From Galactic Halos to the Reimann Zeta Function: An adventure in applied mathematics

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Professor C.C. Lin was internationally renowned for his work in applied mathematics before he made seminal advances in astrophysics. As an homage to his contributions, this talk reverses course, starting with astrophysical orbits on galactic scales and ending with new results in applied mathematics. The gravitational potential for the inner portions of dark matter halos, galactic bulges, and embedded star clusters can all be described by a particular triaxial form. Here we construct orbit solutions for this triaxial potential and show that a robust orbit instability exists: For orbits initially confined to any of the three principal planes, the motion in the perpendicular direction can be unstable. We discuss the range of parameter space for which these orbits are unstable, find the growth rates and saturation levels, and develop analytic model equations that elucidate the essential physics of the instability mechanism. This orbit instability has a number of astrophysical applications, including the formation of dark matter halos, the structure of galactic bulges, the survival of tidal streams, and the early evolution of embedded star clusters. The orbit instability is described by solutions to Hill's equation, where the forcing strength parameters vary from cycle to cycle. Since the differential equation is periodic, the development of the solutions is governed by a discrete map, and the growth rates are determined by multiplication of infinite strings of matrices (with varying parameters). In the regime of large forcing strengths, we find analytic expressions for the growth rates for matrix multiplication, and hence the growth rate for solutions of Hill's equation. Some further implications are discussed, ending with a result involving the Reimann Zeta Function.