKB applies

c-> waves inside corotation

d-> waves beyond corotation

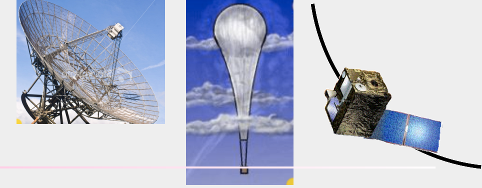
In PTS90 the computation worked only because we started with d=0

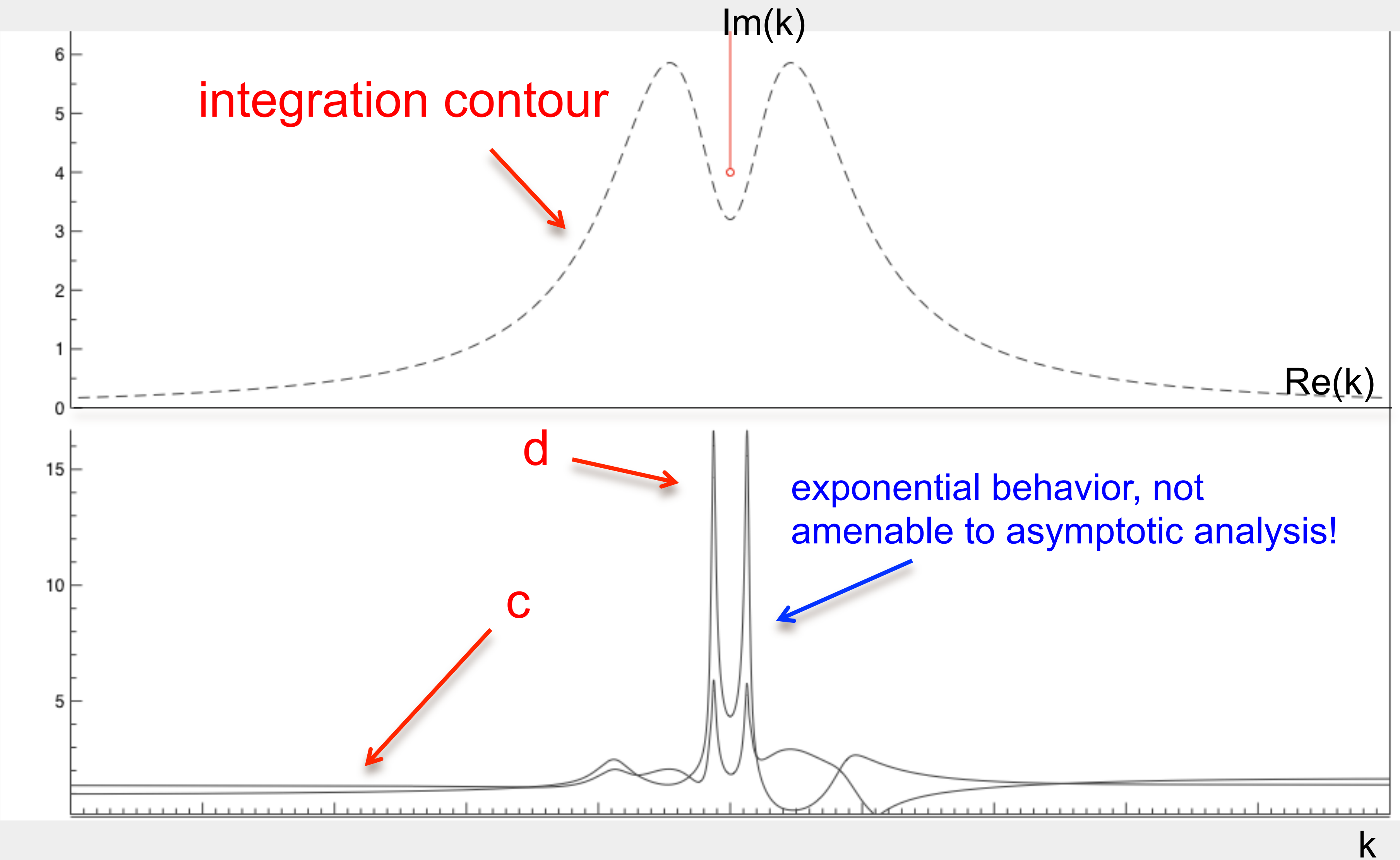
But as soon as d has any amplitude (e.g. there has been Waser to start generating a wave beyond corotation)

it is multiplied by the large exponential

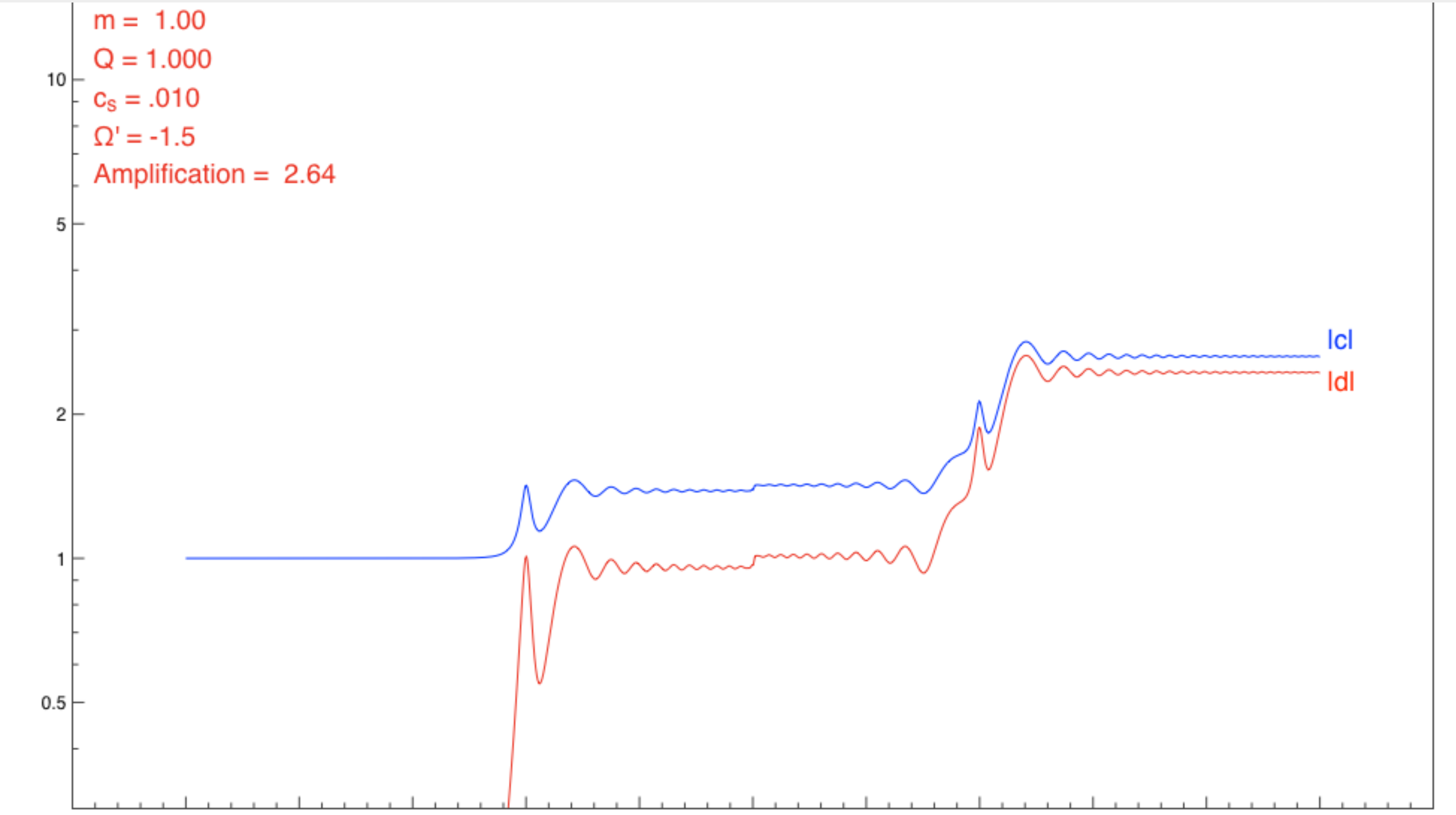
the asymptotic computation breaks down

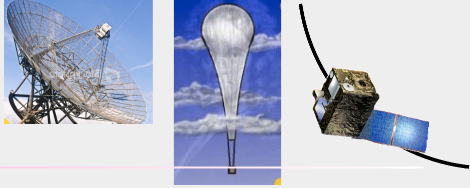
integrating along the contour…



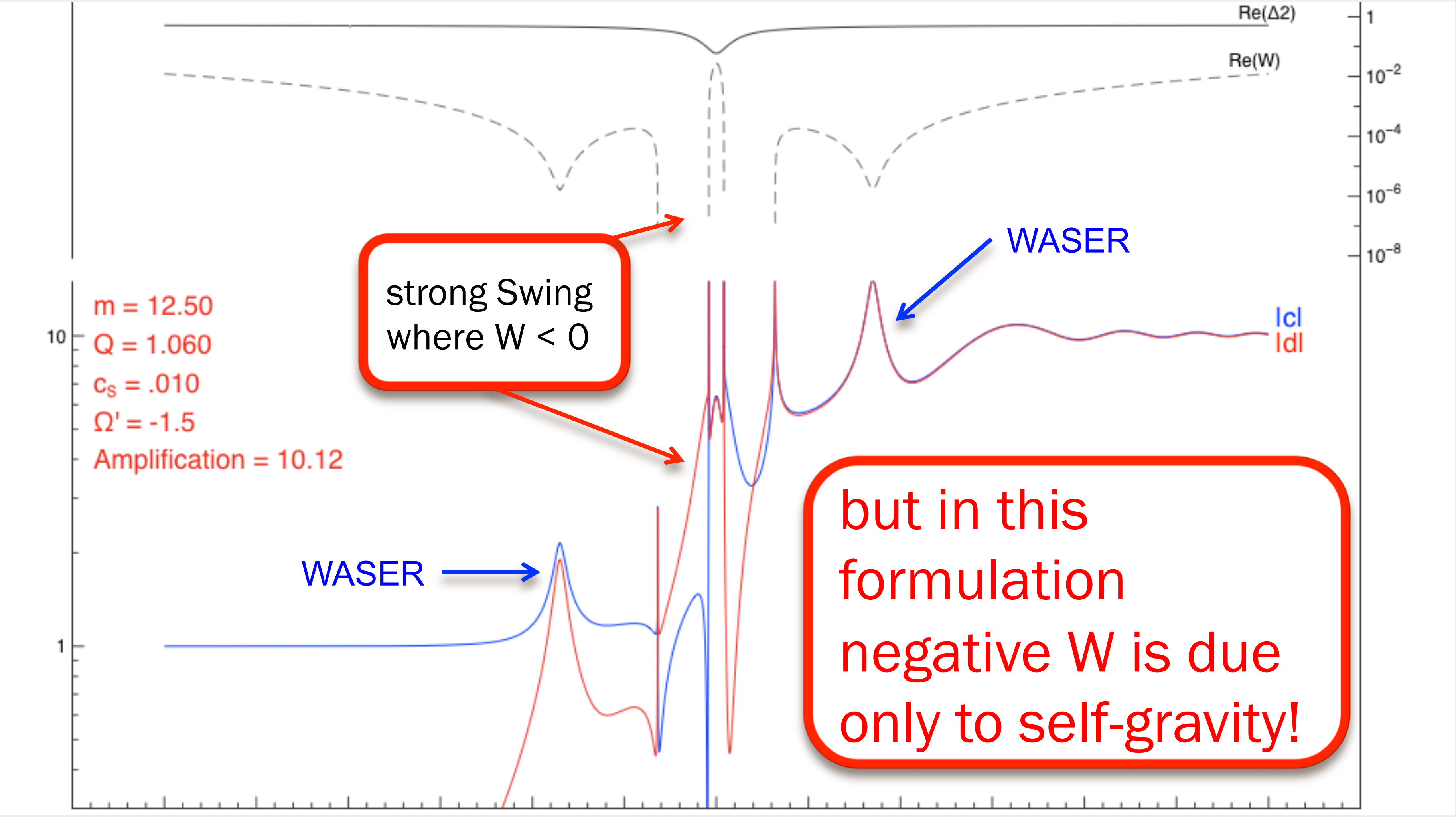


c and d, 2 WASERS, little Swing



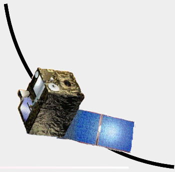


a case with strong Swing



our final word





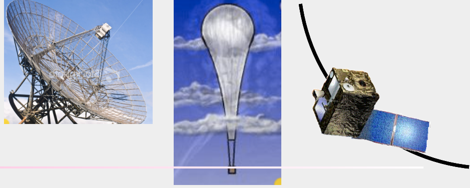
- no details here but the strong amplification is related to a quantity

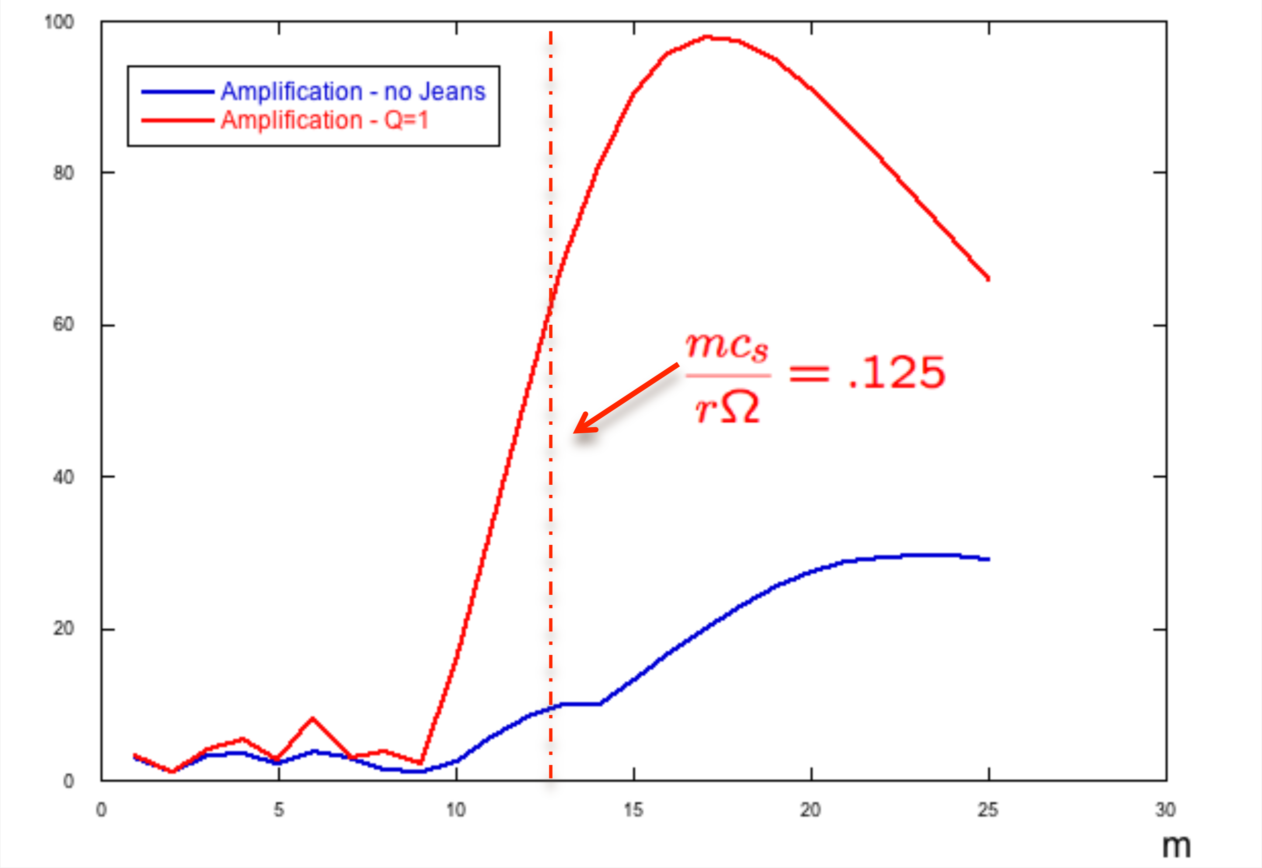
$$\Delta^2 = (2\Omega + \Omega')^2 + m^2 \left(c_s^2 - \frac{c_s \kappa}{Qq} \right)$$

linked to the radial response of the gas

- $\mathcal{W}$ can become negative (but in different k range than Toomre's $\mathcal{S}$) when self-gravity makes Δ^2 small
- this occurs when $\frac{m c_s}{r\Omega} \sim .125$

computed amplification

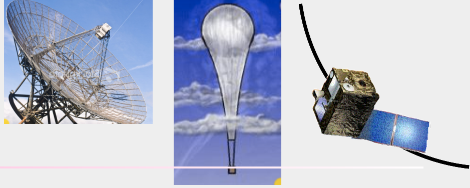




notice : because of small terms Q=1 is not the exact criterion for Jeans -> the blue curve (eg Q=1.035 at m=10)

Thus if not careful some of the amplification is due to Jeans

conclusion



- WASER and SWING amplifiers cooperate, are of comparable strength (though very hard to separate in "realistic" conditions)
- amplification in both cases comes from self-gravity
- whether or not you like modes

you MUST respect boundary conditions