Shockwave

in collapse of singular isothermal sphere

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Outline

- Introduction
- Classical SIS
- Various solutions
- General relativistic case
- General relativistic shock

Self-similarity

- $x \equiv r/t$
- ρ (1/r ^ 2) : singularity
- The only scale factor: isothermal sound speed

Newtonian solutions

- Stellar collapse
- Expansion wave solution
- Shock wave solution
- Wind/Breeze solution

Classical approach

$$\frac{\partial M}{\partial t} + u\frac{\partial M}{\partial r} = 0\,, \quad \frac{\partial M}{\partial r} = 4\pi r^2 \rho$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{a^2}{\rho}\frac{\partial \rho}{\partial r} - \frac{GM}{r^2}$$

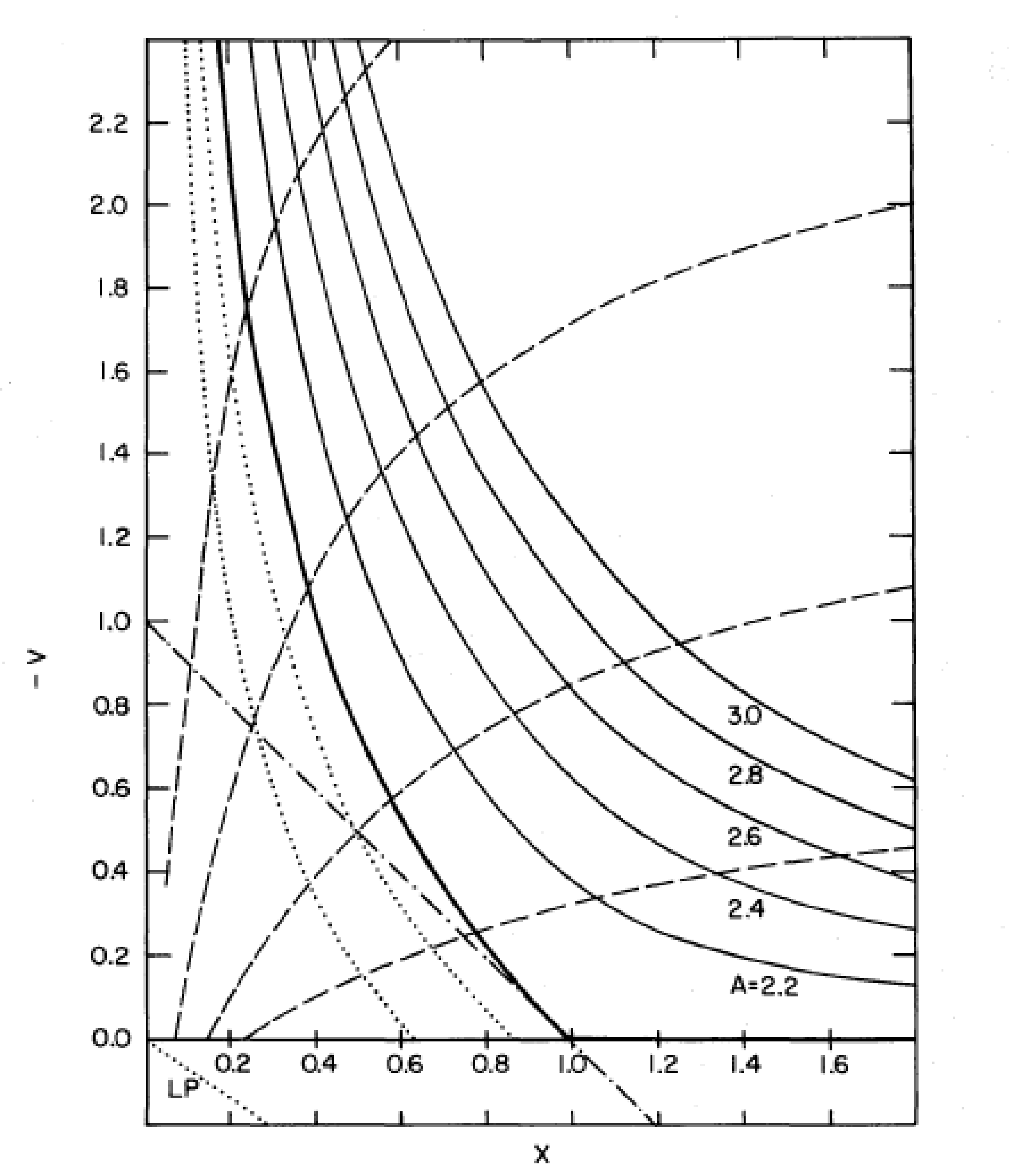
$$\rho(r, t) = \frac{\alpha(x)}{4\pi G t^2}\,, \quad M(r, t) = \frac{a^3 t}{G} m(x)\,,$$

$$u(r, t) = av(x)\,,$$

$$[(x - v)^2 - 1]\frac{dv}{dx} = \left[\alpha(x - v) - \frac{2}{x}\right](x - v)\,, \quad (11)$$

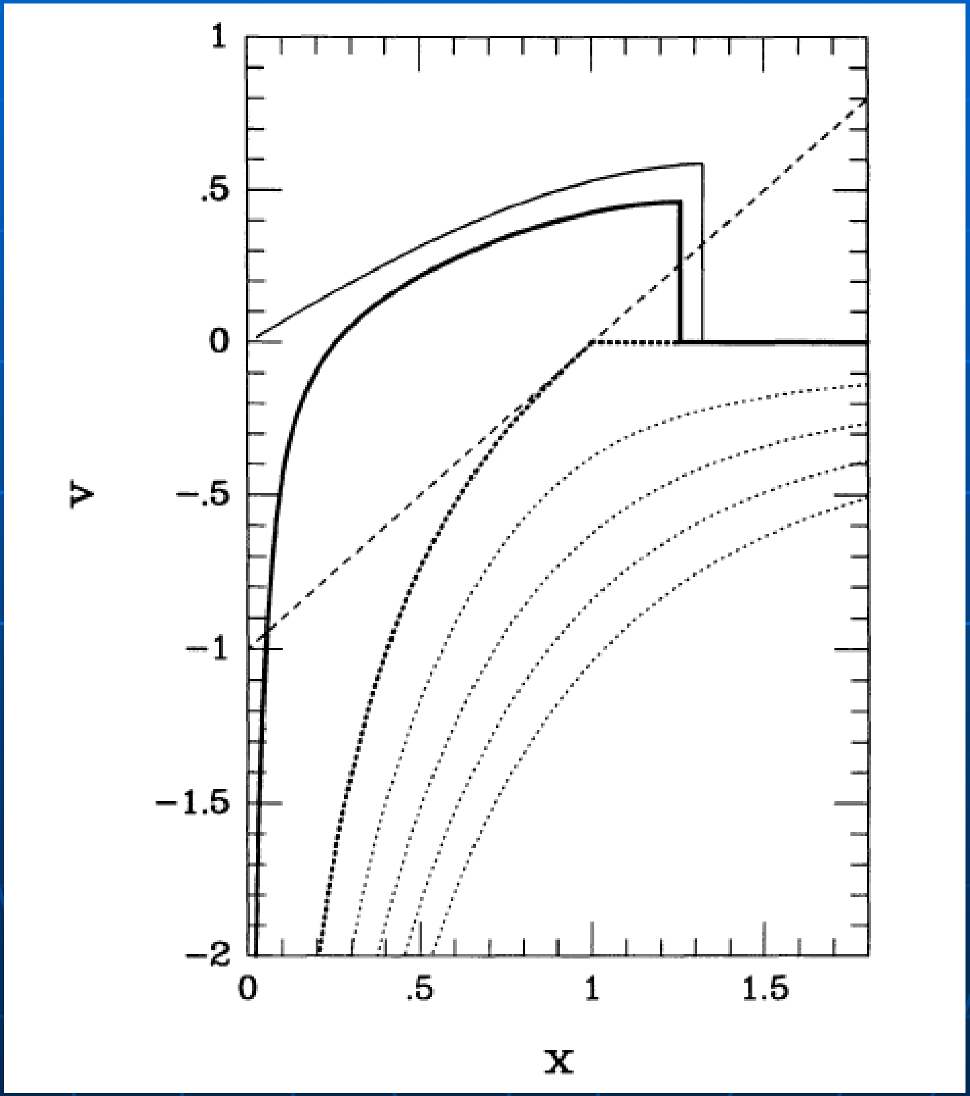
$$[(x - v)^2 - 1]\frac{1}{\alpha}\frac{d\alpha}{dx} = \left[\alpha - \frac{2}{x}(x - v)\right](x - v)\,. \quad (12)$$

Shu, 1977)



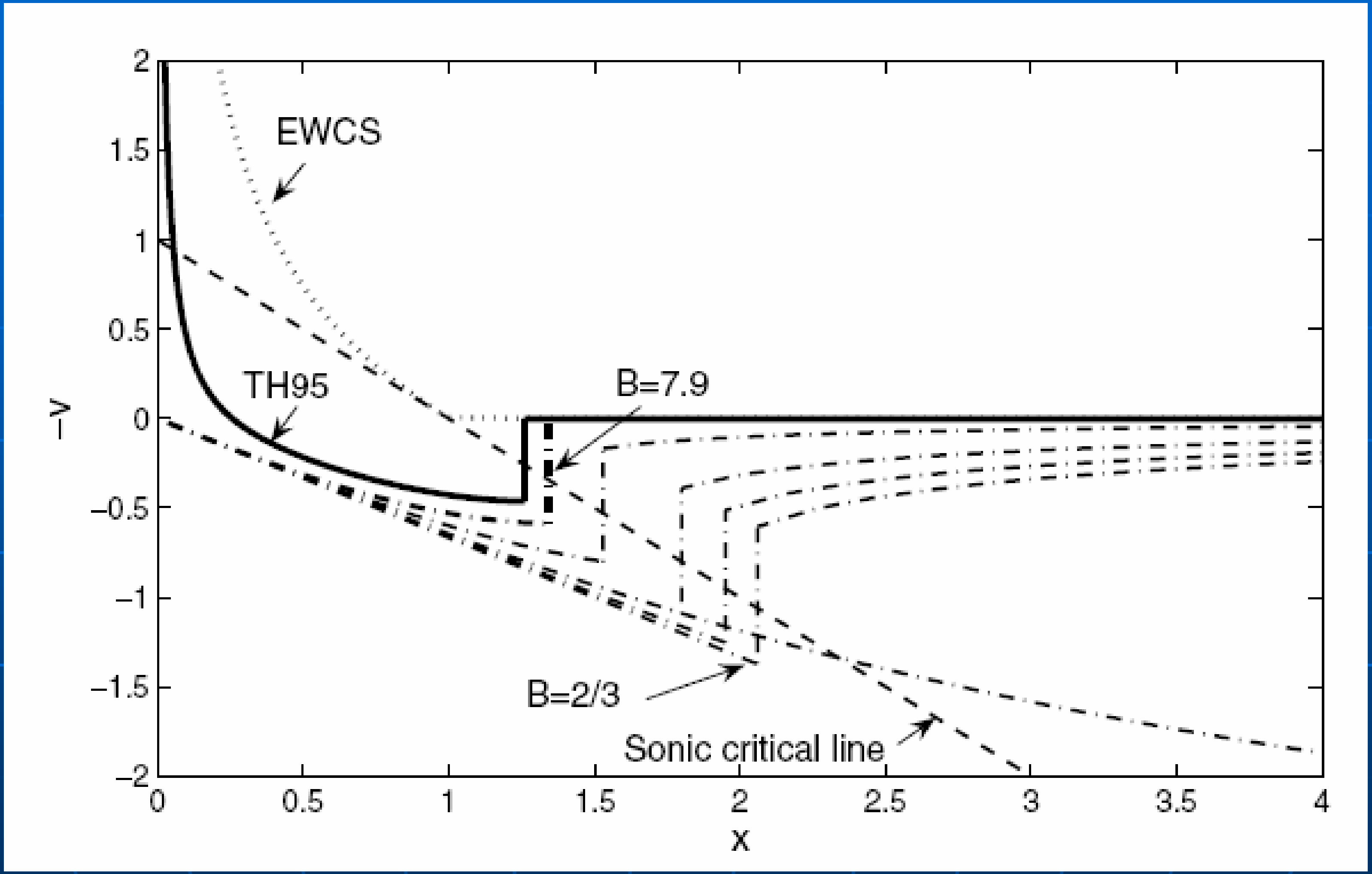
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-V
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
X
LP
3.0
2.8
2.6
2.4
A=2.2

Newtonian Shock



$$u_2 = \frac{u_{sh}^2 - a^2}{u_{sh}}, \quad \frac{\rho_2}{\rho_1} = \left(\frac{u_{sh}}{a}\right)^2$$

Tsai & Hsu, 1995



(Bian & Lou, 2005)

Counterparts in GR

- Formation of super massive & intermediate massive black holes

$$ds^2 = -\alpha^2 dt^2 + a^2 dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2,$$

$$\alpha = \alpha(\zeta), \quad a = a(\zeta), \quad \zeta = \frac{r}{t},$$

$$x^2 = \frac{a^2}{\alpha^2}\zeta^2 = -\frac{g_{rr} r^2}{g_{tt} t^2}$$

$$\varepsilon = 4\pi r^2 (1+\gamma)\rho a^2, \quad \beta = -\frac{2v}{1-v^2} > 0.$$

$$\ln x' = 1 - \varepsilon\sqrt{\beta^2 + 1} - \frac{\beta\varepsilon}{x},$$

$$\ln \varepsilon' = -\varepsilon\frac{\beta}{x}\left(2 - \frac{\Gamma}{D}\right) - \frac{\beta'}{D}\left(\frac{1}{x} + \frac{\beta}{\sqrt{\beta^2 + 1}}\right),$$

$$\beta' = -\frac{\varepsilon\beta\Gamma\left[\beta(x + 1/x) + 2\sqrt{\beta^2 + 1}\right] + 2x(\varepsilon - 1 + \Gamma)D}{(x^2 - 1)/\sqrt{\beta^2 + 1} + \left(2\beta x/\sqrt{\beta^2 + 1} + x^2 + 1\right)\Gamma}.$$

Comoving coordinate

$$ds^2 = -e^{2\Phi}\,dT^2 + e^{2\Lambda}\,dR^2 + e^{2\omega}R^2\,d\Omega^2$$

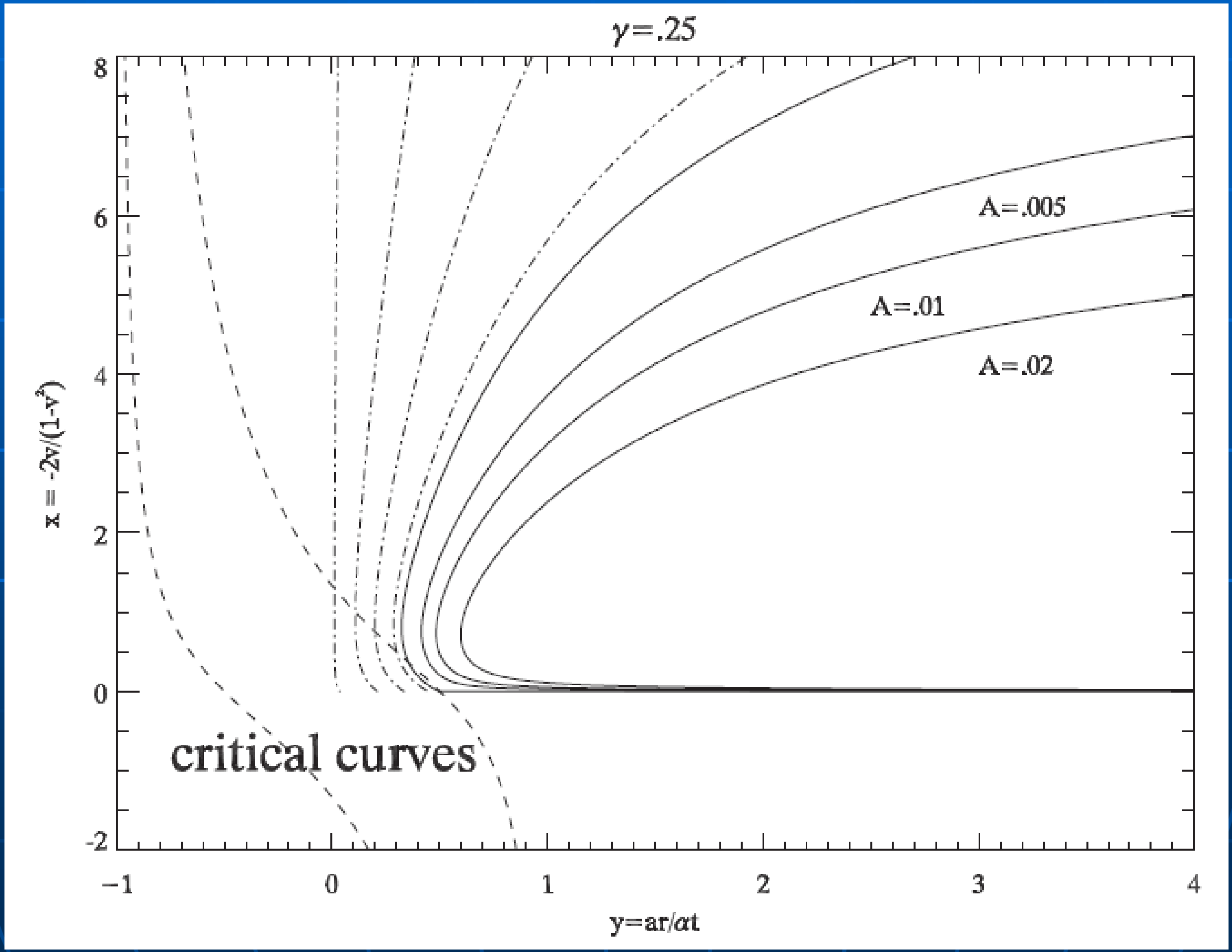
$$\Lambda' = \frac{\mathcal{E} - 1 + \Gamma - 2\Omega\gamma}{\gamma - y^2},$$

$$\ln \mathcal{E}' = -2(\gamma + 1)\Omega + (1 - \gamma)\Lambda',$$

$$\ln y' = (1 - \gamma)\Lambda' + \Gamma - 2\Omega\gamma,$$

$$\Omega' = \Omega^2(2\gamma - 1) + \Lambda'[\Omega(1 + \gamma) + 1] - \Gamma\Omega,$$

Relativistic expansion wave



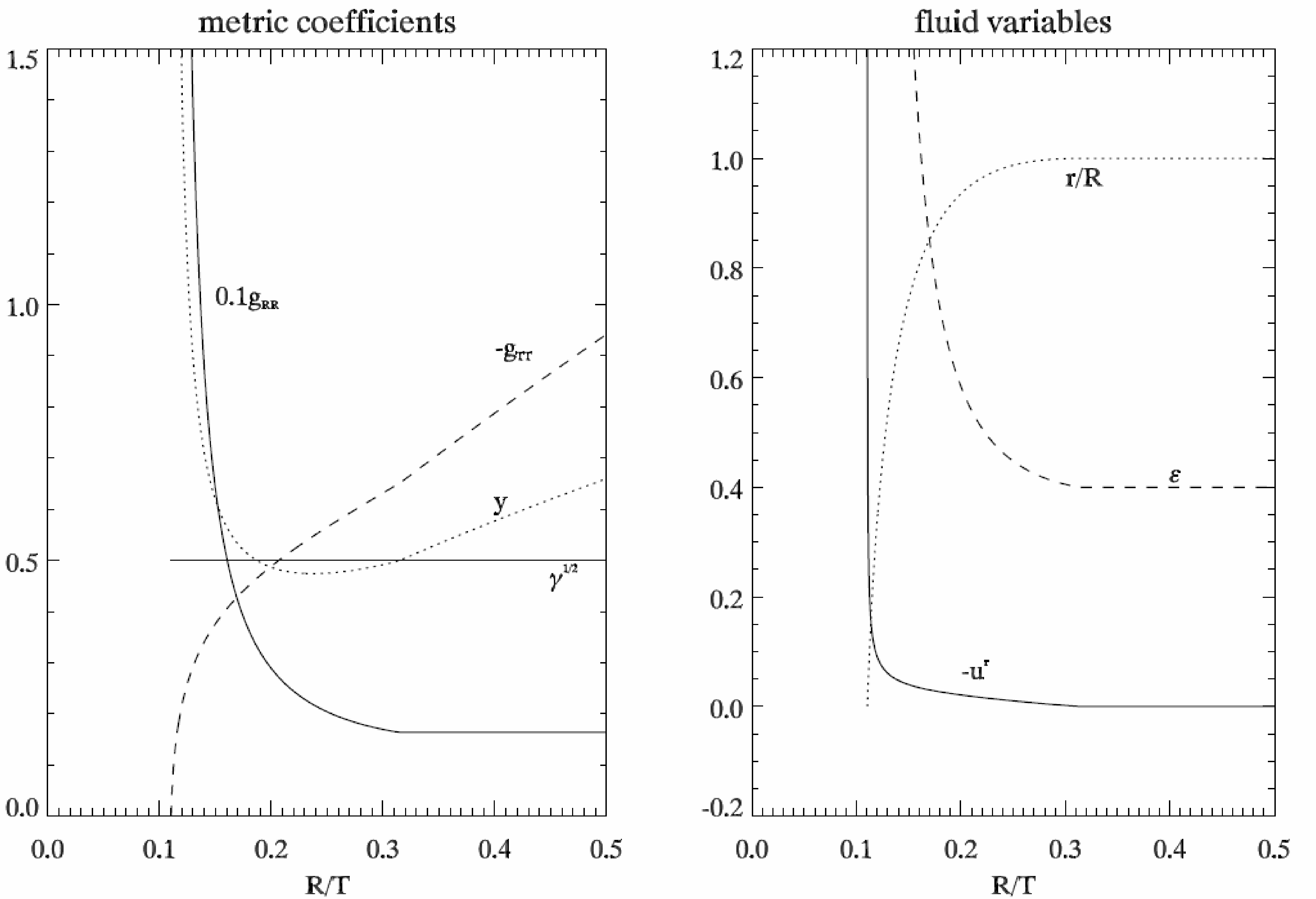
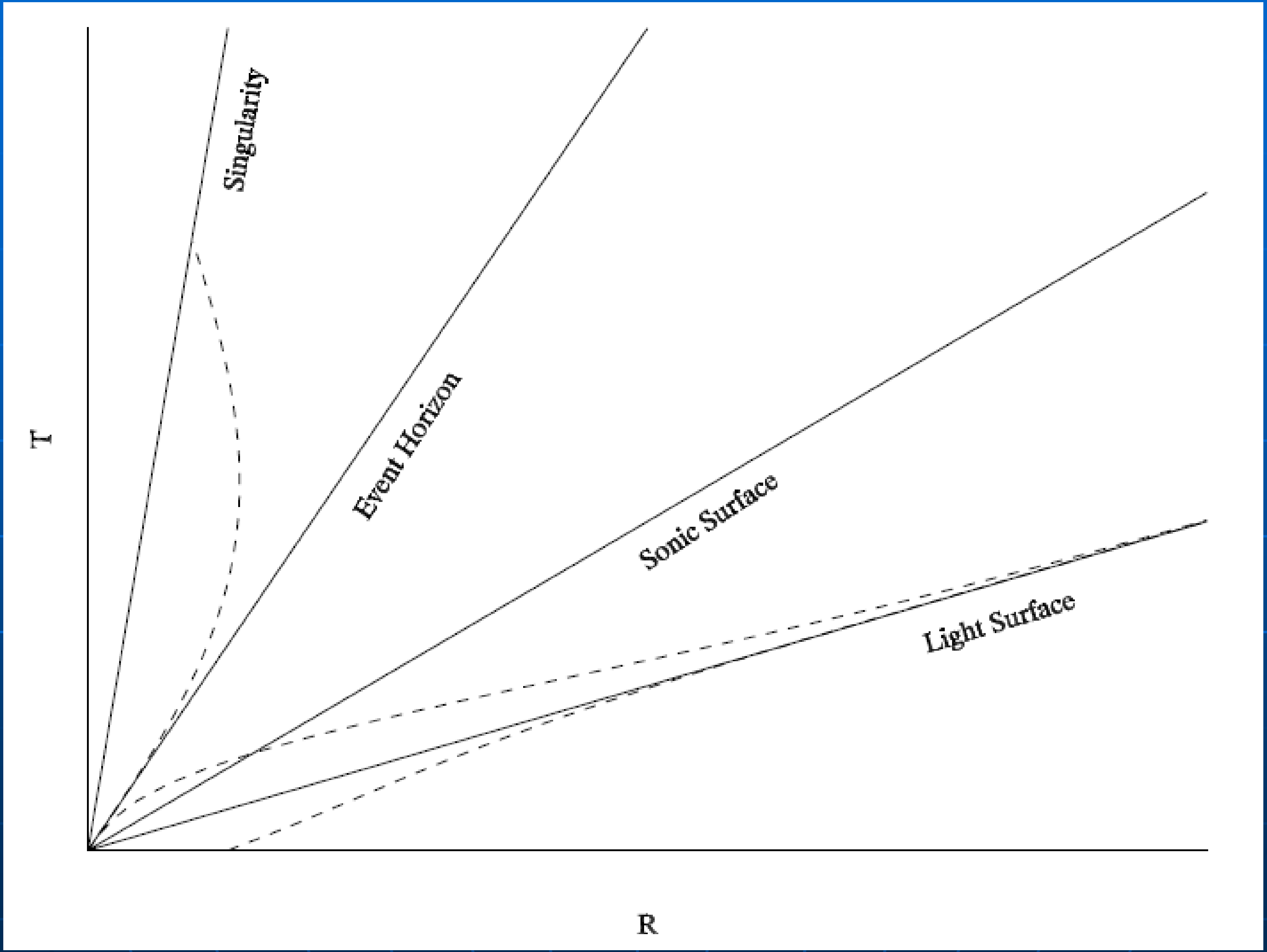
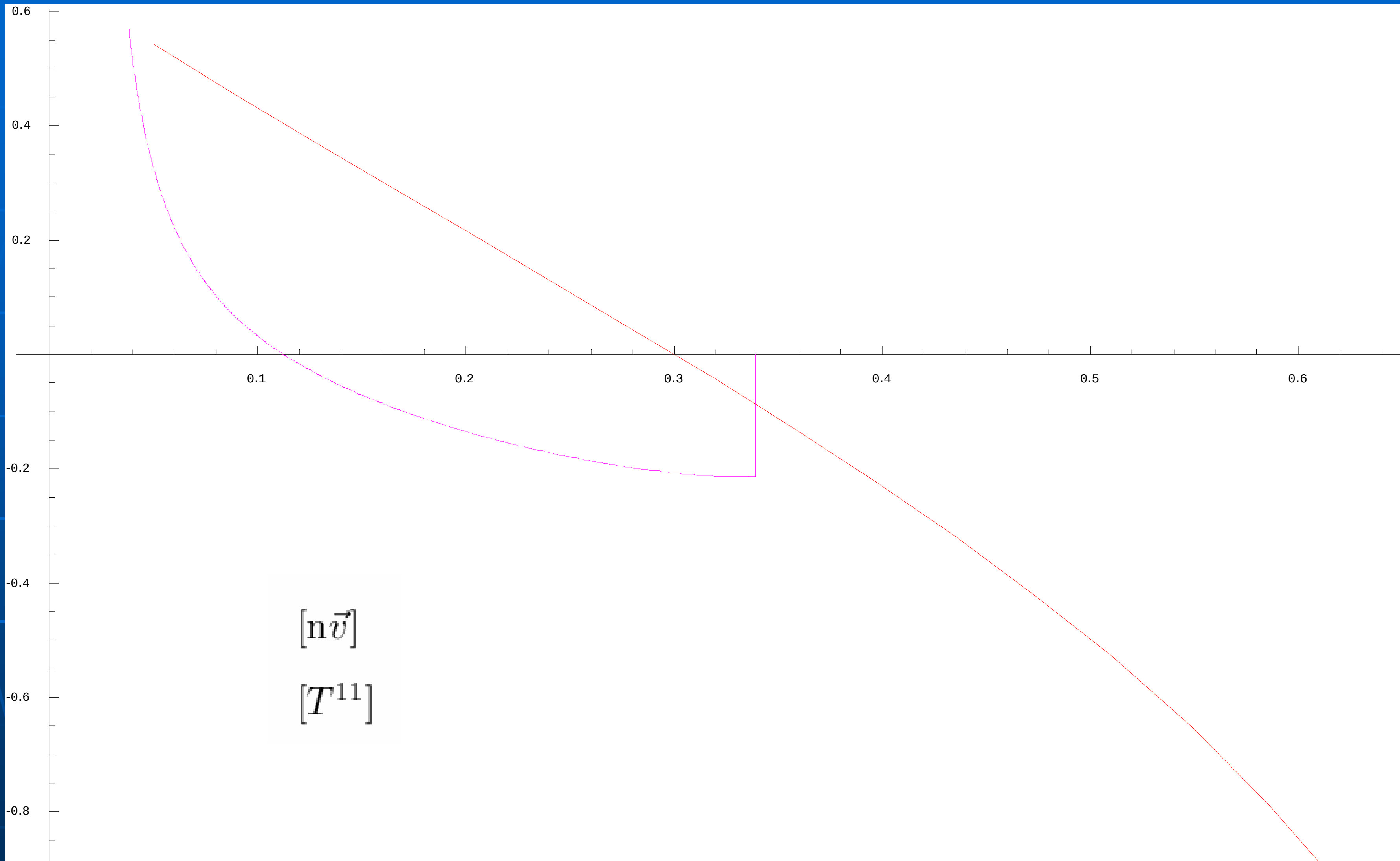


FIG. 2.—Expansion wave solution in comoving coordinates for $\gamma = 0.25$.


Singularity
Event Horizon
Sonic Surface
Light Surface
T
R

What's new in GR?

- Failure of iron sphere theorem
- Shock wave


0.6
0.4
0.2
-0.2
-0.4
-0.6
-0.8
0.1
0.2
0.3
0.4
0.5
0.6
$[\mathrm{n}\vec{v}]$
$[T^{11}]$

Summary

- General relativistic shock wave solution exists
- Still working on: the interplay between shock front and event horizon, energy scale...etc.
- Possible GRBs model?